$$P_{cr} = \frac{\pi^2 EI}{(\underbrace{KL}_{L_e})^2} \Rightarrow \text{(If } P < P_{cr}\text{; The column will not be buckle)}$$

Factor of Safety $= \dfrac{P_{cr}}{P}$; Stress Conc. Factor, $K_t = \dfrac{\sigma_{max}}{\sigma_0}$ #

$$\sigma_{cr} = \frac{P_{cr}}{A}$$

Stress Concentration Axial Member. →

$$\boxed{\sigma_{max} = K\frac{P}{A} \; ; \; K = \frac{\sigma_{max}}{\sigma_{avg}}}$$

Stress Concentration Torsion →

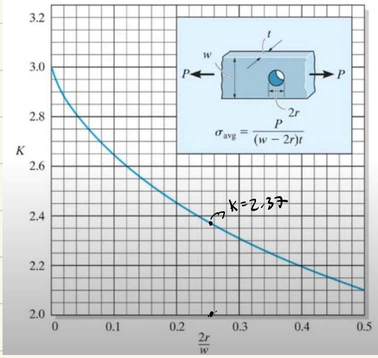
$$\boxed{\tau_{max} = K\frac{Tc}{J}}$$

Stress Concentration for Bending Moment. →

$$\boxed{\sigma_{max} = \frac{KMc}{I}}$$

3. A flat bar with thickness t = 3 mm is loaded as shown. The hole diameter is d = 9 mm the width of the bar is b = 35 mm. What is the maximum allowable tensile load if the allowable tensile stress in the material is σ_t = 50 MPa?

$K = \sigma_m$



Find Stress Concentration.

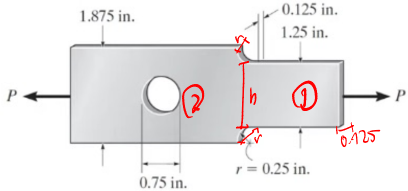
$$K = \frac{2r}{w} = \frac{9}{35} = 0.257$$

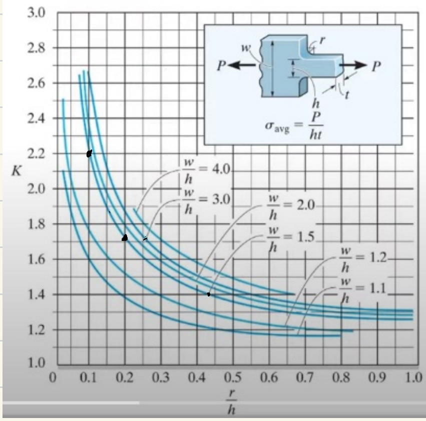
$K = 2.37$ →

$$K = \frac{\sigma_{max}}{\sigma_{avg}} \quad (K = 2.37,\ \sigma_{max} = 50)$$

$$\sigma_{avg} = \frac{P}{(0.035 - 0.009)(0.003)}$$

4. The bar is made from steel and has an allowable stress of 21 ksi. Determine the maximum axial force that can be applied to the bar.





$$\sigma = \frac{P}{A}$$

$$K = \frac{\sigma_{max}}{\sigma_{avg}}$$

$$1.71 = \frac{21\times10^3}{\frac{P}{ht}}$$

$$\sigma_{avg} = \frac{P}{ht}$$

$$\frac{r}{h} = \frac{0.25}{1.25} \Rightarrow 0.2$$

$$\frac{w}{h} \Rightarrow \frac{1.875}{1.25} \Rightarrow 1.5$$

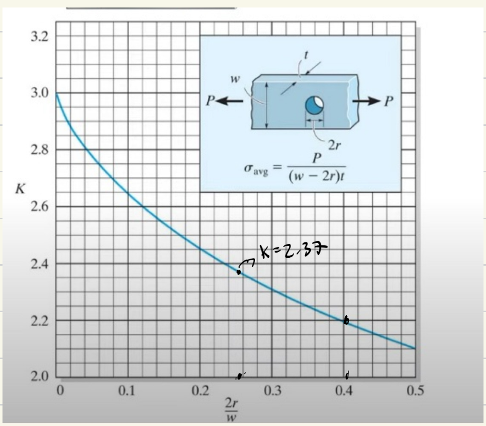
$K = 1.71$

$$\frac{1.71P}{ht} = 21\times10^3$$

$$P = \frac{(21\times10^3)(0.125)(1.25)}{1.71}$$

① section ← $P = 1.9189$ kip

② Section use



$$\frac{2r}{w} = \frac{0.75}{1.875} = 0.4$$

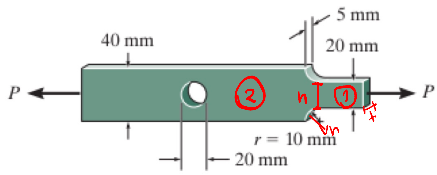
$K = 2.2$

$$2.2 = \frac{21\times10^3}{\frac{P}{(1.875-0.75)(0.125)}}$$

$P \Rightarrow 134816$

9–65. Determine the maximum normal stress developed in the bar when it is subjected to a tension of $P = 8$ kN.

9–66. If the allowable normal stress for the bar is $\sigma_{allow} = 120$ MPa, determine the maximum axial force P that can be applied to the bar.



Probs. 9–65/66

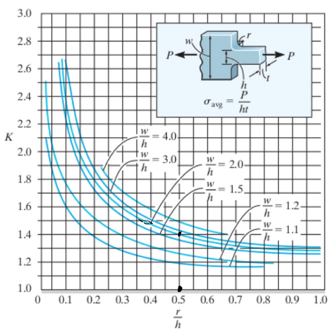


Fig. 9–24

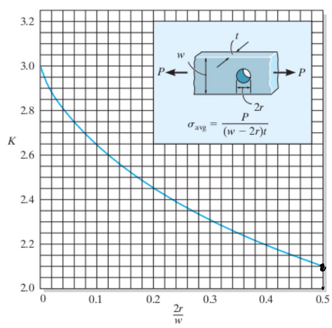


Fig. 9–25

$$K = \frac{\sigma_{max}}{\sigma_{avg}}$$

For the fillet

$$\Rightarrow \frac{r}{h} = \frac{10}{20} = \frac{1}{2} = 0,5$$

$$\frac{w}{h} = \frac{40}{20} = 2 \Rightarrow K = 1.4$$

$$K = \frac{\sigma_{max}}{\sigma_{avg}} \Rightarrow K = \frac{\sigma_{max}}{\sigma_{avg}}$$

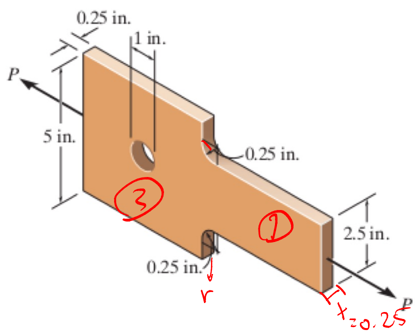
$$\sigma_{max} = 1.4\left(\frac{8 \times 10^3}{20\text{mm}\,(5\text{mm})}\right)$$

$$\sigma_{max} \Rightarrow 112 \text{ MPa}$$

For the hole $\Rightarrow$ $\frac{2r}{w} = \frac{2\cancel{0}}{4\cancel{0}} = \frac{1}{2}$ $K = 2.1$

$$\sigma_{max} = 2.1\left(\frac{8 \times 10^3}{(40-20)5}\right) \Rightarrow \sigma_{max} = 168$$ for hole area

*9–68. Determine the maximum axial force P that can be applied to the steel plate. The allowable stress is $\sigma_{allow} = 21$ ksi.



Prob. 9–68

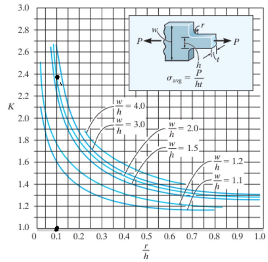


Fig. 9–24

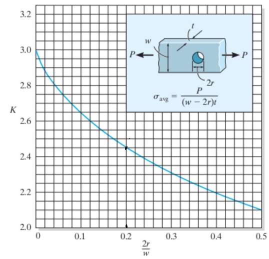


Fig. 9–25

$$K = \frac{\sigma_{max}}{\sigma_{avg}} \Rightarrow$$

for 1 $\longrightarrow$ $\frac{r}{h} = \frac{0.25}{2.5} \approx 0.1$; $\frac{w}{h} \Rightarrow \frac{5}{2.5} = 2$ $\Big\}$ $K = 2.385$

$$K = \frac{\sigma_{max}}{\frac{P}{ht}} \rightarrow P = \frac{\sigma_{max}\, h t}{K}$$

$$P = \frac{(21\times10^3)(2.5)(0.25)}{2.385}$$

$$P = 5.503 \text{ Kip}$$

for 2 $\longrightarrow$ $\frac{2r}{w} = \frac{1}{5}$ $\Big\}$ $K = 2.45$

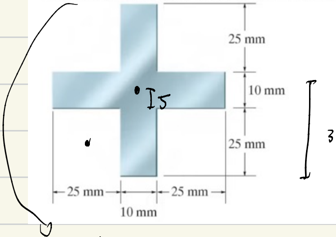
$$K = \frac{\sigma_{max}}{\frac{P}{(w-2r)t}} \rightarrow P = \frac{\sigma_{max}(w-2r)t}{K}$$

$$= \frac{(21\times10^3)(4)(0.25)}{2.45}$$

$$P = 8.571 \text{ Kips}$$

Buckling Example Problem

1. An A-36 steel column has a length of 4 m and is pinned at both ends. If the cross sectional area has the dimensions shown, determine the critical load.



$$P_{cr} = \frac{\pi^2 E I}{(KL)^2}$$

A-36 steel $\Rightarrow$ $E = 200\ GPa$

$$I = \left(\frac{(60)(60)^3}{12}\right) - 4\left(\frac{(25)(25)^3}{12} + (25\times 25)(17.5)^2\right)$$

$$I = 1.8416 \times 10^{-7}\ m^4$$

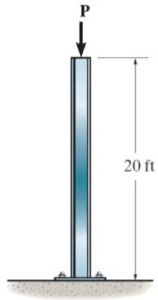
$$P_{cr} = \frac{\pi^2 (200 \times 10^9)(1.8416 \times 10^{-7}\ m^4)}{(1(4m))^2}$$

for pin connected $K = 1$

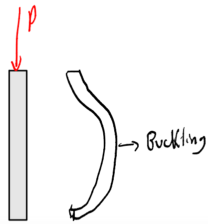
length

$$\boxed{P_{cr} = 22.7\ kN}$$

2. The W14 x 38 column is made of A-36 steel and is fixed supported at its base. If it is subjected to an axial load of P = 15 kip, determine the factor of safety with respect to buckling.



$E = 200\ GPa$



$$F.S \Rightarrow \frac{P_{cr}}{P}$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} \Rightarrow \frac{\pi^2 (29\times10^6)(26.7)}{(2(240))^2}$$

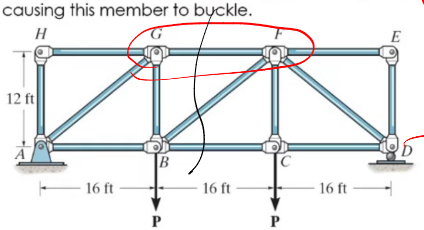
ksi

from Appendix (Weakest axis) for worst case Scenario

$$F.S = \frac{33.17}{15} \Rightarrow 2.21$$

3. The members of the truss are assumed to be pin connected. If member GF is an A-36 steel rod having a diameter of 2 in., determine the greatest magnitude of load P that can be supported by the truss without causing this member to buckle.

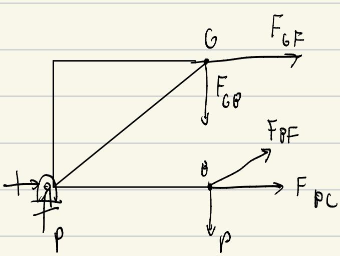
$K = 1$



Symmetry: $R_{Ay} = R_{Dy} = P$

without Causing to buckle $P > P_{cr}$

FBD.



$$\circlearrowleft + \Sigma M_{B} = 0 \quad -F_{GF}(12\,ft) - 16P = 0$$

$$F_{FG} = \frac{4}{3}P \quad (1)$$

$$P_{cr} = \frac{\pi^2 E I}{(KL)^2} \Rightarrow \frac{\pi^2 (29\times10^6)\left(\frac{\pi}{4}(1)^4\right)}{(1(16\times12))^2} \Rightarrow 6098$$

force that can take before it start to buckle.

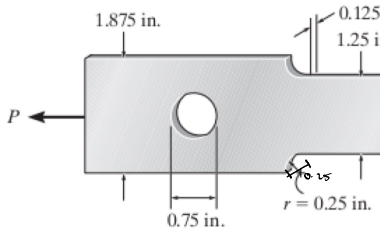
<u>Hence</u> $\rightarrow$ Apply $F_{FG} = \frac{4}{3}P$

$$\frac{4}{3}P = 6098$$

$$P = 4573.5 \text{ lb}$$

9–69. Determine the maximum axial force P that can be applied to the bar. The bar is made from steel and has an allowable stress of $\sigma_{allow} = 21$ ksi.

9–70. Determine the maximum normal stress developed in the bar when it is subjected to a tension of $P = 2$ kip.



Probs. 9–69/70

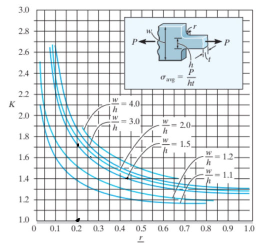


Fig. 9–24

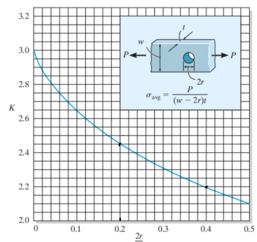


Fig. 9–25

$\frac{w}{h} = \frac{1.875}{1.25}$ $\quad \frac{r}{h} = \frac{0.25}{1.25}$

$\frac{w}{h} = 1.5$ $\quad \frac{r}{h} = 0.2$ $\quad \frac{0.75}{1.875} = 0.4$

Assume failure on the bar.

We know $K = \frac{\sigma_{max}}{\sigma_{avg}}$

$K\,\sigma_{avg} = \sigma_{max}$

$K\left(\frac{P}{ht}\right) = \sigma_{max}$

find K for the bar $\Rightarrow K = 1.72$

$= 1.7\left(\frac{P}{(1.25)(0.125)}\right) = 21\times10^3$

$P = 1.875$ kip

Assume failure for the hole.

$K\left(\frac{P}{(w-2r)t}\right) = \sigma_{max}$

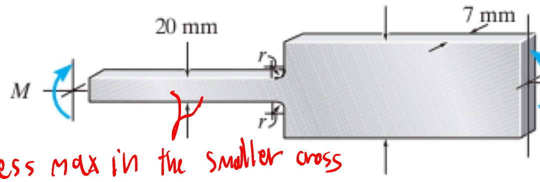
$\frac{2.2P}{(1.875-0.75)(0.125)} = 21\times10^3$

$P = 1342.33 \rightarrow$ or 1.342 kip

Stress Con centration (Bending)

11–83. The bar is subjected to a moment of $M = 40\ \text{N}\cdot\text{m}$. Determine the smallest radius r of the fillets so that an allowable bending stress of $\sigma_{allow} = 124$ MPa is not exceeded.

***11–84.** The bar is subjected to a moment of $M = 17.5\ \text{N}\cdot\text{m}$ If $r = 5$ mm, determine the maximum bending stress in the material.



stress max in the smaller cross section.

Probs. 11–83/84

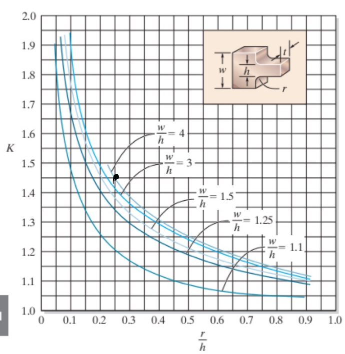


Fig. 11–34

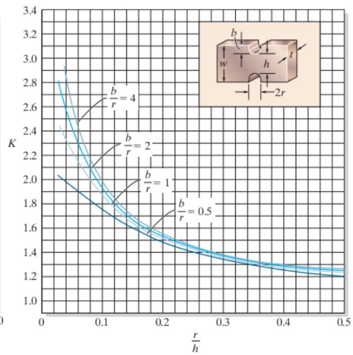
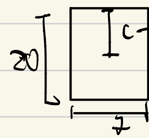


Fig. 11–35

Bending

$$\sigma_{max} = K\frac{(Mc)}{I}$$

find c.



c = 10 mm → 0.01 m

$$I = \frac{bh^3}{12} = \frac{(7)(20)^3}{12}$$

$\Rightarrow 4.166\times10^{-9}\ m^4$

Apply eq. ⇒

$$\frac{(124\times10^6)(4.666\times10^{-9})}{(40\ N\cdot m)(0.01\ m)} = K$$

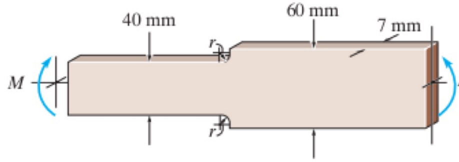
$K = 1.45$ → $\frac{w}{h} = \frac{80}{20} = 4$

$r = 0.25$ → $\frac{r}{h} = 0.25$

$r = 0.25(20) \Rightarrow 5 \text{ mm}$

11–87. The bar is subjected to a moment of $M = 153 \text{ N}\cdot\text{m}$. Determine the smallest radius r of the fillets so that an allowable bending stress of $\sigma_{allow} = 120 \text{ MPa}$ is not exceeded.

***11–88.** The bar is subjected to a moment of $M = 17.5 \text{ N}\cdot\text{m}$. If $r = 6 \text{ mm}$ determine the maximum bending stress in the material.



Probs. 11–87/88

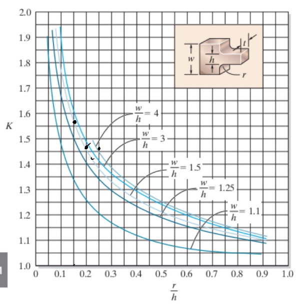


Fig. 11–34

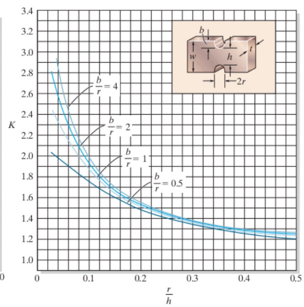
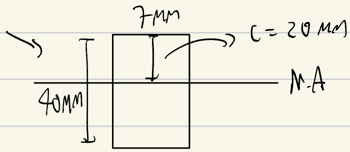


Fig. 11–35

For Bending Moment for Stress Concentration $\Rightarrow$ $\sigma_{max} = \dfrac{K(Mc)}{I}$

From Cross sectional Area.



$; \; I = \dfrac{(7)(40)^3}{12}$

$I = 3.733 \times 10^{-8} \text{ m}^4$

$$K = \frac{(120 \times 10^6)(3.733 \times 10^{-8})}{(153 \text{ N}\cdot\text{m})(0.02)}$$

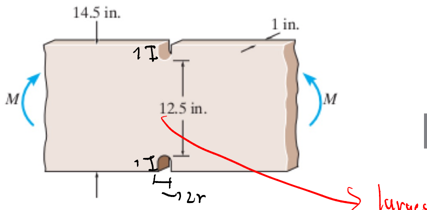
$K \approx 1.464$

$$\frac{w}{h} = \frac{60}{40} = \frac{3}{2}$$

$\dfrac{r}{h} = 0.2 \Rightarrow r = 0.2(40) = 8 \text{ mm}$

11–81. If the radius of each notch on the plate is $r = 0.5$ in., determine the largest moment that can be applied. The allowable bending stress for the material is $\sigma_{allow} = 18$ ksi.

11–82. The symmetric notched plate is subjected to bending. If the radius of each notch is $r = 0.5$ in. and the applied moment is $M = 10$ kip·ft, determine the maximum bending stress in the plate.



Probs. 11–81/82

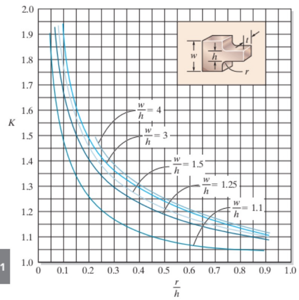


Fig. 11–34

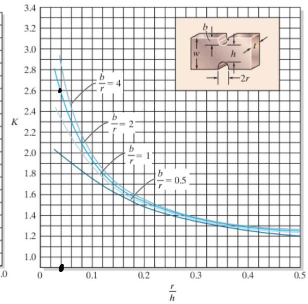


Fig. 11–35

largest moment or force apply → (assume smaller section first)

$$\sigma_{max} = K\left(\frac{Mc}{I}\right) \Rightarrow$$

$$\frac{(18\times10^3)(I)}{K\,c} = M$$

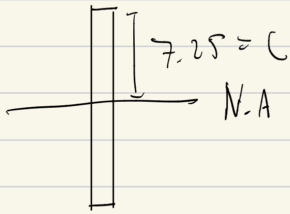
$$= \frac{(18\times10^3)(254.052)}{(2.6)(6.25\text{ in})} = M$$

$M = 15.18$ kip in

$$\frac{r}{h} = \frac{0.5}{12.5} = 0.04$$

$$\frac{b}{r} = \frac{1}{0.5} = 2$$

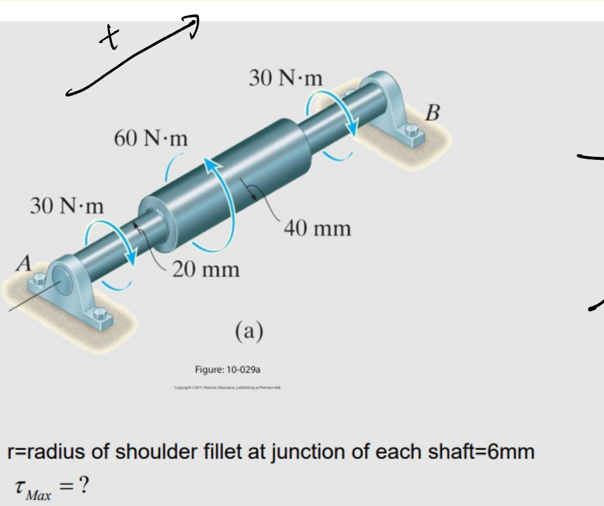
$K = 2.6$



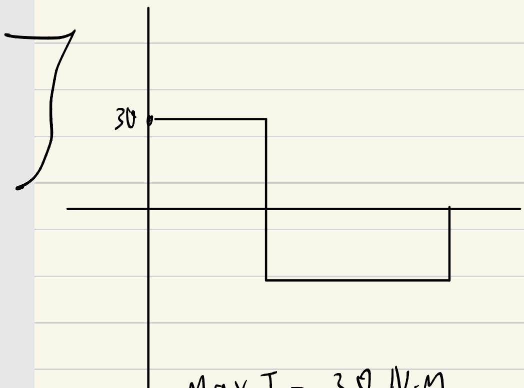
$$I = \frac{(1)(12.5)^3}{12}$$

$I = 162.76$

Stress Concentration : Torsion

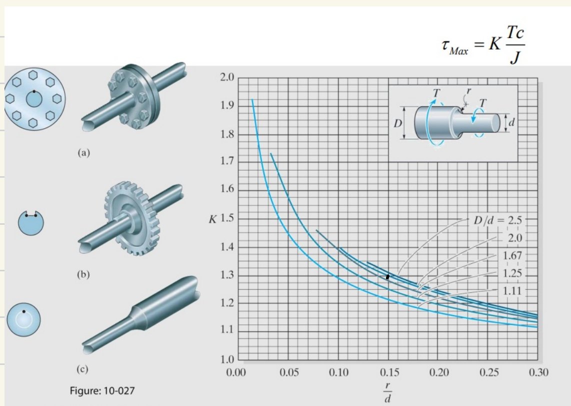


Find Max Torque



Max T = 30 N·M

$$T_{max} = K\left(\frac{Tc}{J}\right)$$



$$T_{max} = 1.3\left(\frac{30\,(0.02)}{\frac{\pi}{2}(0.02)^4}\right)$$

$$r/d = \frac{6}{40} = 0.15 \qquad D/d = 2$$

$$K = 1.3$$

$$T_{max} = 3.103\ \text{MPa}$$