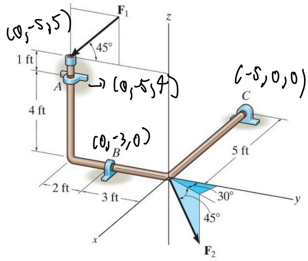
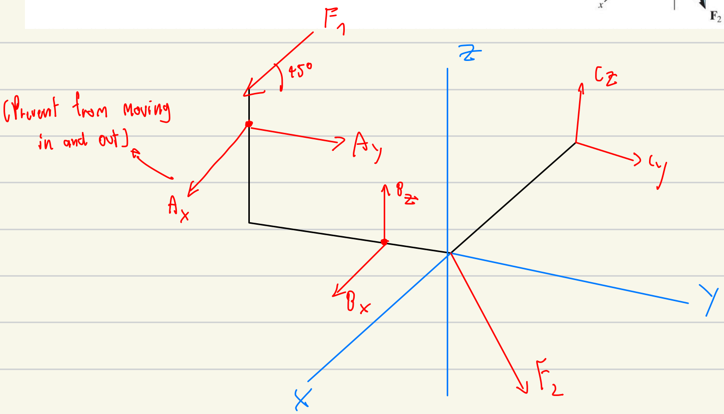
1. The bent rod is supported at A, B, and C by smooth journal bearings. Compute the x, y, z components of reaction at the bearings if the rod is subjected to forces $F_1 = 300\ lb$ and $F_2 = 250\ lb$. F_1 lies in the y–z plane. The bearings are in proper alignment and exert only force reactions on the rod.





$\vec{M}_o F_1 = (-5\hat{j} + 5\hat{k}) \times (-212.1\hat{j} - 212.1\hat{k})$

$\vec{M}_o A = (-5\hat{j} + 4\hat{k}) \times (A_x\hat{i} + A_y\hat{j})$

$\vec{M}_o B = (-3\hat{j}) \times (B_x\hat{i} + B_z\hat{k})$

$\vec{M}_{oC} = (-5\hat{i})$

$\vec{F}_1 = -300\cos 45\,\hat{j} - 300\sin 45\,\hat{k}$

$= -212.1\hat{j} - 212.1\hat{k}$ lb

$\vec{F}_2 = 250\cos 45\sin 30\,\hat{i} + 250\cos 45\cos 30\,\hat{j} - 250\sin 45\,\hat{k}$

$\vec{F}_2 = 88.39\hat{i} + 153.1\hat{j} - 176.8\hat{k}$ lb

$\overset{+}{\rightarrow} \sum F_x = 0 \quad 88.39 + A_x + B_x = 0$

$+\uparrow \sum F_y = 0 \quad -212.1 + 153.1 + A_y + C_y = 0$

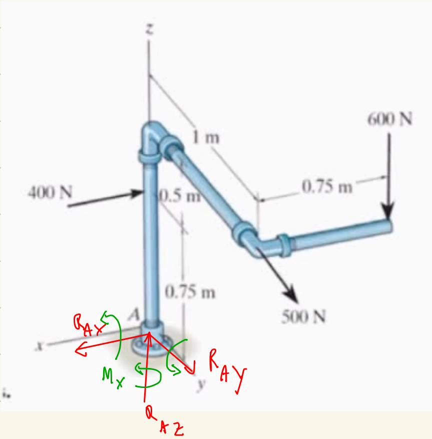
$+\uparrow \sum F_z = 0 \quad -212.1 - 176.8 + C_z + B_z = 0$

$\sum M_x = -B_z(3) - A_y(4) + 212.1(5) + 212.1(5) = 0$

$\sum M_y = C_z(5) + A_x(4) = 0$

$\sum M_z = A_x(5) + B_x(3) - C_y(5) = 0$

1. Determine the components of reaction at the fixed support A. The 400 N, 500 N, and 600 N forces parallel to the x, y, and z axes, respectively.



$\sum F = 0$

$\sum M = 0$

$\xrightarrow{+} \sum F_x = 0;$ $\quad R_{Ax} - 400 = 0$

$\rightarrow R_{Ax} = 400 \text{ N}$

$+\uparrow \sum F_y = 0;$

$R_{Ay} + 500 = 0$

$R_{Ay} - 500 = 0$

$R_{Ay} = -500 \text{ N}$

$+\uparrow \sum F_z = 0; \Rightarrow R_{Az} - 600 = 0$

$R_{Az} = 600 \text{ N}$

$M_x = 0;$ $\quad (M_A)_x - 500(1.25) - 600(1) = 0$

$(M_A)_x = 1225 \text{ N}\cdot\text{M}$

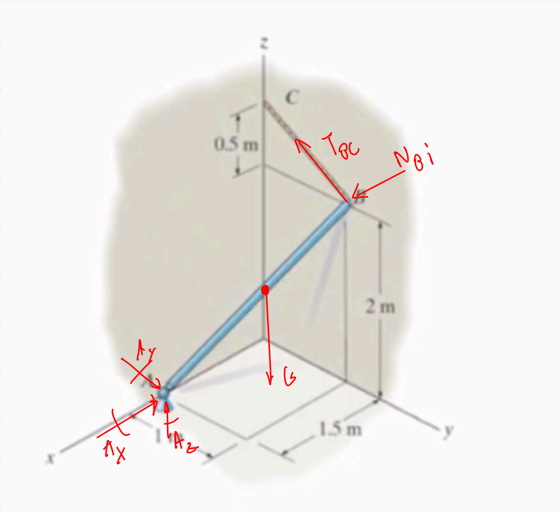
$\sum M_y = 0 \Rightarrow$

$(M)_y - 400(0.75) - 600(0.75) = 0$

$M_{Ay} = 750 \text{ N}\cdot\text{M}$

$\sum M_z = 0;$ $\quad (M_A)_z = 0$

2. The smooth uniform rod AB is supported by a ball-and-socket joint at A, the wall at B, and cable BC Determine the components of reaction at A, the tension in the cable, and the normal reaction at B if the rod has a mass of 20 kg.



Force vector

$\sum F = 0 \Rightarrow \vec{F} = F\hat{v} \rightarrow \frac{\vec{r}}{|r|}$

$\sum M = 0 \Rightarrow \vec{M} = \vec{r} \times \vec{F}$

$\Downarrow$

$M = \vec{r} \times (\vec{T}_{BC} + \vec{N}_B)$

$M = \vec{r}_{GA} \times \vec{W}$

$\vec{r}_{AB} = -1.5\hat{i} + \hat{j} + 2\hat{k}$ M

$\vec{r}_{AG} = -0.75\hat{i} + 0.5\hat{j} + \hat{k}$

$A = (1.5, 0, 0)$, $B(0, 1, 2)$, $C(0, 0, 2.5)$, $G(0.75, 0.5, 1)$

$F_A = -A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$F_{T_{BC}} = T_{BC}\left(\frac{r_{BC}}{r_{BC}}\right) = \frac{(0-1)\hat{j} + (2.5-2)\hat{k}}{\sqrt{1^2 + 0.5^2}}$

$N_B = N_B\hat{j}$ $\quad = \frac{-1}{\sqrt{1.25}} T_{BC}\hat{j} + \frac{0.5 T_{BC}\hat{k}}{\sqrt{1.25}}$

$W = -20(9.81)\hat{k}$ N

$\sum F_x = 0;\quad -A_x + N_B = 0$ eq1

$\sum F_y = 0;\quad A_y - \frac{1}{\sqrt{1.25}} T_{BC} = 0$ eq2

$\sum F_z = 0;\quad A_z + \frac{0.5 T_{BC}}{\sqrt{1.25}} - 20(9.81)$ eq3

$\sum M_A = 0 \quad (\vec{r}_{AG} \times \vec{W}) + (\vec{r}_{AB}) \times (T_{BC} + N_B) = 0$

$$\begin{vmatrix} i & j & k \\ -0.75 & 0.5 & 1 \\ 0 & 0 & -20(9.81) \end{vmatrix} + \begin{vmatrix} i & j & k \\ -1.5 & 1 & 2 \\ N_B & \frac{-1}{\sqrt{1.25}} T_{BC} & \frac{0.5 T_{BC}}{\sqrt{1.25}} \end{vmatrix} = 0$$

$$\hat{i} = -98 + \left(\frac{0.5}{\sqrt{1.25}} T_{BC} + \frac{2}{\sqrt{1.25}} T_{BC}\right) = 0$$

$$\sqrt{5}\, T_{BC} = 98.1$$

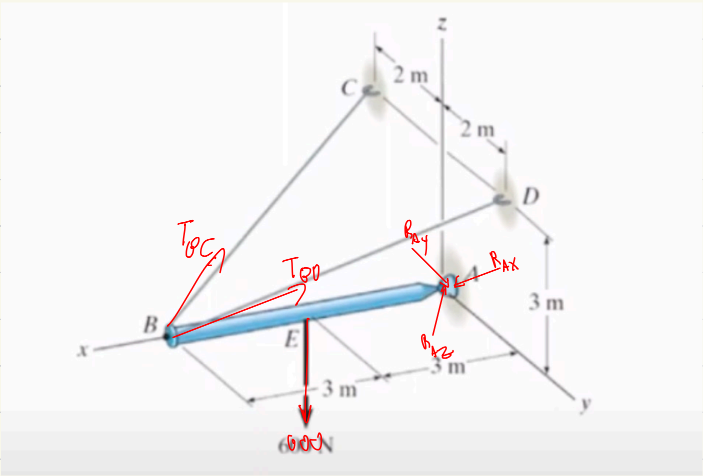
$$T_{BC} = 43.87 \rightarrow \text{Plug to } \underline{\text{eq 2.}}$$

$$A_y = 39.24$$

$\hat{j} = 0iy$, $\hat{k}$ $0iy$ $\Rightarrow$ give $N_D = 58.9\ N$, $A_x = 58.9\ N$

$$A_z = 977N$$

6. Determine the components of reaction at the ball-and-socket joint A and the tension in each cable necessary for equilibrium of the rod.



$$\Sigma F = 0$$
$$\Sigma M = 0$$

$A(0,0,0)$
$B(6,0,0)$
$C(0,-2,3)$
$D(0,2,3)$
$E(3,0,0)$

$$F_{TBC} = T_{BC}\left(\frac{-6\hat{i} - 2\hat{j} + 3\hat{k}}{7}\right) = -\frac{6}{7}T_{BC}\,\hat{i} - \frac{2}{7}T_{BC}\,\hat{j} + \frac{3}{7}T_{BC}\,\hat{k}$$

$$F_{TBD} = T_{BD}\left(\frac{-6\hat{i} + 2\hat{j} + 3\hat{k}}{7}\right) = -\frac{6}{7}T_{BD}\,\hat{i} + \frac{2}{7}T_{BD}\,\hat{j} + \frac{3}{7}T_{BD}\,\hat{k}$$

$\swarrow + \sum F_x = 0 \Rightarrow \quad -\frac{6}{7}T_{BC} - \frac{6}{7}T_{BD} + R_{Ax} = 0$

$\searrow + \sum F_y = 0 \Rightarrow \quad R_{Ay} - \frac{2}{7}T_{BC} + \frac{2}{7}T_{BD} = 0$

$+\uparrow \sum F_z = 0 \quad -600 + \frac{3}{7}T_{BC} + \frac{3}{7}T_{BD} + R_{Az} = 0$

$\circlearrowleft + \sum M = 0 = r \times F \quad , \quad \vec{r}_{AB} = 6\hat{i} + 0\hat{j} + 0\hat{k} \quad , \quad \vec{r}_{AE} = 3\hat{i} + 0\hat{j} + 0\hat{k}$

$$\sum M = r \times F = \left(r_{AB} \times (F_{T_{BC}} + F_{T_{BD}})\right) + \left(r_{AE} \times (-600\hat{k})\right)$$

$$\begin{vmatrix} i & j & k \\ 6 & 0 & 0 \\ -\frac{6}{7}T_{BC} - \frac{6}{7}T_{BD} & -\frac{2}{7}T_{BC} + \frac{2}{7}T_{BD} & \frac{3}{7}T_{BD} + \frac{3}{7}T_{BC} \end{vmatrix} +$$

$$\begin{vmatrix} i & j & k \\ 3 & 0 & 0 \\ 0 & 0 & -600 \end{vmatrix} =$$

$i = 0$

$j = -\left(\frac{18}{7}T_{BD} + \frac{18}{7}T_{BC}\right) + 1800 = 0$

$k = -\frac{12}{7}T_{BC} + \frac{12}{7}T_{BD} = 0$

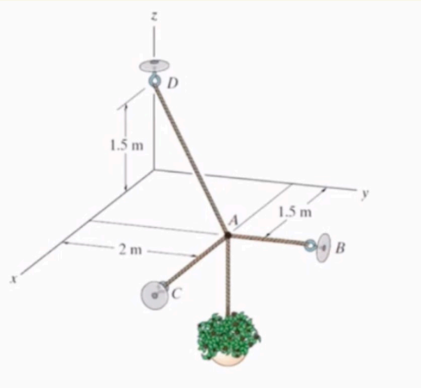
solve $\quad -\frac{18}{7}T_{BD} - \frac{18}{7}T_{BC} = -1800$

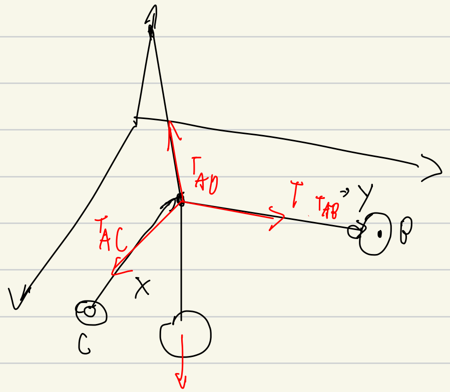
$+\frac{12}{7}T_{BD} - \frac{12}{7}T_{BC} = 0$

$T_{BD} = 350$, $T_{BC} = 350$

$R_{Ax} = 600$, $R_{Ay} = 0$ $R_{Az} = 300$

1. The three cables are used to support the 40-kg flowerpot. Determine the force developed in each cable for equilibrium.





$Mg = 40(9.81) = 392$ N

$A = (1.5, 2, 0)$
$D = (0, 0, 1.5)$

$$\vec{r}_{AD} = (-1.5\hat{i} - 2\hat{j} + 1.5\hat{k})$$

$$F_{AD} = T_{AD}\left(\frac{-1.5\hat{i} - 2\hat{j} + 1.5\hat{k}}{2.915}\right) = \frac{-1.5}{2.915}T_{AD}\hat{i} - \frac{2}{2.915}T_{AD}\hat{j} + \frac{1.5}{2.915}T_{AD}\hat{k}$$

$$\Sigma F_z = 0 \Rightarrow \frac{1.5}{2.915}T_{AD} - 392 = 0$$

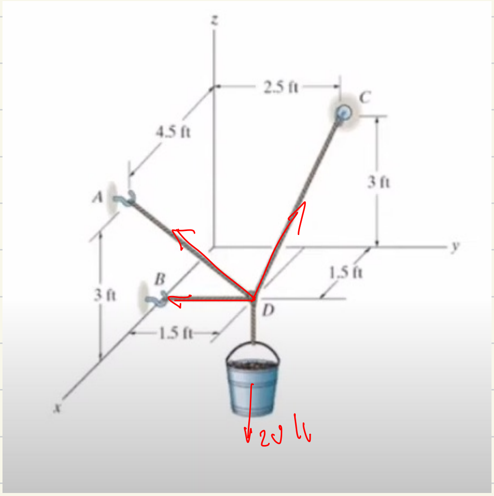
$$T_{AD} = 761.786$$

$$\Sigma F_x = 0 \Rightarrow F_{AC} - 761.786\left(\frac{1.5}{2.915}\right) = 0 \qquad F_{AC} = 391.99\text{ N}$$

$$\Sigma F_y = 0 \Rightarrow F_{AB} - 761.786\left(\frac{2}{2.915}\right) = 0 \Rightarrow F_{AB} = 522.66$$

3. If the bucket and its contents have a total weight of 20 lb, determine the force in the supporting cables DA, DB, and DC.

Tension พุ่งออก



$A = (4.5, 0, 3)$

$B = (1.5, 0, 0)$

$C = (0, 2.5, 3)$, $D = (1.5, 1.5, 0)$

$\vec{r}_{DA} = (+3\hat{i} - 1.5\hat{j} + 3\hat{k})$

$\vec{r}_{DB} = (0\hat{i} + 1.5\hat{j} + 0\hat{k})$

$\vec{r}_{DC} = (-1.5\hat{i} + 1\hat{j} + 3\hat{k})$

$$T_{AD} = T_{AD}\left(\frac{+3\hat{i} - 1.5\hat{j} + 3\hat{k}}{4.5}\right) = +\frac{3}{4.5}T_{AD}\hat{i} - \frac{1.5}{4.5}T_{AD}\hat{j} + \frac{3}{4.5}T_{AD}\hat{k}$$

$$T_{BD} = T_{BD}\left(\frac{0\hat{i} + 1.5\hat{j} + 0\hat{k}}{1.5}\right) = -T_{BD}\hat{j}$$

$$T_{CD} = T_{CD}\left(\frac{1.5\hat{i} - 1\hat{j} + 3\hat{k}}{3.5}\right) = -\frac{1.5}{3.5}T_{CD}\hat{i} + \frac{1}{3.5}T_{CD}\hat{j} + \frac{3}{3.5}T_{CD}\hat{k}$$

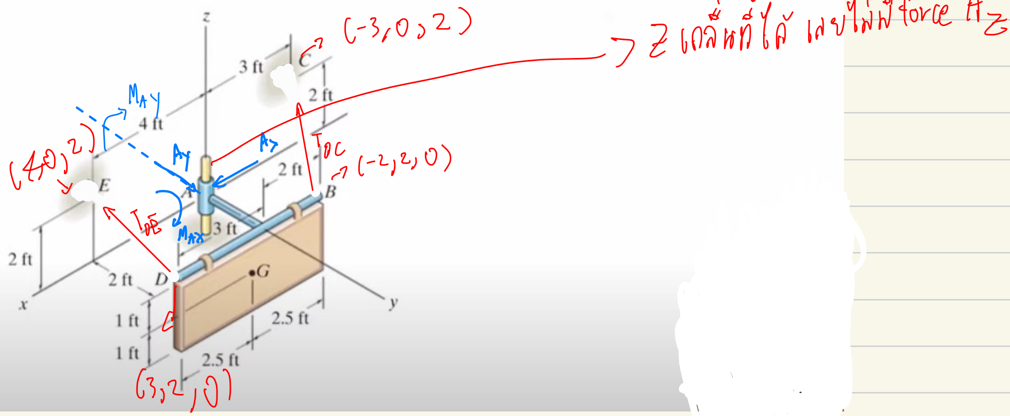
$+\sum F_x = 0 \rightarrow \quad \frac{-3}{4.5}T_{AD} + \frac{1.5}{3.5}T_{CD} = 0$

$+\sum F_y = 0 \rightarrow \quad \frac{1.5}{4.5}T_{AD} - T_{BD} - \frac{1}{3.5}T_{CD} = 0$

$+\uparrow \sum F_z = 0 \quad \frac{-3}{4.5}T_{AD} - \frac{3}{3.5}T_{CD} = 20\ lb$

$\downarrow$ solve DIY Cal

Determine the tension in the cables and the components of reaction on the smooth collar at A necessary to hold the 50-Ib sign tin equilibrium. The center of gravity for the sign is at G.



$$T_{DE} = T_{DE}\left(\frac{4i, -2j, +2k}{3}\right) = T_{DE}\left(\frac{1}{3}i - \frac{2}{3}j + \frac{2}{3}k\right)$$

$$\vec{T}_{BC} = T_{BC}\left(\frac{-1\hat{i} - 2\hat{j} + 2\hat{k}}{3}\right) = -\frac{1}{3}T_{BC}\hat{i} - \frac{2}{3}T_{BC}\hat{j} + \frac{2}{3}T_{BC}\hat{k}$$

$$\Sigma F_x = 0; \quad A_x + \frac{1}{3}T_{DE} - \frac{1}{3}T_{BC} = 0$$

$$\Sigma F_z = 0: \quad \frac{2}{3}T_{DE} + \frac{2}{3}T_{BC} - 50 = 0$$

$$\Sigma F_y = 0 \quad -\frac{2}{3}T_{DE} - \frac{2}{3}T_{BC} + R_{Ay} = 0$$

$$\Sigma M_x = 0; \Rightarrow (M_A)_x + \frac{2}{3}T_{DE}(2) + \frac{2}{3}T_{BC}(2) - 50(2) = 0$$

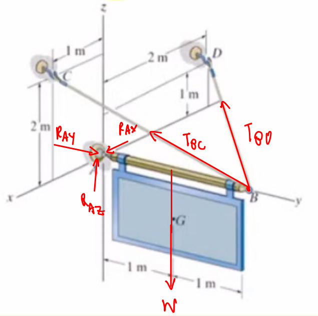
$$\Sigma M_y = 0;$$

$$-(M_A)_y - \frac{2}{3}T_{DE}(3) + \frac{2}{3}T_{BC}(2) + 50(0.5) = 0$$

DIY and Solve

$$\Sigma M_z = 0; \Rightarrow -\frac{1}{3}T_{DE}(2) - \frac{2}{3}T_{DE}(3) + \frac{1}{3}T_{BC}(2) + \frac{2}{3}T_{BC}(2) = 0$$

13. The sign has a mass of 100 kg with center of mass at G. Determine the x, y, z components of reaction at the ball-and socket joint A and the tension in wires BC and BD. $\Sigma \vec{F} = 0$; $\Sigma \vec{M}$ -



$B = (0, 2, 0)$ $A, (0, 0, 0)$

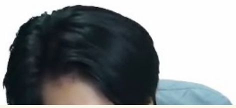
$C = (1, 0, 2)$

$D = (-2, 0, 1)$

$G = (0, 1, 0)$

$\cancel{M}$

Ball and Socket give no moment.



$\vec{r}_{BC} \Rightarrow (1, -2, 2)$, $\vec{r}_{BD} = (-2, -2, 1)$, $\vec{r}_{AB} = (0, 2, 0)$

$\vec{r}_{AG} \Rightarrow (0, 1, 0)$

Force vector $\Rightarrow$ $T_{BC}\left(\frac{1\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{1+4+4}}\right) = \frac{1}{3}T_{BC}\hat{i} - \frac{2}{3}T_{BC}\hat{j} + \frac{2}{3}T_{BC}\hat{k}$

$T_{BD}\left(\frac{-2\hat{i} - 2\hat{j} + 1\hat{k}}{\sqrt{9}}\right) = -\frac{2}{3}T_{BD}\hat{i} - \frac{2}{3}T_{BD}\hat{j} + \frac{1}{3}T_{BD}\hat{k}$

Force summation $\Rightarrow$ $\swarrow + \Sigma F_x = 0 \Rightarrow R_{Ax} + \frac{1}{3}T_{BC} - \frac{2}{3}T_{BD} = 0$

$\xrightarrow{+} \Sigma F_y = 0 \Rightarrow + R_{Ay} - \frac{2}{3}T_{BC} - \frac{2}{3}T_{BD} = 0$

$+\uparrow \Sigma F_z = 0 \Rightarrow$

$R_{Az} + \frac{2}{3}T_{BC} + \frac{1}{3}T_{BD} = W$

Moments $\Rightarrow$ $M_{0,(BC+BD)} = \vec{r}_{AB} \times (T_{BD} + T_{BC}) = \begin{vmatrix} \hat{i} & \hat{j} & K \\ 0 & 2 & 0 \\ \frac{1}{3}T_{BC} - \frac{2}{3}T_{BD} & -\frac{2}{3}T_{BC} - \frac{2}{3}T_{BD} & \frac{2}{3}T_{BC} + \frac{1}{3}T_{BD} \end{vmatrix}$

$$\vec{j} = \frac{4}{3}T_{BC} + \frac{2}{3}T_{BD}$$

$$\vec{k} = -\frac{2}{3}T_{BC} + \frac{4}{3}T_{BD}$$

$$M_{o,W} = \vec{r}_{AG} \times (W) = \begin{vmatrix} i & j & k \\ 0 & 1 & 0 \\ 0 & 0 & -981 \end{vmatrix} = -981\,\hat{j}$$

$$M_{o,(B_C+B_D)} + M_{oW} = \frac{4}{3}T_{BC} + \frac{2}{3}T_{BD} = 981$$

$$-\frac{2}{3}T_{BC} + \frac{4}{3}T_{BD} = 0$$

$$T_{BC} = 588.6\,N,\quad T_{BD} = 294.3\,N$$

Plug to force eq.

$$R_{AX} + \frac{1}{3}(588.6) - \frac{2}{3}(294.3) = 0$$

$$R_{AX} = 0$$

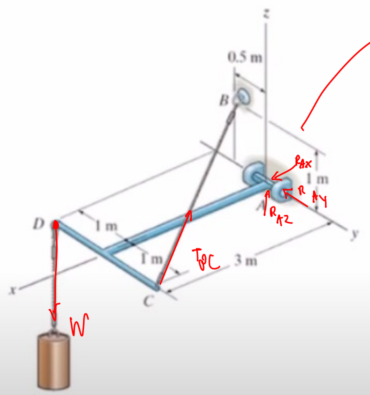
$$R_{AY} - \frac{2}{3}T_{BC} - \frac{2}{3}T_{BD} = 0$$

$$R_{AY} = 588.6\,N$$

$$R_{AZ} + \frac{2}{3}T_{BC} + \frac{1}{3}T_{BD} - W = 0$$

$$R_{AZ} = 490.5\,N$$

11. The member is supported ~~by a pin at A~~ and cable BC. Determine the components of reaction at these supports if the cylinder has a mass of 40 kg.



There is Moment at Pin Support

Moment at M_x and M_z

A (0, 0, 0)

B (0, -0.5, 1)

C (3, 1, 0)

D (3, -1, 0)

$W = -392\,\hat{z}$

$\vec{r}_{CB} = (-3, -1.5, 1)$

$\vec{r}_{AD} = (3, -1, 0)$, $\vec{r}_{AC} = (3, 1, 0)$

Force

$$T_{CB} = T_{CB}\left(\frac{-3\hat{i} - 1.5\hat{j} + 1\hat{k}}{3.5}\right) = -\frac{3}{3.5}T_{CB}\hat{i} - \frac{1.5}{3.5}T_{CB}\hat{j} + \frac{1}{3.5}T_{CB}\hat{k}$$

$W = (0, 0, -392)$ $\overset{+}{\rightarrow}\ \sum F_x = 0 \Rightarrow R_{Ax} - \frac{3}{3.5}T_{CB} = 0$

$+\nearrow\ \sum F_y = 0 \Rightarrow -R_{Ay} - \frac{1.5}{3.5}T_{CB} = 0$

$\uparrow + \sum F_z = 0 \Rightarrow R_{Az} + \frac{1}{3.5}T_{CB} - 392 = 0$

$$\sum M_{A, T_{CB} \text{ and } W} = (\vec{r}_{AC} \times (T_{CB})) + (\vec{r}_{AD} \times (W)) + M_A$$

$\downarrow$

$$\begin{vmatrix} i & j & K \\ 3 & 1 & 0 \\ \frac{-3}{3.5}T_{CB} & \frac{-1.5}{3.5}T_{CB} & \frac{1}{3.5}T_{CB} \end{vmatrix} + \begin{vmatrix} i & j & K \\ 3 & -1 & 0 \\ 0 & 0 & -392 \end{vmatrix}$$

$$i = \frac{1}{3.5}T_{CB} + 392 + M_A(x)$$

$$j = -\frac{3}{3.5}T_{CB} + 1176 = 0$$

$$\hat{K} = \frac{-4.5}{3.5}T_{CB} + \frac{3}{3.5}T_{CB} + M_A(z)$$

$$T_{CB} = 1372\ N$$

$\stackrel{+}{\leftarrow} \sum F_x = 0 \Rightarrow R_{AX} - \frac{3}{3.5}T_{CB} = 0$

$\stackrel{+}{\rightarrow} \sum F_y = 0 \Rightarrow -R_{Ay} - \frac{1.5}{3.5}T_{CB} = 0$

$\uparrow + \sum F_z = 0 \Rightarrow R_{Az} + \frac{1}{3.5}T_{CB} - 392 = 0$

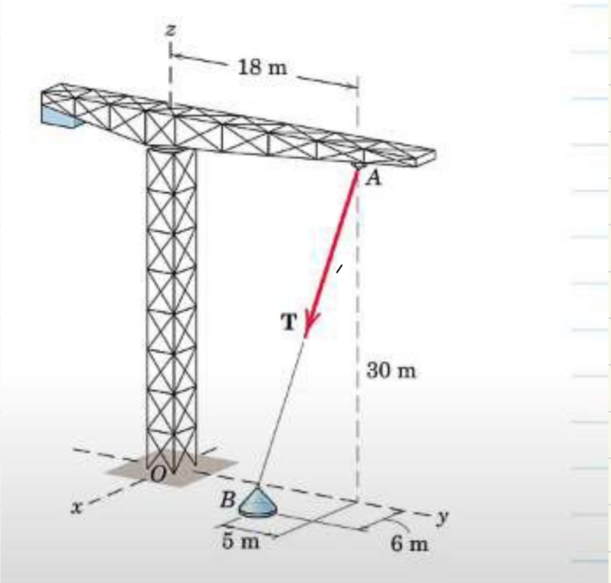
give $F_{CB} = 1.37\ kN$

$R_{AX} = 1.18\ kN$

$R_{AY} = 589\ N$

$A_z = 0$

In picking up a load from position B, a cable tension $\mathbf{T}$ of magnitude 24 kN is developed. Calculate the moment which $\mathbf{T}$ produces about the base O of the construction crane.



$O = (0, 0, 0)$

$A = (0, 18, 30)$

$B = (6, 13, 0)$

$\vec{r}_{AB} = (6, -5, -30)$

$\vec{r}_{OB} = (6, 13, 0)$

Force at $\vec{T} = \left(\frac{6\hat{i} - 5\hat{j} - 30\hat{k}}{31} \right) = 24\left(\frac{6}{31}\right)\hat{i} - 24\left(\frac{5}{31}\right)\hat{j} - 24\left(\frac{30}{31}\right)\hat{k}$

$T_{AB} = 4.645\hat{i} - 3.87\hat{j} - 23.226\hat{k}$

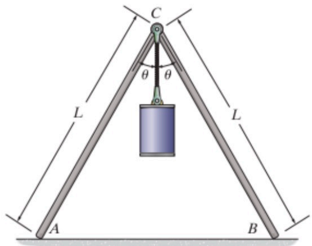
$$M_{O,T} = \vec{r}_{OB} \times \vec{T}_{AB} = \begin{vmatrix} i & j & K \\ 6 & 13 & 0 \\ 4.645 & -3.87 & -23.226 \end{vmatrix}$$

$\sum M_{\text{at } O \text{ from } T} \approx -301.938\hat{i} + 139.356\hat{j} - 83.605\hat{K}$ //

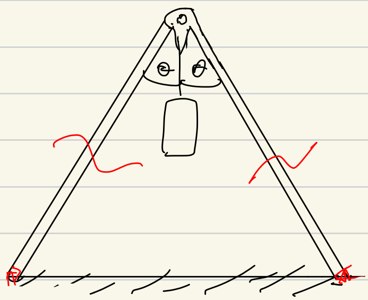
Friction

***4–72.** If $\theta = 30°$, determine the minimum coefficient of static friction at A and B so that equilibrium of the supporting frame is maintained regardless of the mass of the cylinder C. Neglect the mass of the rods.

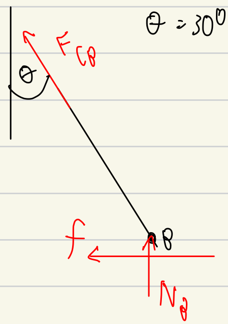
4–73. If the coefficient of static friction at A and B is $\mu_s = 0.6$, determine the maximum angle θ so that the frame remains in equilibrium, regardless of the mass of the cylinder. Neglect the mass of the rods.



FBD 1.



$\theta = 30°$



$f_B = \mu_B \cdot N_B$

$\overset{+}{\rightarrow} \sum F_x = 0 \Rightarrow -F_{CB} \sin\theta - \mu_B \cdot N_B = 0$

$+\uparrow \sum F_y = 0 \Rightarrow N_B + F_{CB} \cos\theta = 0$

$N_B = -F_{CB} \cos\theta$

$-F_{CB} \sin\theta - \mu_B (-F_{CB} \cos\theta) = 0$

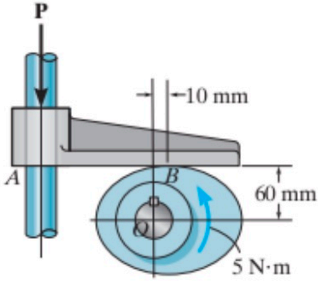
$\downarrow$

$$N_B(F_{CB}\cos\theta) = F_{CB}\sin\theta$$

$$N_B = \tan\theta$$

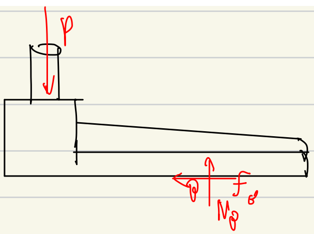
$$N_B = 0.577$$

*4–60. The cam is subjected to a couple moment of 5 N · m. Determine the minimum force P that should be applied to the follower in order to hold the cam in the position shown. The coefficient of static friction between the cam and the follower is $\mu = 0.4$. The guide at A is smooth.



Prob. 4–60

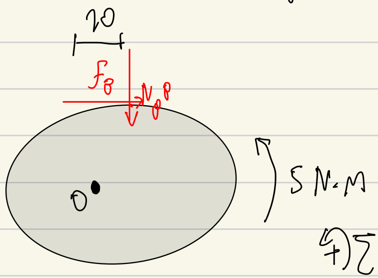
FBD1.



$$+\uparrow\sum F_y = 0 \Rightarrow -P + N_B = 0$$

$$P = N_B$$

FBD2.



$$-P(0.01) - (0.4)(P)(0.06) + 5 = 0$$

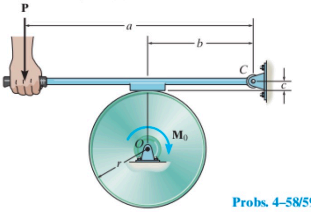
$$-0.034P = -5$$

$$P \approx 147$$

$$\circlearrowleft + \sum M_{O,z} = 0 \Rightarrow -N_B(0.01) - f_B(0.06) + 5\ N_M = 0$$

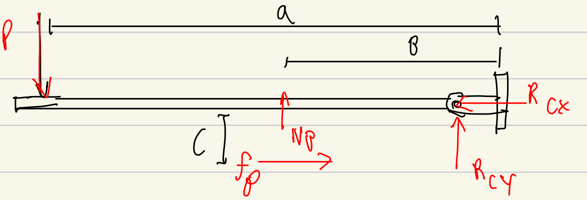
4–58. The block brake is used to stop the wheel from rotating when the wheel is subjected to a couple moment $\mathbf{M}_0$. If the coefficient of static friction between the wheel and the block is μ_s, determine the smallest force P that should be applied.

4–59. Show that the brake in Prob. 4–58 is self-locking, i.e., $P \leq 0$, provided $b/c \leq \mu_s$.



Probs. 4–58/59

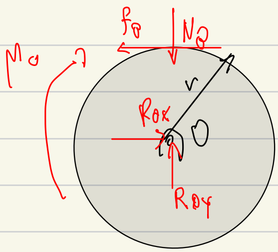
FBD 1.



$$\circlearrowleft + \sum M_{C,z} = 0 \Rightarrow \quad P(a) - N_B(b) + f_B(c) = 0$$

$$P(a) = N_B(b) - \mu_s N_B(c) = 0$$

FBD 2.



$$\circlearrowleft + \sum M_{O,z} = 0 \Rightarrow -M_O + f_B(r) = 0$$

$$M_O = f_B(r)$$

$$M_O = \mu_s(N_B)\, r$$

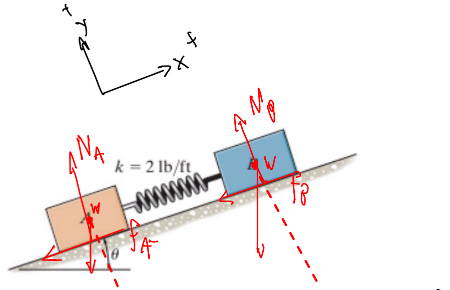
$$N_B = \frac{M_O}{\mu_s r}$$

$$P(a) = \frac{M_O}{\mu_s r}(b) - \cancel{\mu_s}\left(\frac{M_O c}{\cancel{\mu_s} r}\right)$$

$$\boxed{= \frac{M_O b}{\mu_s r a} - \frac{M_O c}{r a}}$$

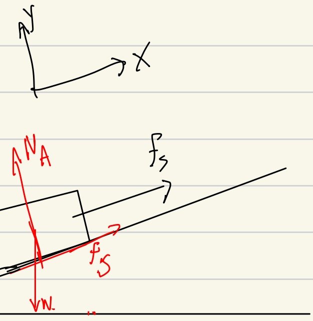
$$= \frac{M_O}{\mu_s r a}(b - \mu_s c)$$

*4–52. Two blocks A and B have a weight of 10 lb and 6 lb, respectively. They are resting on the incline for which the coefficients of static friction are $\mu_A = 0.15$ and $\mu_B = 0.25$. Determine the angle θ which will cause motion of one of the blocks. What is the friction force under each of the blocks when this occurs? The spring has a stiffness of $k = 2$ lb/ft and is originally unstretched.



Friction => แยก เป็น สอง FBD

FBD 1.



$+\uparrow \sum F_y = 0 \Rightarrow N_A - W_A\cos\theta = 0$

$N_A = W_A\cos\theta \rightarrow N_A = 10\cos\theta$

$+\nearrow \sum F_x = 0 \Rightarrow f_s - W_A\sin\theta + F_s = 0$

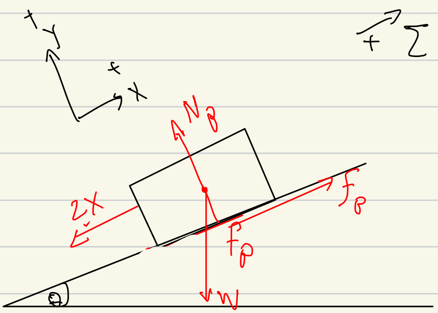
$0.15N_A - 10\sin\theta + 2x = 0$

$2x = 10\sin\theta - 0.15N_A$

$x = \dfrac{10\sin\theta - 0.15N_A}{2}$

$x = \dfrac{10\sin\theta - 0.15(10\cos\theta)}{2}$

FBD 2



$+\nearrow \sum F_x = 0 \Rightarrow 0.25N_B - 2x - 6\sin\theta = 0$

$+\uparrow \sum F_y = 0 \Rightarrow N_B - 6\cos\theta = 0$

$2x = 0.25N_B - 6\sin\theta$

$x = \dfrac{0.25(6\cos\theta) - 6\sin\theta}{2}$

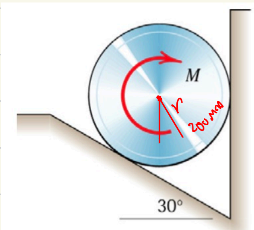
$= \dfrac{\frac{3}{2}\cos\theta - 6\sin\theta}{\cancel{2}} = \dfrac{10\sin\theta - \frac{3}{2}\cos\theta}{\cancel{2}}$

$$3\cos\theta = 16\sin\theta$$

$$\frac{3}{16} = \frac{\sin\theta}{\cos\theta}$$

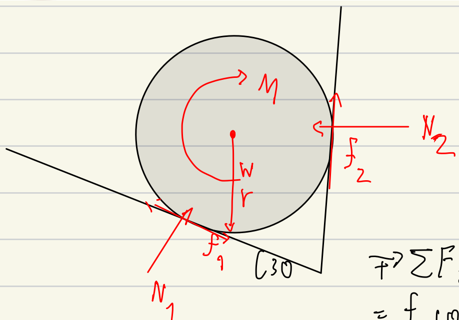
$$\tan\theta = \frac{3}{16}$$

$$\theta = 10.619 \quad //$$



6/1 The 30-kg homogeneous cylinder of 400-mm diameter rests against the vertical and inclined surfaces as shown. If the coefficient of static friction between the cylinder and the surface is 0.3, calculate the applied clockwise couple M which would cause the cylinder to slip. [Engineering Mechanics Statics 5th edition, Meriam & Kraige, prob.6/8]

ทำพร้อมกันได้เลย



$$W = mg = (30)(9.81)$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0$$

$$= f_1\cos 30 + N_1 \sin 30 - N_2 = 0 \rightarrow \text{eq1}$$

$$+\uparrow \Sigma F_y = 0 \Rightarrow \quad f_2 + N_1\cos 30 - f_1\sin(30) - (30)(9.81) = 0 \rightarrow \text{eq2}$$

$$\overset{+}{\circlearrowleft} \Sigma M_o \quad -M + f_1(0.2) + f_2(0.2) = 0$$

$$M = f_1(0.2) + f_2(0.2)$$

$$f_1 = \mu_s N_1 = 0.3N_1 \qquad f_2 = \mu_s N_2 = 0.3\,N_2$$

sub to eq 1 / eq 2

$$0.3N_1\cos 30 + N_1\sin 30 - N_2 = 0$$

$$0.3N_2 + N_1\cos 30 - 0.3N_1\sin 30 - 294.3 = 0$$

$$N_1(0.3\cos(30) + \sin(30)) - N_2 = 0 \quad \text{eq.3}$$

$$N_1(\cos(30) - 0.3\sin(30)) + 0.3N_2 = 294.3 \quad \text{eq.4}$$

Solve $\Rightarrow$ $N_1 \Rightarrow 311.7697$

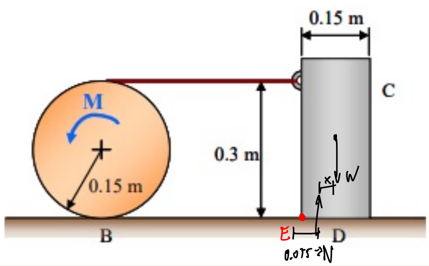
$N_2 = 236.8846.$

sub to $\Rightarrow$ $M = f_1(0.2) + f_2(0.2)$

$$M = (0.3)(311.7697)(0.2) + (0.5)(236.88)(0.2)$$

$$\boxed{M = 32.4 \text{ N/m}}$$

6/2 Determine the smallest couple moment which can be applied to the 20-N (≈2-kg) wheel that will cause impending motion. The cord is attached to the 30-N (≈3-kg) block, and the coefficients of static friction are $\mu_B = 0.2$ and $\mu_D = 0.3$. [Engineering Mechanics Statics 11th edition, R.C.Hibbeler, prob.8/46]



Block will not cause Impending motion as

$F_{Dmax} = \mu_D N_D$

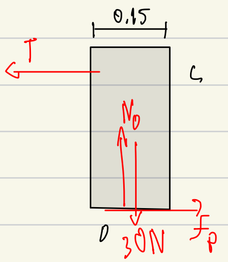
$= (0.3)(30) = 9$

and $F_{Bmax} = \mu_B N_B$

$= (0.2)(20) = 4$

as $9 > 4$ hence wheel will be more and likely to cause impending motion

FBD 1.



$+\uparrow \sum F_y = 0 \Rightarrow N_D - 30 = 0$

$N_D = 30\,N$

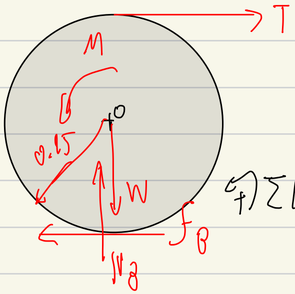
$\overset{+}{\rightarrow} \sum F_x = 0 \Rightarrow -T + f_D = 0$

$f_D = T$

$\mu_D N_D = T$ $\quad F_{max} = 0.3$

$T = (0.3)(30) = 9\,N$

FBD 2



$\circlearrowleft + \sum M_{O,z} = M - T(0.15) - f_B(0.15) = 0$

$M = 4(0.15) + 4(0.15)$

$M = 1.2$

$+\uparrow \sum F_y = 0 \Rightarrow -W + N_B = 0$

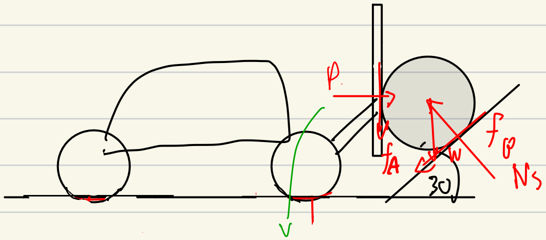
$N_B = W$

$N_B = 20$

$\overset{+}{\rightarrow} \sum F_x = 0 \quad T - f_B = 0$

$T = 20(0.2) = 4\,N$

6/3 The industrial truck is used to move the solid 1200-kg roll of paper up the 30° incline. If the coefficients of static and kinetic friction between the roll and the vertical barrier of the truck and between the roll and the incline are both 0.40, compute the required tractive force P between the tires of the truck and the horizontal surface.
[Engineering Mechanics Statics 5th edition, Meriam & Kraige, prob.6/43]



รวมแรง P เป็นแรงจาก Truck.

$$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow \quad P - N_s \sin 30 - F_B \cos(30) = 0 \rightarrow \text{eq1}$$

$$+\uparrow \Sigma F_y = 0 \quad N_s \cos(30) - f_A - f_B \sin 30 - Mg = 0 \rightarrow \text{eq2.}$$

$$\circlearrowleft \Sigma M_o = -f_B r + f_A(r) = 0$$

$$f_B = f_A$$

Assume slip at B first.

$$f_B = N_s N_B$$

$$N_s \cos(30) - N_s N_B - N_s N_B \sin 30 = Mg$$

$$N \cos(30) - 0.4 N_B - 0.4 N_B \sin 30 = Mg$$

$$N = 44257.476 \text{ N}$$

$$P = N \sin 30 + N N \cos 30$$

$$P = 37454.8456 \text{ N}$$

Check if our assumption of slip at surface B first is true such that

$$F_A < F_{A(max)}$$

$$\downarrow$$

$$f_A = f_B$$

$$\downarrow$$

$$\mu N < \mu P$$

$$(0.4)(44251.416) \not< (0.4)(37457.4456)$$

Not possible so it not slip at B first

Hence slip should start at A first.

$$\text{and } F_A = (F_A)_{max} = \mu P$$

From eq 1.

$$N \sin 30 = P - F_B \cos 30^0 \rightarrow ⑤$$

eq 2.

$$N \cos 30 = F_A + F_B \sin 30^0 + Mg \rightarrow ⑥$$

$$\frac{5}{6} = \quad \tan 30 = \frac{P - F_B \cos 30}{F_A + F_B \sin 30 + Mg}$$

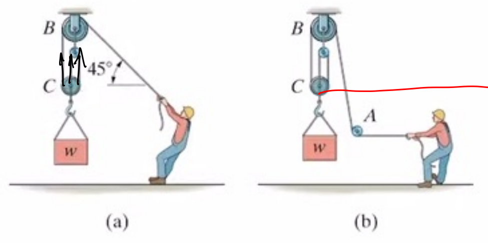
$$\tan 30 = \frac{P - \mu P \cos 30}{\mu P + \mu P \sin 30 + Mg}$$

$$P = 22125.71 N$$

//

8–11. Determine the maximum weight W the man can lift with constant velocity using the pulley system, without and then with the "leading block" or pulley at A. The man has a weight of 200 lb and the coefficient of static friction between his feet and the ground is $\mu_s = 0.6$.

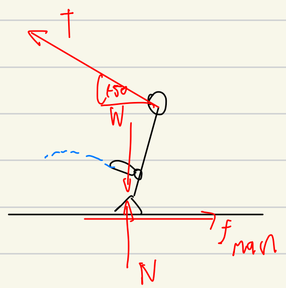
Note: Tension in pulley should be equal.



→ T ดูจากจุดเริ่มต้นอย่างเดียว.

a. without Pulley

FBD. 1 (a).



$\xrightarrow{+} \sum F_x = 0 \Rightarrow -T\cos 45 + f_{man} = 0$

$+\uparrow \sum F_y = 0 \Rightarrow N - W + T\sin 45 = 0$

$N = W - T\sin 45$

$-T\cos 45 = -f_{man}$

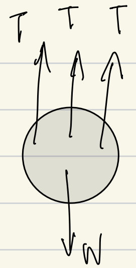
$T\cos 45 = N(0.6)$

$T\cos 45 = (200 - T\sin 45)\,0.6$

$T\cos 45 = 120 - 0.424\,T$

$\Rightarrow \quad 1.131\,T = 120$

FP02.



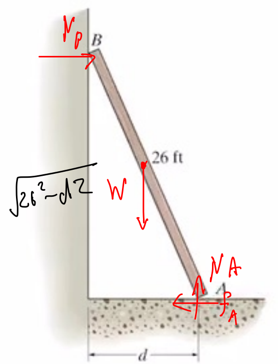
$T = 106.100$

$+\uparrow\Sigma F_y = 0 \quad 3T - W = 0$

$W = 3(106.100)$

$W = 318.3\ lb$

8–25. The uniform pole has a weight of 30 lb and a length of 26 ft. Determine the maximum distance d it can be placed from the smooth wall and not slip. The coefficient of static friction between the floor and the pole is $\mu_s = 0.3$.



$+\uparrow\Sigma F_y = 0$

$= -W + N_A = 0$

$N_A = W$

$N_A = 30\ lb$

$\xrightarrow{+}\Sigma F_x = 0 \quad N_B - f_A = 0$

$N_B = \mu_s N_A$

$N_B = (0.3)(30)$

$N_B = 9\ lb$

Suppose slip and Tip at Point A.

$\curvearrowleft + \Sigma M_{A,z} = 0; \ -9(\sqrt{26^2 - d^2}) + W\left(\frac{d}{2}\right) = 0$

$9(\sqrt{26^2 - d^2}) = \frac{30d}{2}$

$\left(\sqrt{26^2 - d^2}\right)^2 \left(\frac{15}{9} d\right)^2$

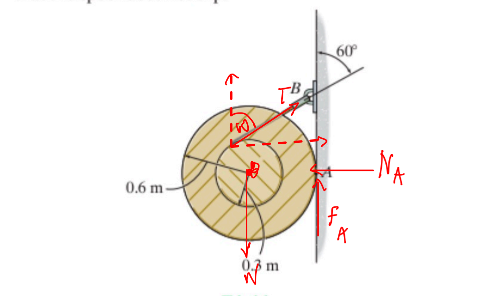
$$26^2 - d^2 = \frac{25}{9} d^2$$

$$676 = \frac{34}{9} d^2$$

$$178.94 = d^2 \qquad 4(676 - d^2)$$

$$\boxed{d = 13.38 \text{ ft}} \qquad 6084$$

F4–16. Determine the minimum coefficient of static friction between the uniform 50-kg spool and the wall so that the spool does not slip.



$$W = (50)(9.8)$$

No of unknown = 3 (N_A, F_A, T)

No of eq = 3

FBD spool

Type 1. No impending Motion

$$F_A = \mu_s N_A$$

$$\xrightarrow{+} \sum F_x = 0 \Rightarrow T\sin 60 - N_A = 0 \rightarrow \text{eq1}$$

$$N_A = T\sin 60$$

$$+\uparrow \sum F_y = 0 \searrow \quad T\cos 60 - W + F_A = 0 \rightarrow \text{eq2}$$

$$\circlearrowleft \sum M_{O,z} = 0 \Rightarrow -T(0.3) + f_A(0.6) = 0 \rightarrow \text{eq3}$$

eq 2 + eq 3

$$T\cos 60 + F_A = W$$

$$-T(0.3) + f_A(0.6) = 0$$

$$T = 490.5\text{ N}, \quad F_A = 245.25\text{ N}$$

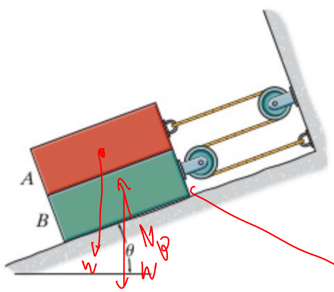
$$N_A = 490.5\sin 60$$

$$\rightarrow \boxed{N_A = 424.79\text{ N}}$$

$$F_A = \mu_s N_A$$

$$\mu_s = \frac{F_A}{N_A} \Rightarrow \boxed{0.58}$$

F4–17. If the coefficient of static friction at all contacting surfaces is μ_s, determine the inclination θ at which the identical blocks, each of weight W, begin to slide.



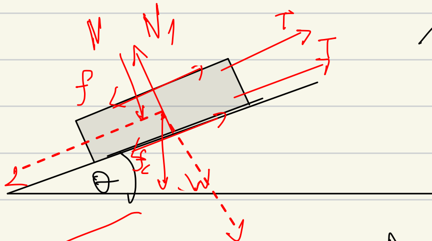
F4–17

began to slide

$$F_\theta < \mu_s N_\theta$$

reaction force ระหว่าง block ต้อง เท่ากัน

FBD 1 Block B



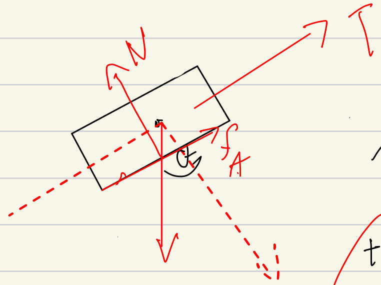
$$\nearrow + \sum F_x = 0$$

$$2T - W\sin\theta - f_A - f_B = 0$$

!!! (Weight แต่ละ Block ต้องเป็นอันเดียว)

$$\nwarrow + \sum F_y = 0 \quad N_1 - N - W\cos\theta = 0$$

FQ0 2. Block A.



$\overset{+}{\nearrow}\Sigma F_x = 0 \Rightarrow T + f_A - W\sin\theta = 0$

$+\nwarrow \Sigma F_y = 0 \Rightarrow N - W\cos\theta = 0$

$N = W\cos\theta$

$T = W\sin\theta - \mu_s N$

$T = W\sin\theta - \mu_s W\cos\theta$

Hence $N_1' - W\cos\theta - W\cos\theta = 0$

$N_1 = 2W\cos\theta$

Now $2T - W\sin\theta - \mu_s W\cos\theta - \mu_s 2W\cos\theta = 0$

$2W\sin\theta - 2\mu_s W\cos\theta - W\sin\theta - \mu_s W\cos\theta - \mu_s 2W\cos\theta = 0$

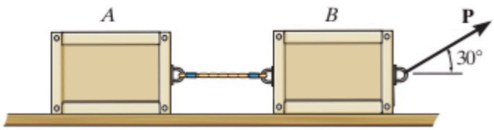
$W\sin\theta - 5\mu_s W\cos\theta = 0$

$\sin\theta = 5\mu_s\cos\theta$

$\tan\theta = 5\mu_s$

$\boxed{\theta = \tan^{-1}(5\mu_s)}$

F4–15. Determine the maximum force P that can be applied without causing the two 50-kg crates to move. The coefficient of static friction between each crate and the ground is $\mu_s = 0.25$.

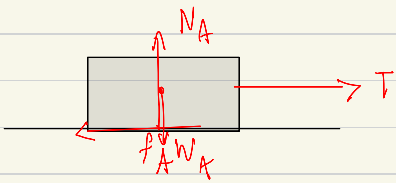


F4–15

Max P

static $P = \mu_s N$

FBD 1. Block A.



$\xrightarrow{+} \sum F_x = 0 \Rightarrow T - f_A = 0$

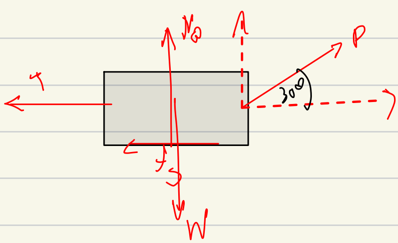
$T = f_A = \mu_s (N_A) \rightarrow T = (0.25)(490.5)$

$T = 122.625\ N$

$+\uparrow \sum F_y = 0 \Rightarrow N_A - W_A = 0$

$N_A = W_A = 50(9.81) = 490.5\ N$

FBD 2.



$\xrightarrow{+} \sum F_x = 0 \Rightarrow P\cos 30 - f_s - T = 0$

$+\uparrow \sum F_y = 0 \Rightarrow N_B - W + P\sin 30 = 0$

$$N_p = W - P\sin 30$$

from ΣF_x $\quad P\cos 30 - N_s(W - P\sin 30) - T = 0$

$$P\cos 30 - N_s W + N_s P\sin 30 - T = 0$$

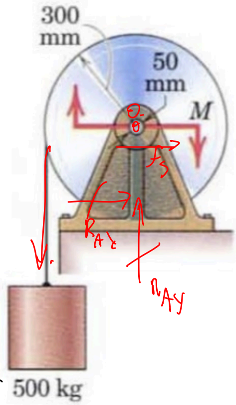
$$P\cos 30 + N_s P\sin 30 = T + N_s W$$

$$P = \frac{T + N_s W}{\cos 30 + N_s \sin 30}$$

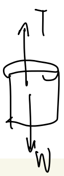
$$P = \frac{122.625 + (0.25)(490.5)}{\cos(30) + (0.25)\sin 30}$$

$$P = 247.48\ N$$

A torque M of 1510 Nm must be applied to the 50-mm diameter shaft of the hoisting drum to raise the 500-kg load at constant speed. Calculate the coefficient of friction for the bearing.



FBD 2



$+\uparrow \sum F_y = 0$

$T - W = 0$

$T = 500(9.81)$

$T = 4905$

$R_{Ay} = N$

$\xrightarrow{+} \sum F_x = 0 \Rightarrow R_{AX} + f_s = 0$

$R_{AX} = -f_s$

$R_{AX} = -(N_s)(R_{Ay}) \leftarrow$ Move left

$+\uparrow \sum F_y = 0 \Rightarrow R_{Ay} - T = 0$

$R_{Ay} = 4905$

$R_{AX} = -N_s(4905)$ $\quad f_s = -R_{AX}$

$+\circlearrowleft \sum M_{0,2} = 0 \Rightarrow +T(300) + f_s(50) - 1510 = 0$

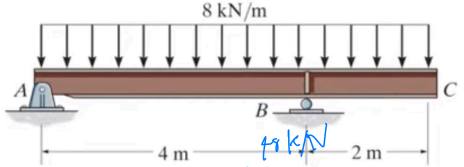
$T(0.3) + N_s(4905)(0.05) - 1510 = 0$

$N_s = \dfrac{38.5}{(4905)(0.05)}$

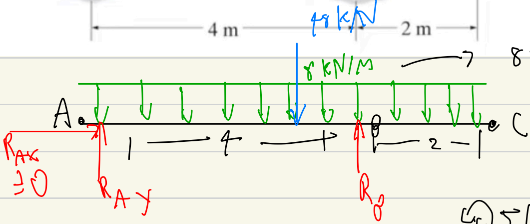
$\boxed{N_s = 0.157}$

Shear and Moment Diagram

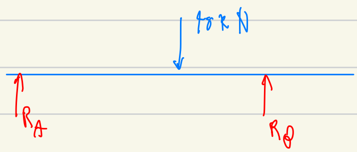
Draw the shear and moment diagrams for the overhang beam.



$8 \times 6 = 48$ N/M



$\sum M_{A,z} = 0 \Rightarrow R_{By}(4) - 48(3) = 0$

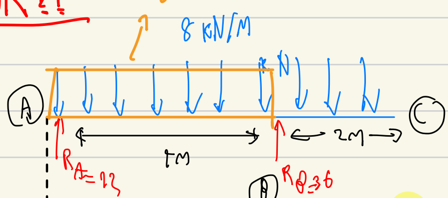


$R_{By} = 36$ kN

$+\uparrow \sum F_y = 0 \Rightarrow R_{Ay} + 36 - 48 = 0$

$R_{Ay} = 12$ kN

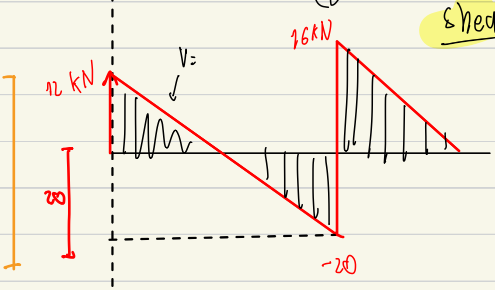
Trick!!

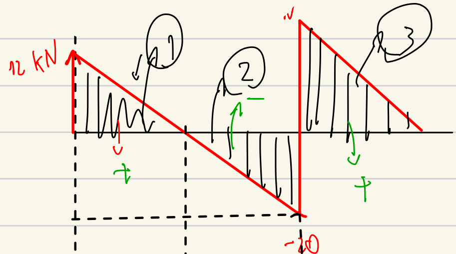


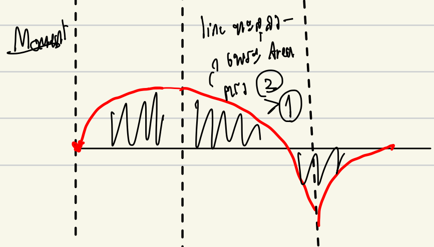
Load

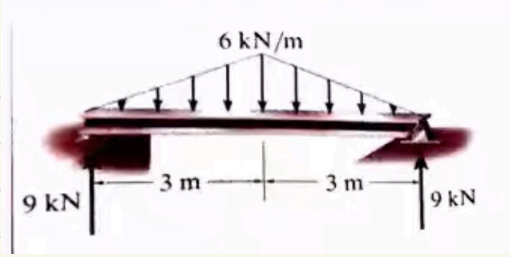
Moment

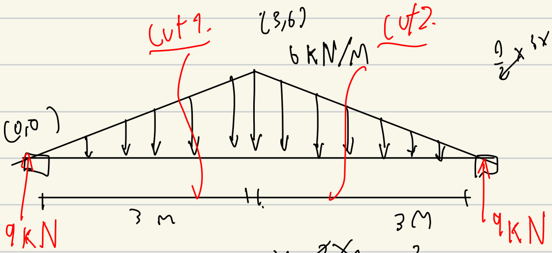
Shear









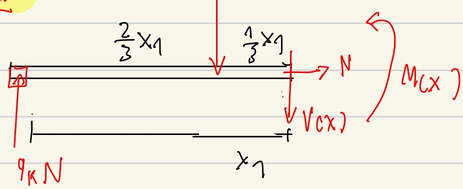


$\cancel{\frac{1}{2} \times 3 \times 6}$

$\frac{6-0}{3-0} = \quad y = 2x$

$\frac{1}{2} \times x_1 \times 2x_1 = x_1^2$

Cut 1.



$\circlearrowleft + \sum M_{x_1, z} = 0 \quad -9\,kN\,(x_1) + (x_1^2)\left(\frac{1}{3}x_1\right) + M(x) = 0$

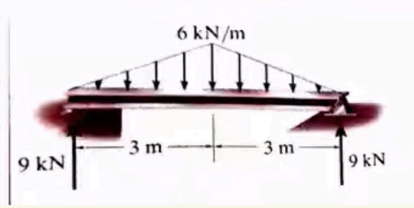
$-9x_1 + \frac{1}{3}x_1^3 + M(x) = 0$

$\boxed{M(x) = 9x - \frac{1}{3}x^3}$

$+\uparrow \sum F_y = 0 \Rightarrow 9\,kN - x^2 - V(x) = 0$

$\boxed{V(x) = 9 - x^2}$

* Symmetry => Cut 2 = cut just different direction



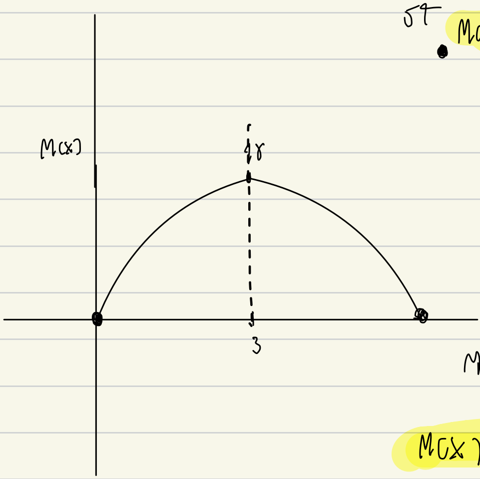
$V'(x) = -2x + 3$

$V''(x) = -2$

Moment Diagram

54

$M(x)\ 9x - \frac{1}{3}x^3$



$M'(x) = 9 - x^2$

$9 - x^2 = 0$

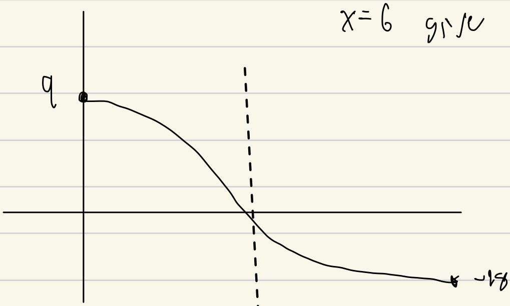
$x^2 = 9$

$x = \pm 3 \rightarrow$ Max at $x=3$

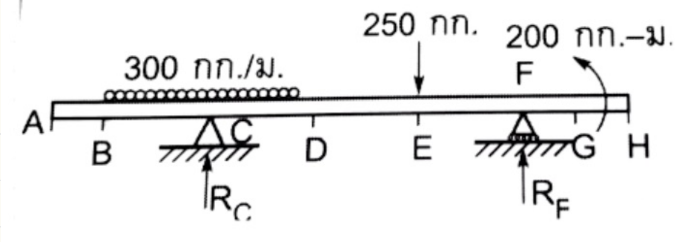
$M''(x) = -2x \rightarrow$ concave down

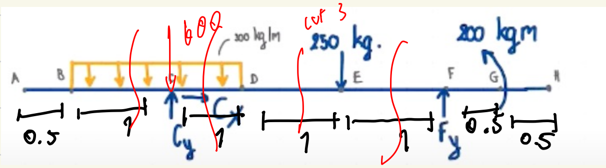
$M(x) = -\frac{2}{3}x^3 + 4x^2 - 6x + 18$

$x=6$ give $= -18$



05 SFD BMD





cut 1 cut 2 cut 4

$\xrightarrow{+} \sum F_x = 0$

$C_x = 0$

$\circlearrowleft + \sum M_{C,z} = 0$

$\Rightarrow -250(2) + F_y(3) + 200 = 0$

$\vec{F}_y = 100$ kg (↑)

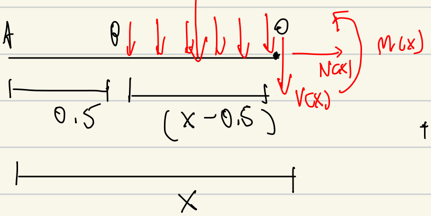
Cut 1.

$\circlearrowleft + \sum \vec{M}_{F,z} = 0$

$\Rightarrow -C_y(3) + 250(1) + 200 + 600(3) = 0$

$\vec{C}_y = 750$ kg

load = $(x-0.5)(300)$



$+\uparrow \sum \vec{F}_y = 0 \Rightarrow -V(x) - 300(x-0.5) = 0$

$\vec{V} = -300(x-0.5)$

$\circlearrowleft + \sum M_{O,z} = 0 \Rightarrow M(x) + \left(300(x-0.5)\cdot\frac{1}{2}(x-0.5)\right) = 0$

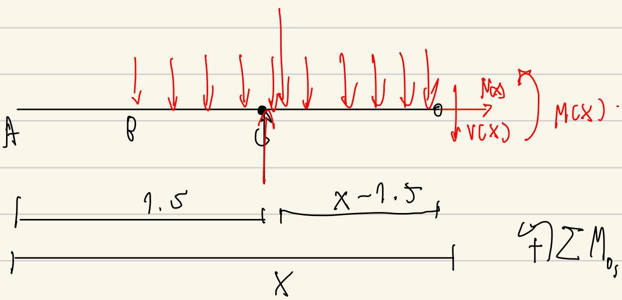
$M(x) = -\frac{300(x-0.5)^2}{2}$

at B $x = 0.5 \Rightarrow \vec{V}_B = 0$, $\vec{M}_B = 0$

at C : $x = 1.5 \Rightarrow \vec{V}_C = -300$ kg ↑, $\vec{M}_C = -150$ kg m ↺

Cut 2

Total load = $300(x - 0.5)$



$\circlearrowleft + \sum M_{O,z} = 0 \Rightarrow M(x) - C_y(x - 1.5) + 300(x - 0.5)\dfrac{(x - 0.5)}{2} = 0$

$+\uparrow \sum F_y = 0 \Rightarrow -V(x) - 300(x - 0.5) + 750 = 0$

$\vec{M}(x) = 750(x - 1.5) - 300\dfrac{(x - 0.5)^2}{2}$

$\vec{V}(x) = 750 - 300(x - 0.5)$

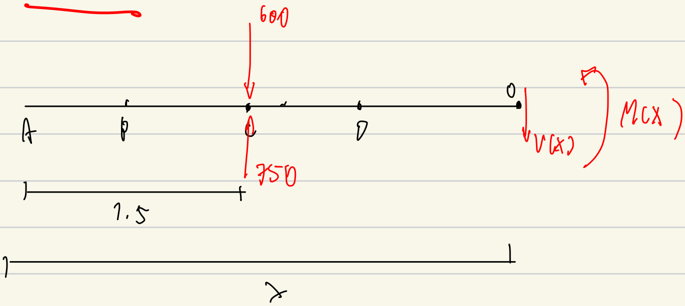
at C when $x = 1.5 \Rightarrow \vec{V}_C = 750 - 300(1.5 - 0.5) = 450$ Kg

$\vec{M}_C = -150$ KgM

at d $x = 2.5$: $\vec{V}_D = 750 - 300(2.5 - 0.5) = 150$ Kg $= 150$ Kg

$\vec{M}_D = 150$ KgM

Cut 3.



$+\uparrow \sum \vec{F}_y = 0$

$-V(x) - 600 + 750 = 0$

$\vec{V} = 150$ Kg

$\sum \vec{M}_O = 0 \circlearrowleft; \Rightarrow M(x) - 750(x - 1.5) + 600(x - 1.5) = 0$

$\vec{M} = 150(x - 1.5)$

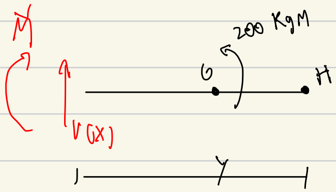
at D: $x = 2.5$, $\vec{V}_D = 150$ Kg, $\vec{M}_D = 150$ KgM

at E: $x = 3.5 \Rightarrow \vec{V}_E = 150$ Kg

$\vec{M}_E = 300$ KgM

ช่วง GH ไม่มีแรงภายนอก แรงเฉือนและโมเมนต์ดัดเป็น 0.

Cut 4. ตัดจากซ้าย ระบุได้ช่วง FG ($0 \le y < 1$) ให้ y เป็นระยะที่วัดจากจุด H ไปทางซ้ายของคาน



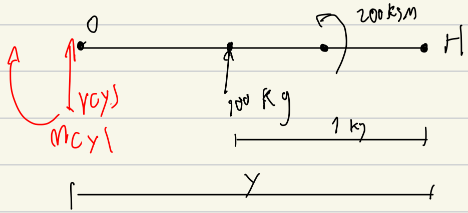
$+\uparrow \sum \vec{F}_y = 0; \Rightarrow \vec{V} = 0$

$\sum M_G = 0 \; +\circlearrowleft;$

$= 200 - \vec{M}(y) = 0$

$\vec{M}(y) = 200$ KgM

Cut 5. ช่วง EF ($1 < y < 2$)



$\sum \vec{F}_y = 0 \; \uparrow +;$

$\vec{V} + 100 \text{ kg} = 0$

$\vec{V} = -100 \text{ kg}$

$\circlearrowleft + \sum \vec{M}_O = 0; \; -\vec{M} + 100(y-1) + 200 = 0$

$\vec{M} = 200 + 100(y-1)$

at F; $y = 1$

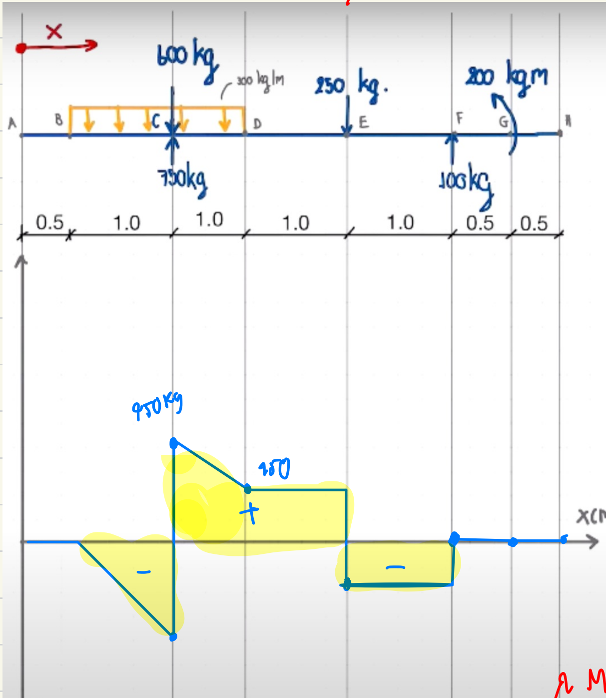
$\vec{M}_F = 200 + (100)(1-1) = 200$ KgM

at E; $y = 2$ m

$\vec{M}_E = 200 + 100(2-1) = 300$ KgM

Graph.

shear



$\vec{V}_{AC} = -300(x-0.5)$; สมการเส้นตรง

$\vec{V}_B = 0$

$\vec{V}_C = -300$ kg

$\vec{V}_{CD} = 750 - 300(x-0.5)$: สมการเส้นตรง

$\vec{V}_C = 450$ kg

$\vec{V}_D = 150$ kg

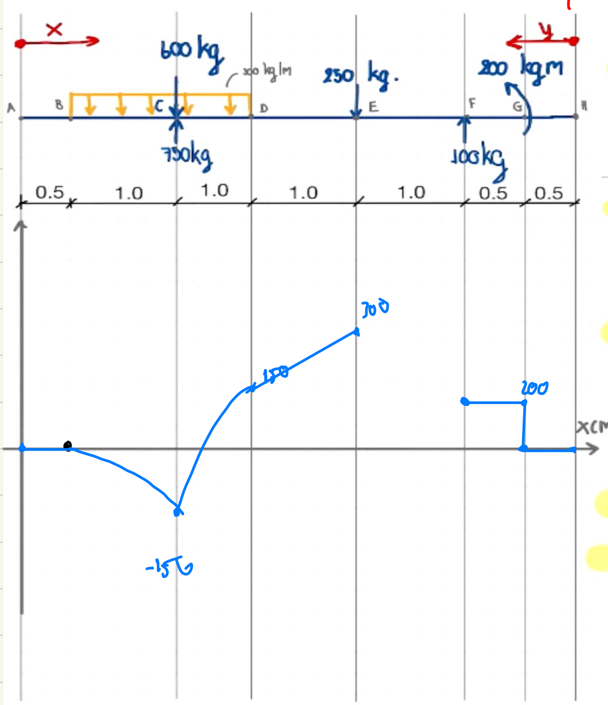
$\vec{V}_{DE} = 150$ kg

ช่วง GH ไม่มีแรงภายนอก ∴ แรงเฉือน = 0 !

$\vec{V}_{FG} = 0$

$\vec{V}_{EF} = -100$ kg

Moment



$\vec{M}_{AC} = -300\,\frac{(x-0.5)^2}{2}$: พาลาโบล่า

$\vec{M}_B = 0$

$\vec{M}_C = -150$ kgm

$\vec{M}_{CD} = 750(x-1.5) - 300\,\frac{(x-0.5)^2}{2}$ ∴ สมการพาลาโบล่า

$\vec{M}_C = -150$ kgm

$\vec{M}_D = 150$ kgm

$\vec{M} = 150(x-1.5)$

$\vec{M}_D = 150$ kgm

$\vec{M}_E = 300$ kgm

ช่วง GH $\vec{M} = 0$

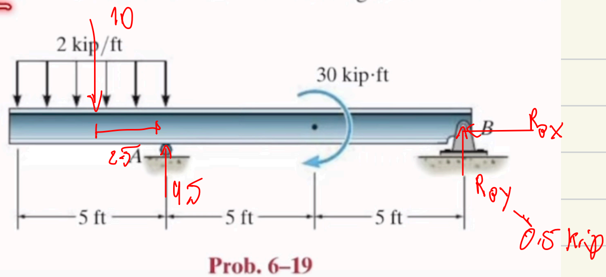
$\vec{M}_{FG} = 200$ kgm

$\vec{M}_{EF} = 200 + 100(y-1)$

$\vec{M}_F = 200$ kg m

$\vec{M}_E = 300$ kgm

6–19. Draw the shear and moment diagrams for the beam.



Prob. 6–19

$$\circlearrowleft + \Sigma M_{B,z} = 0 \Rightarrow -R_{Ay}(10) - 30 \text{ kip ft} + 10(12.5) = 0$$

$$-R_{Ay}(10) = -95$$

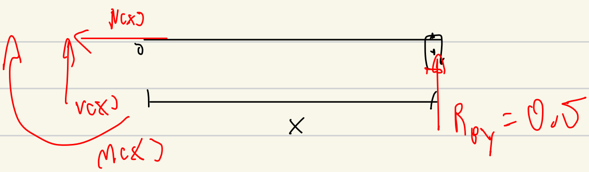
$\xrightarrow{+} \Sigma F_x = 0 \quad R_{Bx} = 0 \qquad R_{Ay} = 9.5 \text{ kip}$

$$\circlearrowleft + \Sigma M_{A,z} = 0 \Rightarrow 10(2.5) - 30 \text{ kip ft} + R_{By}(10) = 0$$

$$R_{By}(10) = 5$$

$$R_{By} = 0.5 \text{ kip}$$

Cut 1.



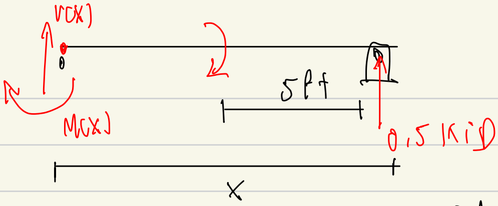
$+\uparrow \Sigma F_y = 0$

$\Rightarrow V(x) + 0.5 = 0$

$V(x) = -0.5$

$\circlearrowleft + \Sigma M_{o,z} = 0 \quad 0.5(x) - M(x) = 0$

$M(x) = 0.5(x)$



at O A

$+\uparrow \sum F_y = 0 \Rightarrow V(X) + 0.5 = 0$

$V(X) = -0.5$ kip

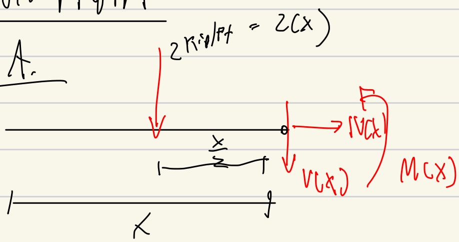
$\circlearrowleft + \sum M_{O,z} = 0 \Rightarrow -M(X) - 30 + 0.5(X) = 0$

$-M(X) = -0.5(X) + 30$

$M(X) = 0.5(X) - 30$

cut from right

At O to A.

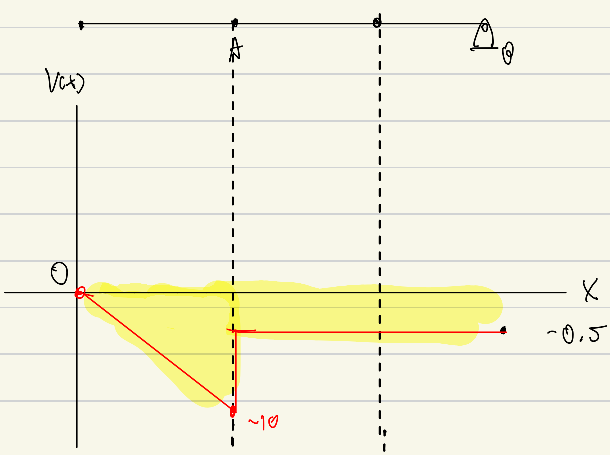


$+\uparrow \sum F_y = 0 \Rightarrow -V(X) - 2X = 0$

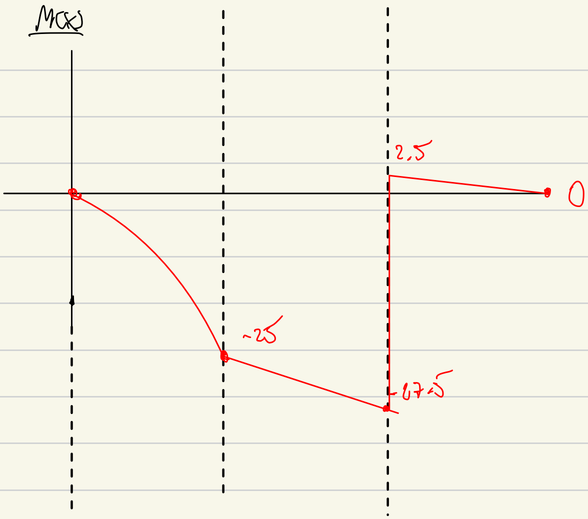
$V(X) = -2X$

$\circlearrowleft + \sum M_{O,z} = 0 = 2x\left(\frac{X}{2}\right) + M(X) = 0$

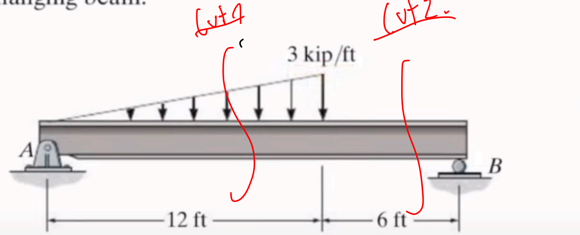
$M(X) = -X^2$



$V(X) = -0.5$



*6–20. Draw the shear and moment diagrams for the overhanging beam.

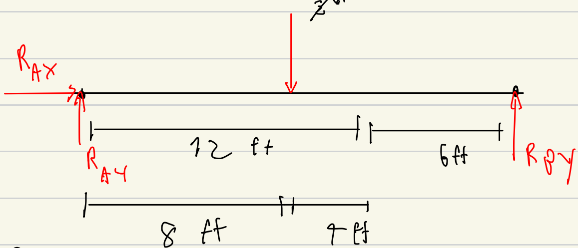


Prob. 6–20

(12, 3) (0, 0)

$$\frac{3-0}{12-0} = \frac{1}{4}$$

$\frac{1}{2}(12)(3) = 18$ kip/ft



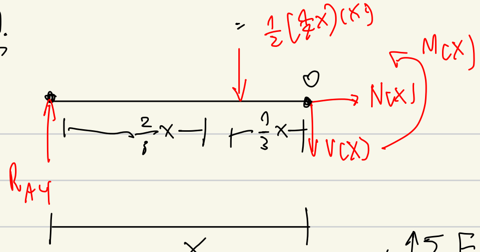
$\circlearrowleft + \sum M_{A,z} = 0 \Rightarrow R_{By}(18) - 18(8) = 0$

$R_{By} = 8$ kip/ft

$+\uparrow \sum F_y = 0 \quad 8 - 18 + R_{Ay} = 0$

$R_{Ay} = 10$ kip/ft

Cut 1.



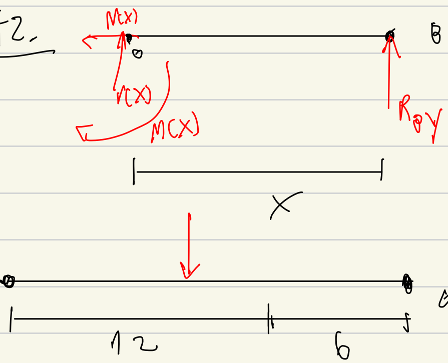
$$+\uparrow \Sigma F_y = 0 \Rightarrow \quad 10 - \frac{1}{2}\left(\frac{1}{4}x\right)(x) - V(x) = 0$$

① V $\quad V(x) = 10 - \frac{1}{8}x^2$

$$\circlearrowleft + \Sigma M_{O,z} = 0 : \quad -10(x) + \frac{1}{8}x^2\left(\frac{1}{3}x\right) + M(x) = 0$$

$M(x) = 10x - \frac{1}{24}x^3$ ① M

Cut 2.



$$\Rightarrow +\uparrow \Sigma F_y = 0 \Rightarrow R_{By} + V(x) = 0$$

$$V(x) = -R_{By}$$

$V(x) = -8$ kip/ft ② V

$$\circlearrowleft + \Sigma M_{O,z} = 0 \Rightarrow R_{By}(x) - M(x) = 0$$

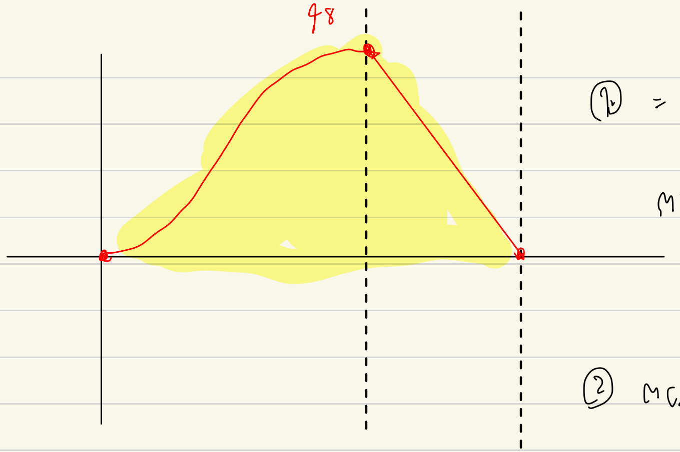
$M(x) = 8x$ ② M

V(x) shear graph.

① $V(x) = 10 - \frac{1}{8}x^2$

② $V(x) = -8$ kip

M(x)



$10 - \frac{1}{8}x^2 = 0$

(1) $\Rightarrow M(x) = 10(x) - \frac{1}{24}x^3$

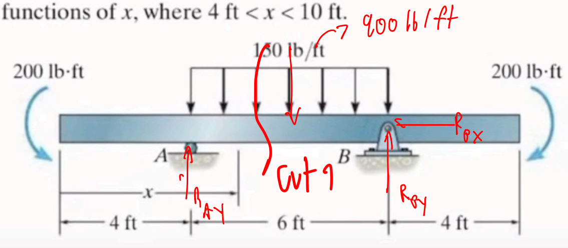
$M'(x) = 10 - \frac{1}{8}x^2$

$M''(x) = -\frac{1}{4}$

concave down

(2) $M(x) = 8(x)$

6–25. Draw the shear and moment diagrams for the beam and determine the shear and moment in the beam as functions of x, where $4 \text{ ft} < x < 10 \text{ ft}$.



Prob. 6–25

$\xrightarrow{+} \sum F_x = 0 \quad R_{Bx} = 0$

$+\circlearrowleft \sum M_{B,z} = 0$

$200 - 200 - R_{Ay}(6) + 900(3) = 0$

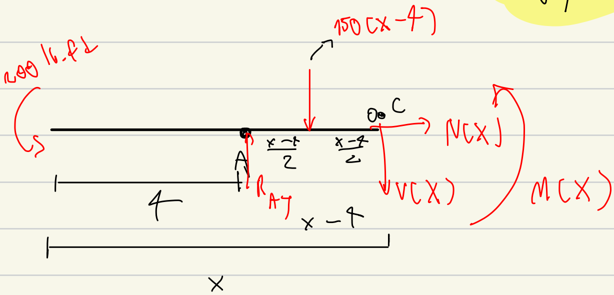
$-R_{Ay}(6) = -2700$

$R_{Ay} = 450$

$+\uparrow \sum F_y = 0 \Rightarrow R_{Ay} + R_{By} - 900 = 0$

$450 + R_{By} - 900 = 0$

$R_{By} = 450$



$+\uparrow \sum F_y = 0$

$\Rightarrow -(150(x-4)) + 450 - V(x) = 0$

(A) $V(x) = 450 - 150x + 600$

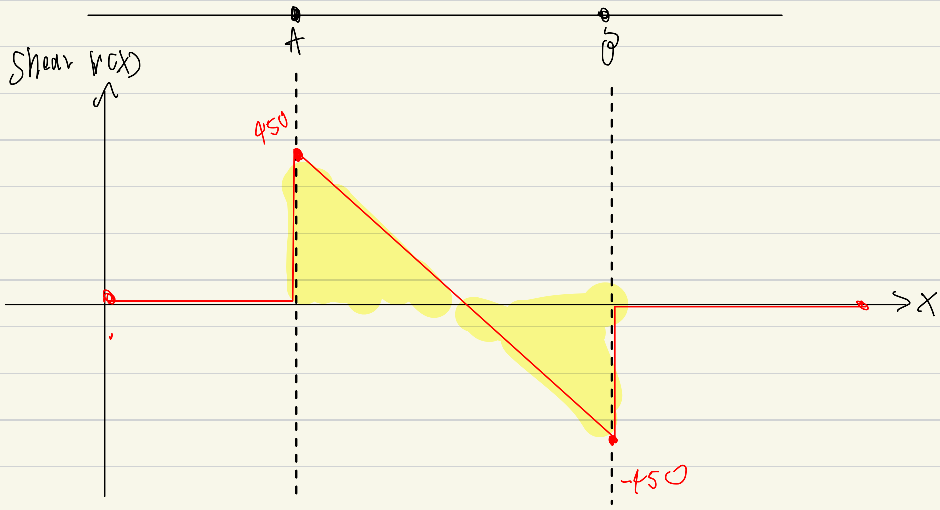
$V(x) = -150x + 1050$

$4) \overset{+}{\circlearrowleft} \sum M_{C,Z} = 0 \Rightarrow -450(x-4) + 150(x-4)\left(\frac{x-4}{2}\right) + 200 + M(x) = 0$

B

$$M(x) = 450(x-4) - 75(x-4)^2 - 200$$

graph



Moment.

$M(x) = 450(x-4) - 75(x-4)^2 - 200$

Max when $M'(x) = 0$

$450 - 150(x-4) = 0$

$x - 4 = 3$

$x = 7$

$M(7) =$

Determine the values and draw the diagrams for shear force and bending moment due to the imposed load on overhanging beam shown in figure 5-4(a) and find the position of point of contra-flexure, if any.

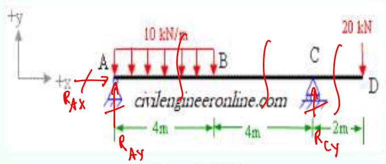
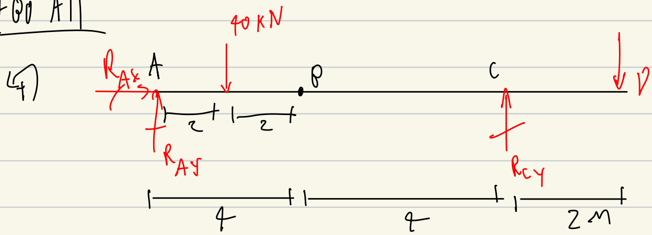


Figure 5-4(a)

FBD All



4) $\sum M_{A,z} = 0 \Rightarrow -40(2) + R_{cy}(8) - 20(10) = 0$

$\Rightarrow 8R_{cy} = 280$

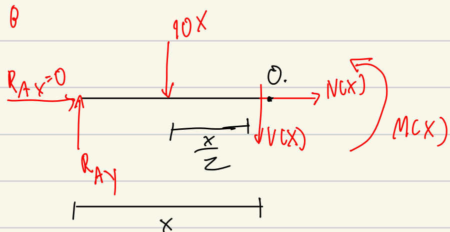
$\Rightarrow R_{cy} = 35 \text{ kN}$

$+\uparrow \sum F_y = 0 \Rightarrow R_{Ay} - 40 \text{ kN} + 35 - 20 = 0$

$R_{Ay} = 25 \text{ kN}$

$\xrightarrow{+} \sum F_x = 0 \Rightarrow R_{Ax} = 0$

Cut 1. A to B



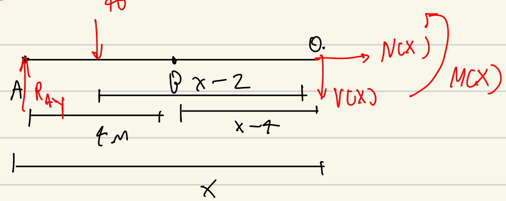
$+\uparrow \sum F_y = 0 \qquad 25 - 10X - V(x) = 0$

$V(x) = 25 - 10X$

$\sum M_{O,z} = 0 \Rightarrow M(x) + 10(x)\left(\frac{x}{2}\right) - 25(x) = 0$

$M(x) = 25X - 5X^2$

Cut 2. B to C



$$+\uparrow \sum F_y = 0 \Rightarrow R_{Ay} - 40 - V(x) = 0$$

$$\boxed{V(x) = -15}$$

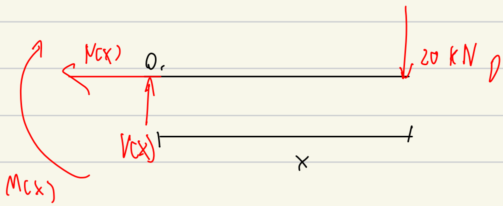
$$\circlearrowleft \sum M_{0,z} = 0$$

$$M(x) + 40(x-2) - 25(x) = 0$$

$$M(x) = 25x - 40x + 80$$

$$\boxed{M(x) = -15x + 80}$$

Cut 3 from right to left.



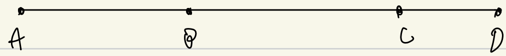
$$+\uparrow \sum F_y = 0 \Rightarrow V(x) - 20 = 0$$

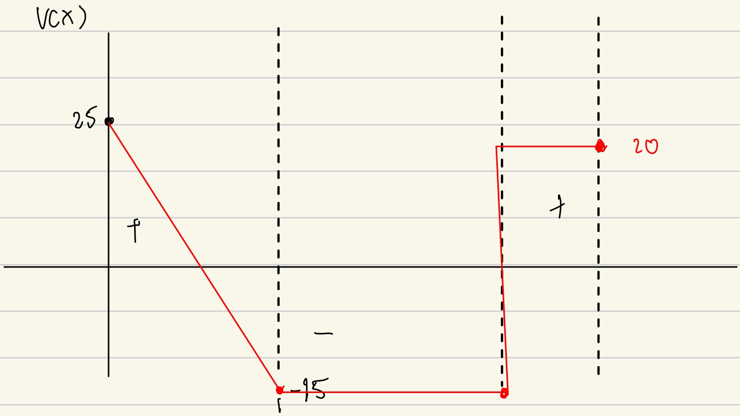
$$\boxed{V(x) = 20}$$

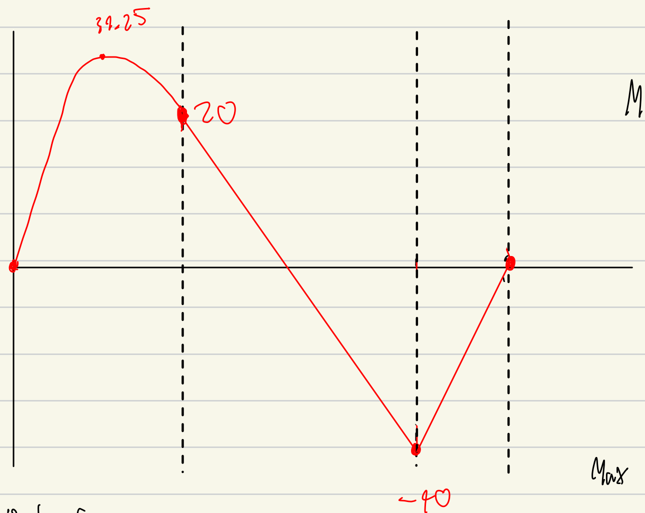
$$\circlearrowleft \sum M_{0,z} = 0 \Rightarrow -M_{(x)} - 20(x) = 0$$

$$\boxed{M(x) = 20(x)}$$

Diagram







V(x) A to B

$= 25 - 10x$

$= 25 - 10(0)$

$= 25$

at $x = 4$

$= 25 - 10(4)$

$=$

V(x) B to C

$= -15$

V(x) at C to D = 20

M(x) at A to B

$25x - 5x^2$ Concave down

at $x = 0 = 0$

at $x = 4$

$= 20$

Max $M'(x) = 0$

$25 - 10x = 0$

$10x = 25$

$x = 2.5$

M(x) at B to C

$= -15x + 80$

at $x = 4 = 20$

at $x = 8 = -40$

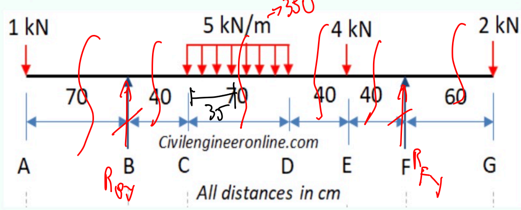
M(x) at C to D

$= M(x) = -20x$

Problem 5-5

Determine the values and draw the diagrams for shear force and bending moment due to the imposed load on overhanging beam shown in figure 5-5(a) and find the position of point of contra-flexure, if any.

Convert to meter



$$↺\sum M_{B,z}=0 \Rightarrow 1(70)-350(75)-4(150)+R_{Fy}(190)-2(250)=0$$

$$R_{Fy}(190)=27280$$

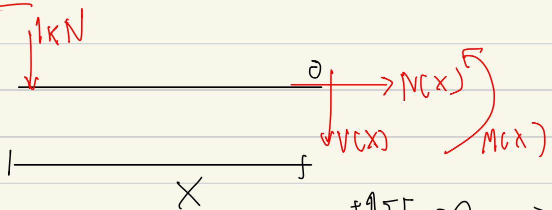
$$R_{Fy}=143.58 \text{ kN}$$

$$+\uparrow\sum F_y=0 \Rightarrow -1+R_{By}-350-4+143.58-2=0$$

$$R_{By}=213.42 \text{ kN}$$

Cut 1.

A to B



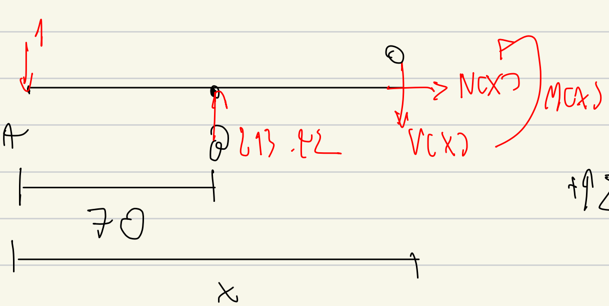
$$+\uparrow\sum F_y=0 \Rightarrow -V(x)-1=0$$

$$V(x)=-1$$

$$↺\sum M_{O,z}=0 \Rightarrow M(x)+1(x)=0$$

$$M(x)=-x$$

Cut 2. B to C



$$+\uparrow\sum F_y=0 \Rightarrow -V(x)-1+213.42=0$$

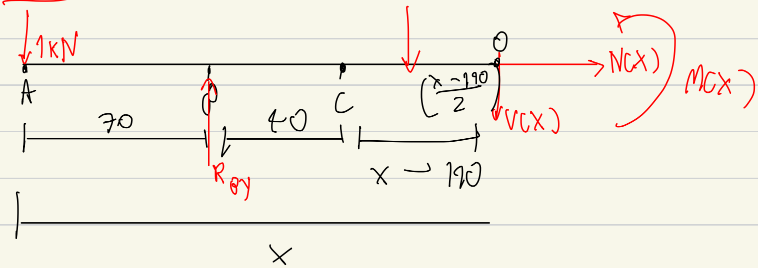
$$-V(x)=-212.42$$

$$V(x)=212.42$$

$$\circlearrowleft + \sum M_{(x)} = 0 \Rightarrow M(x) - 213.42(x-70) + x = 0$$

$$M(x) = 212.42x - 14939.4$$

Cut 3. c to D.



$$+\uparrow \sum F_y = 0 \Rightarrow -1 + 213.42 - 5(x-110) - V(x) = 0$$

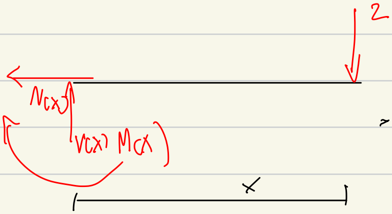
$$V(x) = -5x + 550 - 1 + 213.42$$

$$V(x) = -5x + 712.42$$

$$\circlearrowleft + \sum M_{0,z} = 0 \Rightarrow M(x) + 5(x-110)\left(\frac{x-110}{2}\right) - R_{By}(x-70) + x = 0$$

$$M(x) = 213.42x - 14939.4 - x - \frac{5}{2}(x-110)^2$$

Cut 4 from right side. G to F



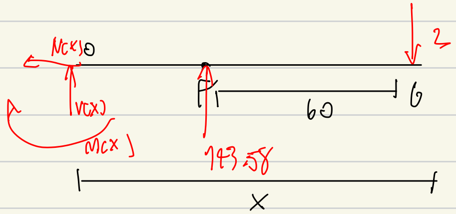
$$+\uparrow \sum F_y = 0 \Rightarrow V(x) - 2 = 0$$

$$V(x) = 2$$

$$\circlearrowleft + \sum M_{(x)} = 0 \Rightarrow -M(x) - 2(x) = 0$$

$$M(x) = -2x$$

Cut 5 from right side: F to E



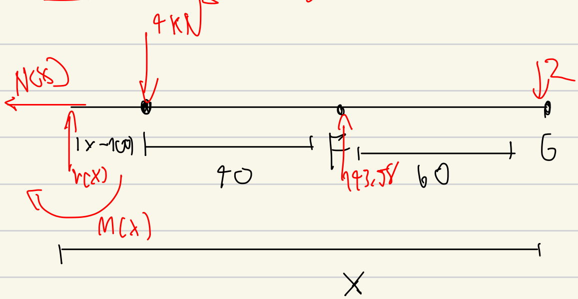
$$+\uparrow \sum F_y = 0 \Rightarrow V(X) + 143.58 - 2 = 0$$

$$V(X) = -141.58 \text{ N}$$

$$\circlearrowleft \sum M_{0,z} = 0 \Rightarrow -2(X) + 143.58(X-60) - M(X) = 0$$

$$M(X) = 141.58X - 8614.8$$

Cut 6. E to D



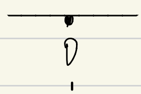
$$+\uparrow \sum F_y = 0 \Rightarrow V(X) - 4 + 143.58 - 2 = 0$$

$$V(X) = -137.58$$

$$\circlearrowleft \sum M_{(x)} = 0 \Rightarrow -M(X) - 4(X-100) + 143.58(X-60) - 2(X) = 0$$

$$M(X) = -4X + 400 + 143.58X - 8614.8 - 2X = 0$$

$$M(X) \quad 137.58X - 8214.8$$



Beam loaded as shown in Fig. P-404.

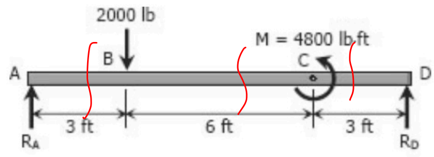


Figure P-404

$$\circlearrowleft + \sum M_{A,z} = 0 \Rightarrow -2000(3) + 4800 + R_D(12) = 0$$

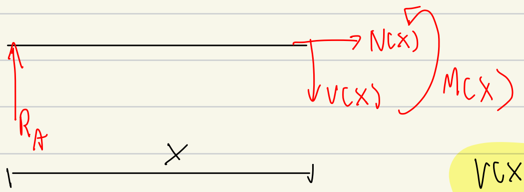
$$R_D(12) = 1200$$

$$R_D = 100 \text{ lb}$$

$$+\uparrow \sum F_y = 0 \Rightarrow R_{Ay} - 2000 + 100 = 0$$

$$R_{Ay} = 1900 \text{ lb}$$

Cut 1. A to B

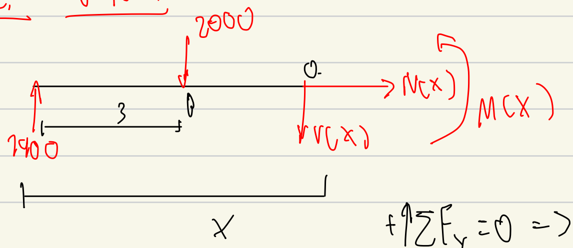


$$V(x) = 1900 \text{ lb}$$

$$\circlearrowleft + \sum M_{(x)} = 0 \Rightarrow -1900(x) + M(x) = 0$$

$$M(x) = 1900(x)$$

Cut 2. B to C.



$$+\uparrow \sum F_y = 0 \Rightarrow 1900 - 2000 - V(x) =$$

$$V(x) = -100 \text{ lb}$$

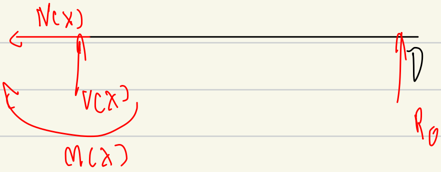
$$\circlearrowleft + \sum M_{(x)} = 0 \Rightarrow -1900(x) + M(x) + 2000(x-3) = 0$$

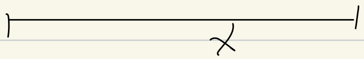
$$M(x) = 1900x - 2000x + 6000$$

$\downarrow$

$M(x) = -100x + 6000$

Cut 3. D to C



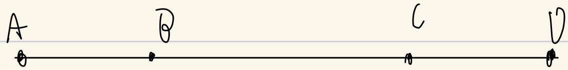


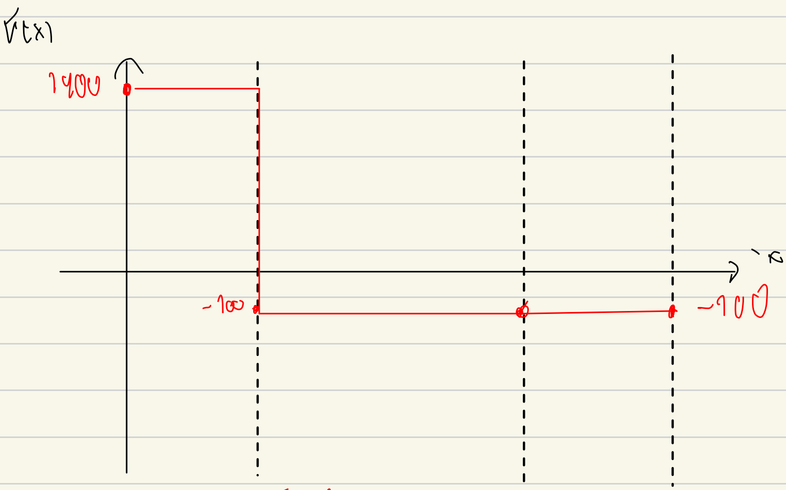
$+\uparrow \sum F_y = 0 \Rightarrow R_D + V(x) = 0$

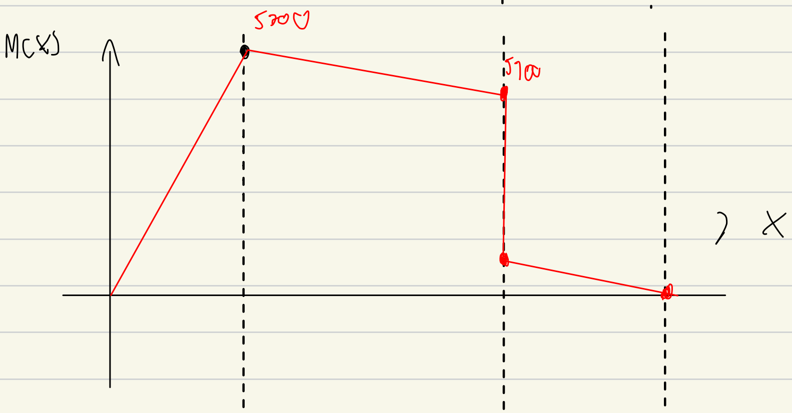
$V(x) = -100$

$\circlearrowleft + \sum M_{cut} = 0 \Rightarrow -M(x) + R_D(x)$

$\Rightarrow M(x) = 100(x)$







Problem 406

Beam loaded as shown in Fig. P-406.

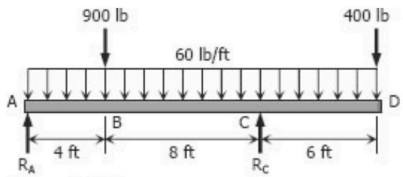
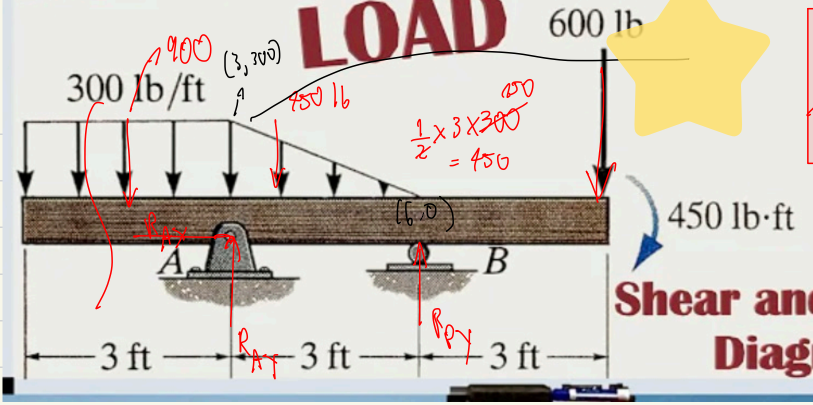


Figure P-406



Note Moment = Integrate avg shear

$$\frac{300-0}{3-6} = -100$$

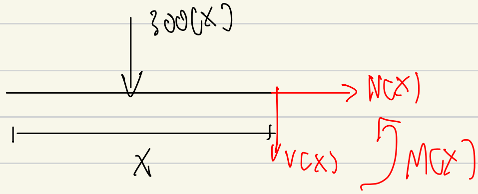
$$\circlearrowleft + \sum M_{A,z} = 0 \Rightarrow 900(1.5) - 450(1) + R_{By}(3) - 600(6) - 450$$

$$R_{By} = 1050 \text{ lb}$$

$$+\uparrow \sum F_y = 0, \quad -900 - 450 + R_{Ay} + 1050 - 600 = 0$$

$$R_{Ay} = 1900 \text{ lb}$$

Cut 1 from 0 to A.



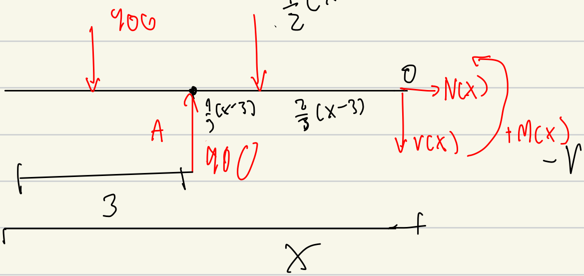
$$+\uparrow \sum F_y = 0 \Rightarrow -300x - V(x) = 0$$

$$V(x) = -300x$$

$$\circlearrowleft + \sum M_{(x)} = 0 \Rightarrow M(x) + 300(x)\left(\frac{x}{2}\right) = 0$$

$$M(x) = -150x^2$$

Cut 2 from A to B



$$\frac{1}{2}(-100x^2+300x+300x-900)=0$$
$$\frac{1}{2}(-100x^2+600x-900)=0$$
$$-50x^2+300x-450$$

$$+\uparrow\Sigma F_y=0$$

$$-V(x)-900+900-\frac{1}{2}(x-3)(-100x+300)=0$$

$$V(x)=-50x^2+300x-450$$

$$\circlearrowleft + \Sigma M_{(x)}=0 \Rightarrow$$

$$900(x-1.5)-900(x-3)+\frac{1}{2}(x-3)(-100x+300)\left(\frac{2}{3}x-2\right)+M(x)=0$$

$$M(x)+\cancel{900x}-1350-\cancel{900x}+2700+\frac{1}{2}(x-3)\left(-\frac{200}{3}x^2+200x+200x-600\right)=0$$

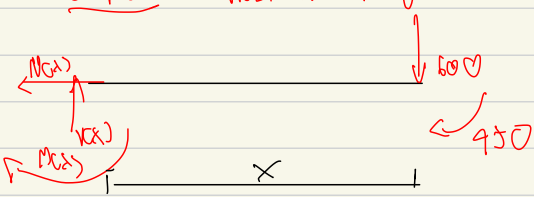
$$M(x)+1350+\frac{1}{2}(x-3)\left(-\frac{200}{3}x^2+400x-600\right)=0$$

$M(x)$

$$M(x)=-\frac{100}{3}x^3+300x^2-900x-450$$

หลัง solve สมการข้างบน

Cut 3 from end to B



$$+\uparrow\Sigma F_y=0 \Rightarrow -600+V(x)=0$$

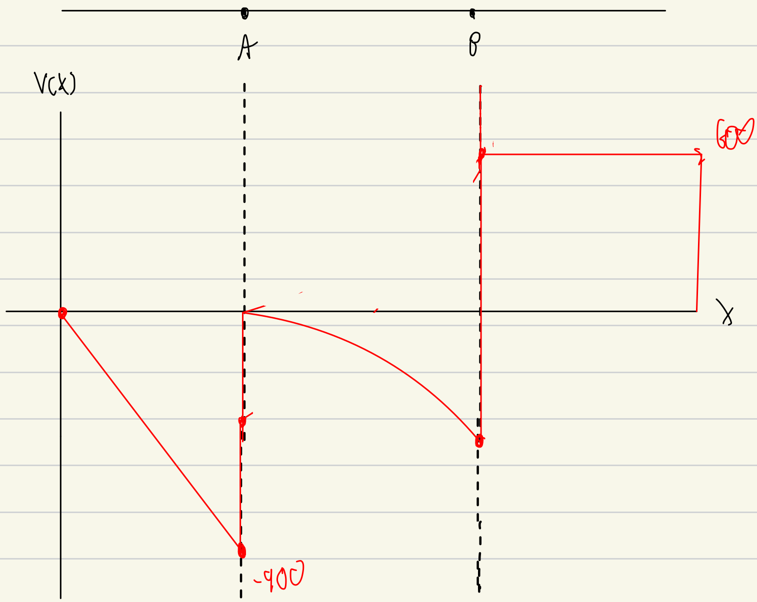
$$V(x)=600$$

$$\circlearrowleft + \Sigma M_{(x)}=0 \Rightarrow -M(x)-600(x)-450=0$$

$$M(x)=-600x-450$$

Graph.

Shear.



For A to B

Shear max/min at $V(x) = 0$

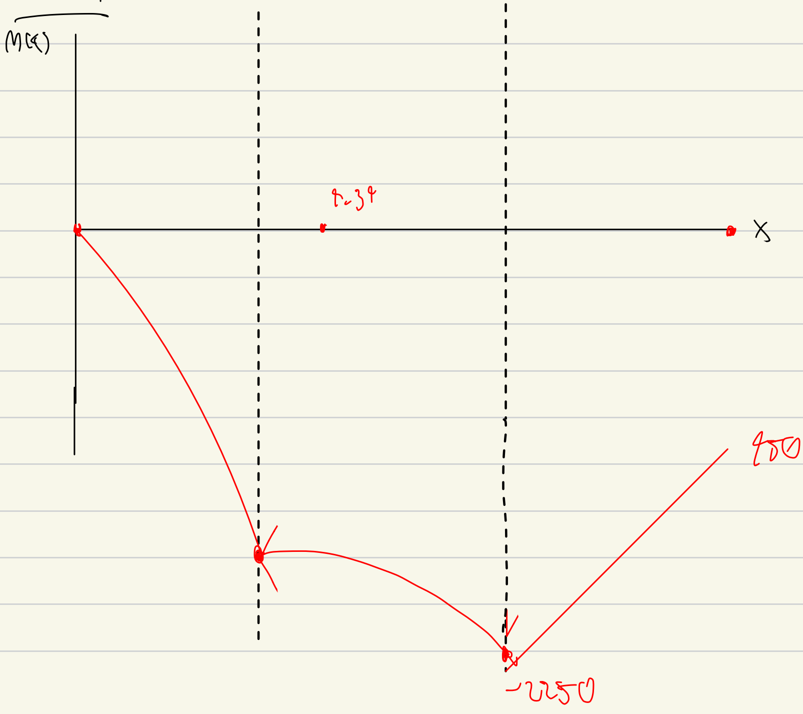
$$100x - 150 = 0$$

$$x = \frac{150}{100}$$

$x = 1.5$

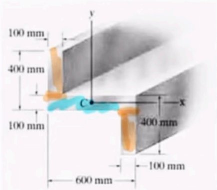
min

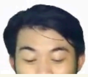
Moment

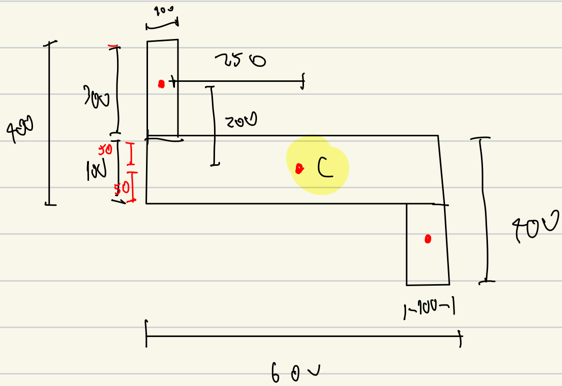


Centroid and Moment of Inertia.

21. Determine the moments of inertia for the cross-sectional area of the member showing Fig. about the x and y centroidal axes.







A, d

$$I_x = \bar{I}_x + Ad^2 = \frac{1}{12}bh^3 + Ad^2$$

$$= \frac{1}{12}(100)(300)^3 + (100)(300)(200)^2$$

$$= 1.425(10^9)\ mm^4$$

$$I_y = \bar{I}_y + Ad^2$$

$$= \frac{1}{12}(300)(100)^3 + (300)(100)(250)^2$$

$$I_y = 1.9(10^9)\ mm^4$$

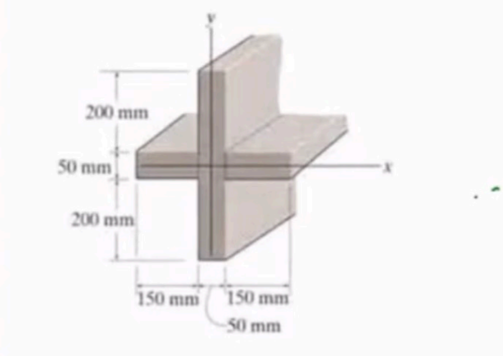
ของ C.

$$I_x = \frac{1}{12}(600)(100)^3 = 0.05(10^9)\ mm^4$$

$$I_y = \frac{1}{12}(600)^3(100) = 1.80(10^9)\ mm^4$$

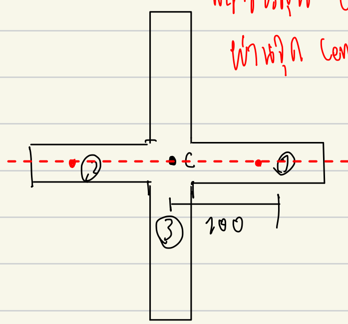
$$\Sigma I_x = 2(1.425(10^9)) + 0.05(10^9), \quad \Sigma I_y = 2(1.9(10^9)) + 1.80(10^9)$$

22. Determine the moment of inertia of th



$I_x + I_y$

Paravell Axis theorem ใช้ได้ต่อเมื่อ แกนหมุน (x or y axis) ไม่ผ่านจุด Centroid.



1 and 2 are both symmetry

1

$$I_x = \frac{bh^3}{12} + AD^2 = \frac{150\,(50)^3}{12}$$

$$I_y = \frac{b^3h}{12} + AD^2 = \frac{(150)^3(50)}{12} + (150)(50)(100)^2$$

3

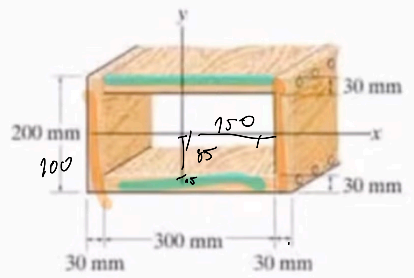
$$I_x = \frac{bh^3}{12} = \frac{(50)(400)^3}{12} = 2.66 \times 10^8 \text{ mm}^4$$

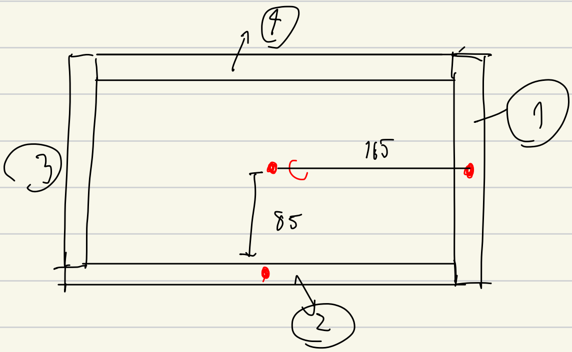
$$I_y = \frac{(50)^3(400)}{12} = 4.16 \times 10^6 \text{ mm}^4$$

$$I_x = 2\left(\frac{150\,(50)^3}{12}\right) + 2.66 \times 10^8 = 383 (10^6) \text{ mm}^4$$

$$I_y = 2\left(\frac{(150)^3(50)}{12} + (150)(50)(100)^2\right) + 4.16 \times 10^6 = 183 \times (10^6) \text{ mm}^4$$

23. Determine the moment of inertia of the beam's cross-sectic





1 = 3 and 2 = 4 due to Symmetry.

1 ⇒

$$I_x = \frac{(30)(200)^3}{12} + A(0)^2 =$$

$$I_y = \frac{(30)^3(200)}{12} + (30)(200)(165)^2$$

2

$$I_x = \frac{(300)(30)^3}{12} + (300)(30)(85)^2$$

$$I_y = \frac{(300)^3(30)}{12} + A(0)^2$$

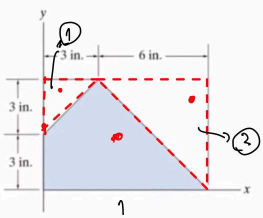
Total Inertia ⇒ $I_x = 2\left(\frac{(30)(200)^3}{12}\right) + 2\left(\frac{(300)(30)^3}{12} + (300)(30)(85)^2\right)$

$$I_y = 2\left(\frac{(30)^3(200)}{12} + (30)(200)(165)^2\right) + 2\left(\frac{(300)^3(30)}{12}\right)$$

$$I_x \approx 1.714 \times 10^8 \ mm^4$$

$$I_y \approx 4.626 \times 10^8 \ mm^4$$

25. Determine the moment of inertia of the composite area about the x axis.



$$I_{x\,total} = I_{total} - (I_1 + I_2)$$

$$I_x\,total = \frac{(9)(6)^3}{12} + (9)(6)(3)^2$$

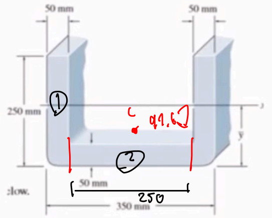
$$I_{x \Rightarrow 1} \Rightarrow \frac{(3)(3)^3}{36} + \left(\frac{1}{2}\right)(3)(3)(5)^2$$

$$I_{x} \Rightarrow 2 \Rightarrow \frac{(6)(6)^3}{36} + \frac{1}{2}(6)(6)(4)^2$$

$$= I_x\,total = \left(\frac{(9)(6)^3}{12} + (9)(6)(3)^2\right) - \left(\left(\frac{(3)(3)^3}{36} + \left(\frac{1}{2}\right)(3)(3)(5)^2\right) + \left(\frac{(6)(6)^3}{36} + \frac{1}{2}(6)(6)(4)^2\right)\right)$$

$$I_x\,total \approx 209.25 \ in^4$$

26. Determine the location $\bar{y}$ of the centroid of the channel's cross-sectional area and then calculate the moment of inertia of the area about this axis.



$$\bar{y} = \frac{2(\tilde{y}_{A_1} A_1) + \tilde{y}_{A_2} A_2}{A_1 + A_2 + A_3} = \frac{2(125(250(50))) + 25(250(50))}{2(250(50)) + 250(50)}$$

$$\bar{y} = 99.67$$

$I = I_c + Ad^2 \Rightarrow$ segment : 1+2

$$= 2\left(\frac{(50)(250)^3}{12}\right) + 2((50)(250))(d = 33.33)^2$$

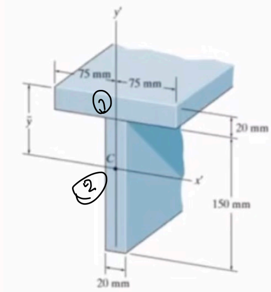
+

Segment 3. $= \dfrac{(250)(50)^3}{12} + (250)(50)(66.67)^2$

$$I_{1+2} + I_3 = 157.99(10)^6 + 58.16(10)^6 \text{ mm}^4$$

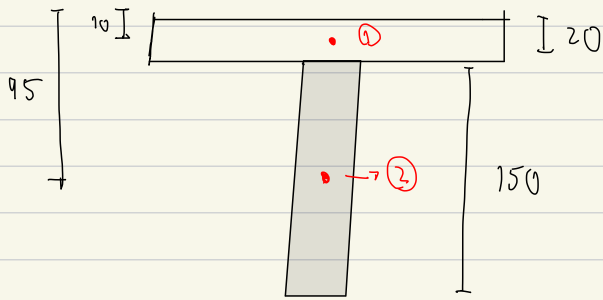
$$= 216(10^6) \text{ mm}^4$$

27. Determine $\bar{y}$, which locates the centroidal axis x' for the cross-sectional area of the T-beam, and then find the moments of inertia $I_{x'}$ and $I_{y'}$.



$I_c + A(d)^2$

ระยะจากจุด centroid แต่ละ object ถึงจุด centroid รวม



Location

$$\bar{y} = \frac{\tilde{y}A_1 + \tilde{y}A_2}{A_1 + A_2} = \frac{10(20(150)) + 95(150(20))}{(20(150)) + (20(150))}$$

$$\bar{y} = 52.5 \text{ mm}$$

$$I = \bar{I}_c + AD^2$$

I_x

Segment 1. →

$$I_{x_1} = \frac{(150)(20)^3}{12} + (150)(20)(42.5)^2$$

$$I_{y_1} = \frac{(150)^3(20)}{12} + (150)(20)(0)$$

Segment 2.

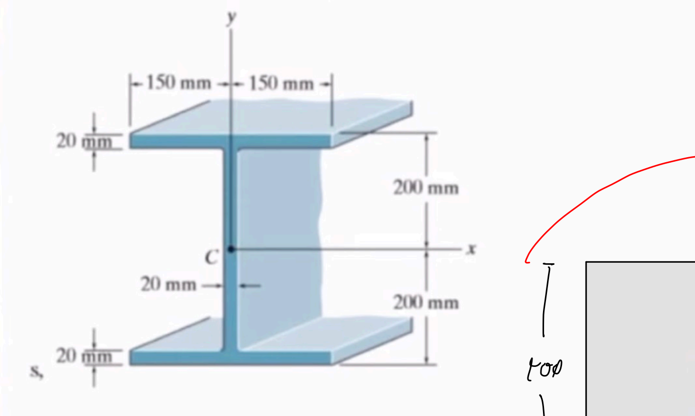
$$I_{x_2} = \frac{(20)(150)^3}{12} + (20)(150)(42.5)^2$$

$$I_{y_2} = \frac{(20)^3(150)}{12} + 0$$

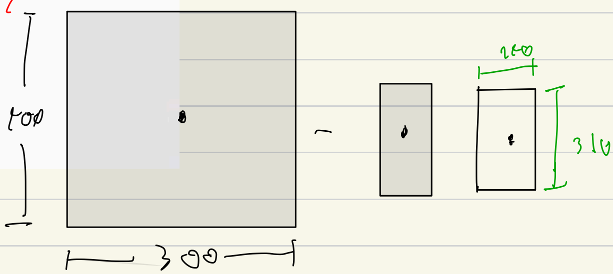
Segment $I_{x-x} \Rightarrow (1.65 \times 10^7)$

$I_y \Rightarrow (5.725 \times 10^6)$

28. Determine the moment of inertia about the x, y axis.



รูปใหญ่ − 2 รูปเล็ก

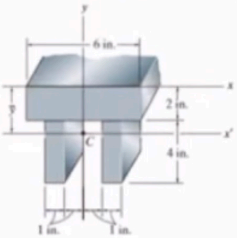


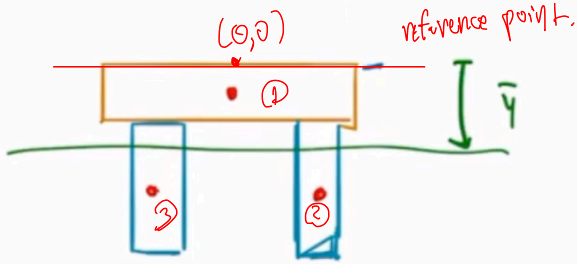
$$I_x = \frac{1}{12}(300)(400)^3 - 2\left(\frac{1}{12}(140)(360)^3\right) = 511(10^6)\ mm^4$$

$$I_y = \frac{1}{12}(360)(20)^3 + 2\left(\frac{1}{12}(20)(300)^3\right) = 90.2(10^6)\ mm^4$$

477.6×10^6

29. Determine the distance $\bar{y}$ to the centroid of the beam's cross-sectional area; then find the moment of inertia about the x', x, y axis.





2 and 3 are symmetry

→ วัดจนถึง centroid แต่ละชิ้น แล้วรวมกัน

$$\bar{y} = \frac{\sum \bar{y}A}{\sum A} = \frac{1(6\times2) + 2(4(4\times1))}{6\times2 + 2(4\times1)} = 2.20\text{ in}$$

$$I_{x'} = \left(\frac{1}{12}(6)(2)^3 + (6)(2)(1.2)^2\right) + 2\left[\frac{1}{12}(1)(4)^3 + (4\times1)(4-2.2)^2\right]$$

$$= 57.9\text{ in}^4$$

พิจารณาผ่านจุด $\bar{y}$ location

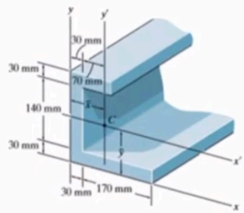
$$I_x = \frac{1}{12}(6)(2)^3 + (6\times2)(1)^2 + 2\left[\frac{1}{12}(1)(4)^3 + (4\times1)(4)^2\right]$$

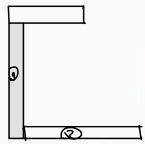
$$= 155\text{ in}^4$$

$$I_y = \frac{1}{12}(6^3)(2) + 2\left[\left(\frac{1}{12}(1)^3(4)\right) + (4)(1.5)^2\right]$$

$$= 54.7\text{ in}^4$$

8. Determine the distance y to the centroid C of the beam's cross-sectional area and the compute the moment of inertia $I_{x'}$ about the x' axis.





$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{170 \times 30(15) + 170 \times 30(85) + 100(30)(185)}{170 \times 30 + 170 \times 30 + 100 \times 30}$$

$$= 80.66\,mm$$

$$I_{x'} = \frac{1}{12}(170)(30)^3 + (170 \times 30)(80.68 - 15)^2$$

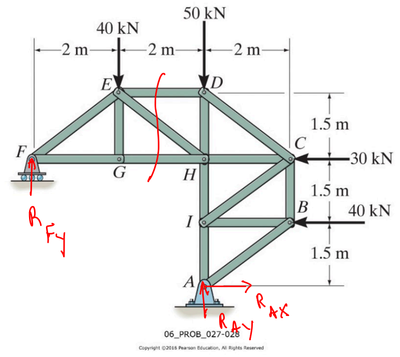
$$+ \frac{1}{12}(30)(170)^3 + (30)(170)(85 - 80.68)^2$$

$$+ \frac{1}{12}(100)(30)^3 + (100 \times 30)(185 - 80.68)^2$$

$$I_{x'} = (67.6)(10^6)\,mm^4$$

$A_x = 633\ lb$, $A_y = -141\ lb$, $B_x = -721\ lb$, $B_z = 895\ lb$, $C_y = 200\ lb$,
$C_z = -500\ lb$

2. Determine the force in members *ED*, *EH*, and *GH* of the truss, and state if the members are in tension or compression.



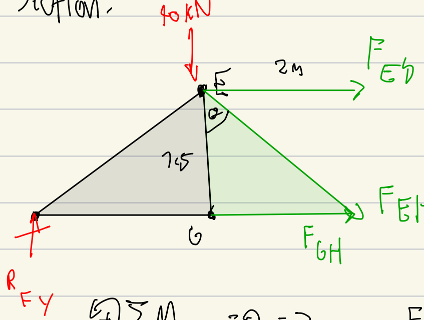
Full FBD. $\circlearrowleft + \sum M_A = 0 \Rightarrow$

$$-R_{Fy}(4) + 40\,kN(2) + 30\,kN(3) + 40\,kN(1.5) = 0$$

$$R_{Fy}(4) = 230$$

$$R_{Fy} = 57.5\ kN$$

FBD cut Method of Section.



$\tan\theta = \frac{2}{1.5}$

$\theta = 53.13$

$\circlearrowleft + \sum M_{H,z} = 0 \Rightarrow \quad -F_{ED}(1.5) - R_{Fy}(4) + 40(2) = 0$

$= \quad \boxed{F_{ED} = -100} \Rightarrow$ Compression

$+\uparrow \sum F_y = 0 \quad R_{Fy} - 40\,kN - F_{EH}\cos(53.13) = 0$

$57.5 - 40\,kN = F_{EH}\cos(53.13)$

$17.5 = F_{EH}$

$\boxed{F_{EH} = 29.166\ kN}$

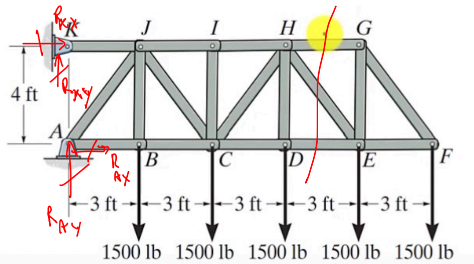
Tension

$\overset{+}{\rightarrow} \sum F_x = 0 \quad F_{ED} + F_{EH}\sin(53.13) + F_{GH} = 0$

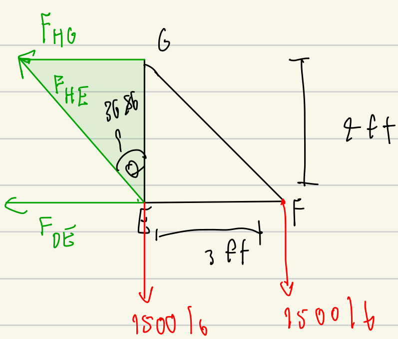
$\boxed{F_{GH} = 76.7\ kN}$

Tension

6–29. Determine the force in members *HG*, *HE* and *DE* of the truss, and state if the members are in tension or compression.



$\tan\theta = \frac{3}{4} \quad \theta \approx 36.86$



$\circlearrowleft + \sum M_{H,z} = 0$

$= -1500(6\,ft) - 1500(3\,ft) - F_{DE}(4) = 0$

$-F_{DE}(4) = 13500$

$\boxed{F_{DE} = -3375\ lb/ft}$

Compression

$+\uparrow \sum F_y = 0 \Rightarrow F_{HE}\cos 36.86 - 2500 - 1500 = 0$

$F_{HE}\cos 36.86 = 3000$

$\boxed{F_{HE} = 3749.57}$ → Tension

$\xrightarrow{+} \sum F_x = 0 \Rightarrow -F_{HG} - F_{HE}\sin(36.86) - F_{DE} = 0$

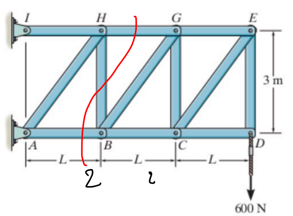
$-F_{HG} - 3749.57\sin(36.86) + 3375 = 0$

$\boxed{F_{HG} = 1125.87}$ → Tension

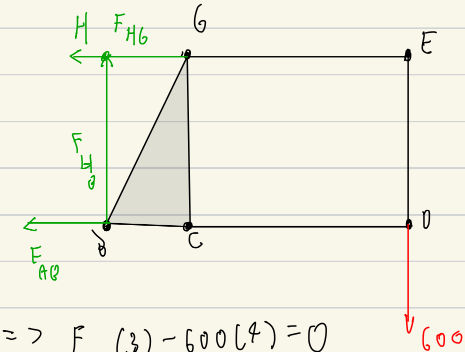
***5–40.** The truss supports the vertical load of 600 N. If $L = 2$ m, determine the force on members HG and HB of the truss and state if the members are in tension or compression.

5–41. The truss supports the vertical load of 600 N. Determine the force in members BC, BG, and HG as the dimension L varies. Plot the results of F (ordinate with tension as positive) versus L (abscissa) for $0 \le L \le 3$ m.

** (Quite tricky)



Probs. 5–40/41



$\circlearrowleft + \sum M_{B,z} = 0 \Rightarrow F_{HG}(3) - 600(4) = 0$

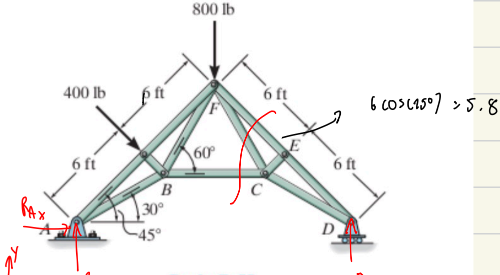
$F_{HG} = 800$ → Tension

$+\uparrow \sum F_y = 0 : F_{HB} - 600 = 0$

$F_{HB} = 600$ N → Tension

Truss Stress / Shear.

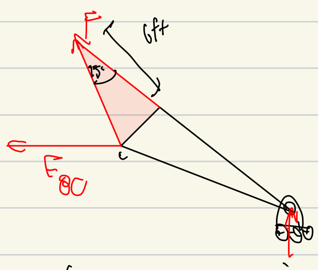
7–55. The truss is used to support the loading shown. Determine the required cross-sectional area of member BC if the allowable normal stress is $\sigma_{allow} = 24$ ksi.



R_{Ay} **Prob. 7–55** R_{Dy}

$$(+)\ \Sigma M_{A,z} = 0 \Rightarrow R_{Dy}\big(2(12\cos(45))\big) - 800(12\cos(45)) - 400(6) = 0$$

$$R_{Dy} = 541.42\ \text{lb}$$



$$(+)\ \Sigma M_{F,z} = 0$$

$$= -F_{BC}(6.21\sin(60)) + 541.42(12\cos(45)) = 0$$

$$F_{BC} = 854.1\ \text{lb}$$

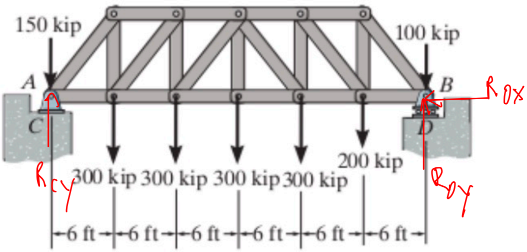
$$\cos(15) = \frac{6}{F_{FC}}$$

$$F_{FC} = \frac{6}{\cos 15} = 6.21$$

$$\sigma = \frac{854.1}{A}$$

$$A = \frac{854.1}{(24\times 10^3)} = \boxed{0.0355}$$

7–78. The pin support A and roller support B of the bridge truss are supported on concrete abutments. If the bearing failure stress of the concrete is $(\sigma_{\text{fail}})_b = 4$ ksi, determine the required minimum dimension of the square bearing plates at C and D to the nearest $\frac{1}{16}$ in. Apply a factor of safety of 2 against failure.



$$\text{ksi} = \frac{\sigma_{fail}}{\sigma_{allow}} \quad 2 \quad 4 \text{ ksi}$$

find σ_{allow}

$$\circlearrowleft \Sigma M_{D,z} = 0 = -R_{Cy}(36\text{ ft}) + 150\text{ kip}(36\text{ ft}) + 300(30) + 300(24) + 300(18) + 300(12) + 200(6) = 0$$

$$R_{Cy} = 883.3\bar{3} \text{ kip}$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0 \Rightarrow R_{Dx} = 0$$

$$+\uparrow \Sigma F_y = 0 \quad 883.3\bar{3} + R_{Dy} - 100 - 150 - 300(4) - 200 = 0$$

$$R_{Dy} = 766.67 \text{ kip}$$

At C. $\sigma_{allow} = \dfrac{883.33}{(L^2)}$

$$2 = \frac{(4\times10^3)}{\frac{883.33}{L^2}} = 2 = \frac{4 L^2}{883.33} \quad (4\times10^3)$$

$\boxed{L = 21.01}$ ft at C. → $21\left(\frac{1}{16}\right)$ in

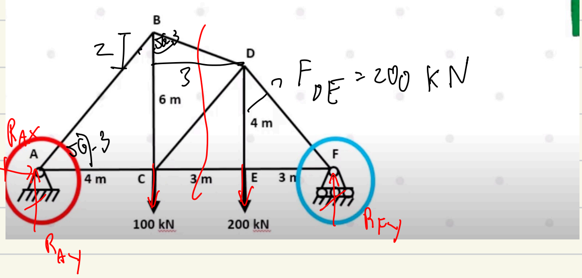
dimension = 21.01 ; 21.01

At D. $\sigma_{allow} = \dfrac{766.67}{L^2} = 2 = \dfrac{4L^2}{766.67} = \boxed{19.579 \text{ ft}}$

→ $19\left(\frac{9}{8}\right)$

* Can Use similar triangle to find angle

For the truss shown in the figure, calculate the stresses in members DE, CE, and BD. The cross-sectional area of each member is 1,200 mm². Indicate tension (T) or compression (C)

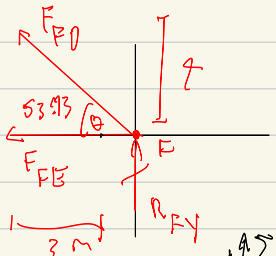


$+\uparrow \sum F_y = 0 \Rightarrow R_{Ay} = 120$

$\overset{\curvearrowleft}{+} \sum M_{A,z} = 0 \Rightarrow R_{Fy}(10m) - 200kN(7m) - 100kN(4m) = 0$

$R_{FY} = 180$ kN

Joint F



$\tan\theta = \left(\frac{4}{3}\right) \quad \theta = 53.13$

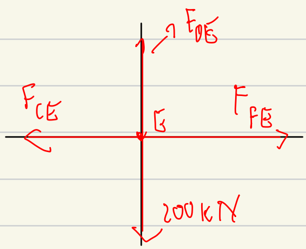
$+\uparrow \sum F_y = 0 \Rightarrow R_{Fy} + F_{FD}\sin(53.13) = 0$

$F_{FD} = -225 \quad (C)$

$\overset{+}{\to} \sum F_x = 0 \Rightarrow -F_{FE} - F_{FD}\cos(53.13) = 0$

$F_{FE} = 135 \quad (T)$

Joint E

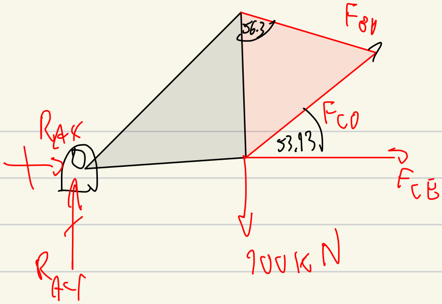


$\to F_{DE} = 200$ kN

$\to$ Tension

$F_{FE} = F_{CE} = 135$ kN $\to$ T

Cut



$$\circlearrowleft + \sum M_{O,Z} = 0 \quad \left(F_{CD} \cos(53.13)(6)\right) + 135(6) - 120(4) = 0$$

$$F_{CD} = -91.66$$

$$\xrightarrow{+} \sum F_x = 0 \Rightarrow \quad -91.66\left(\cos(53.13)\right) + 135\,kN + F_{BD} \sin(56.3)$$

$$F_{BD} = -96.163 \quad (C)$$

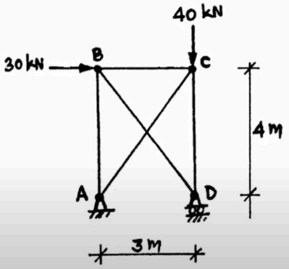
$$F_{DF} = \frac{(225 \times 10^3)}{1200} = 187.5 \text{ Mpa}$$

$$F_{FE} = \frac{(135 \times 10^3)}{1200} = 112.5 \text{ Mpa}$$

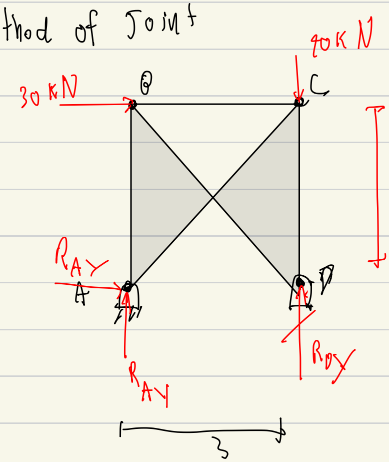
$$F_{BD} = \frac{(96.163 \times 10^3)}{1200} = 80.108 \text{ Mpa}$$

4. จงวิเคราะห์หาแรงในชิ้นส่วนต่างๆ ของโครงข้อหมุนรับแรงดังแสดงในรูป โดยใช้วิธี Method of Joint

(ปีการศึกษา 51)



Method of Joint



$\circlearrowleft + \sum M_{A,z} = 0 \Rightarrow$

$4m \quad -30(4) - 40(3) + R_{Dy}(3) = 0$

$3R_{Dy} = 240$

$R_{Dy} = 80\ kN$

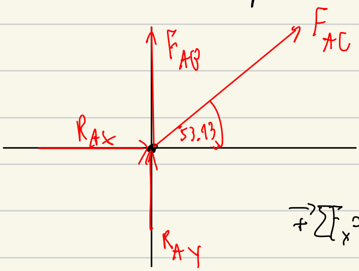
$\xrightarrow{+} \sum F_x = 0 \quad R_{Ax} + 30 = 0$

$R_{Ax} = -30 \leftarrow$

$+\uparrow \sum F_y = 0 \Rightarrow \quad R_{Ay} + 80 - 40 = 0$

$R_{Ay} = -40 \downarrow$

Joint A.



$\xrightarrow{+} \sum F_x = 0 = F_{AC}\cos 53.13 + R_{Ax} = 0$

$F_{AC} = \frac{30}{\cos(53.13)}$

$F_{AC} \approx 50$ Tension $\rightarrow 49.9\bar{9}$

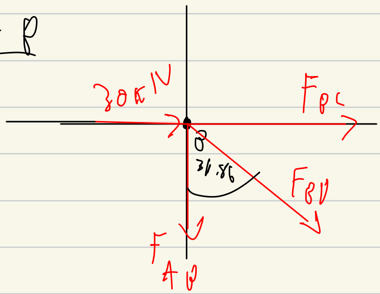
$+\uparrow\sum F_y = 0 \quad F_{AB} + R_{Ay} + F_{AC}\sin(53.13) = 0$

$F_{AB} - 40 + 50\sin(53.13) = 0$

$F_{AB} = 5.359\times10^{-5}$ kN

Tension

Joint B



$+\uparrow\sum F_y = 0 \quad -F_{AB} - F_{BD}\cos(36.86) = 0$

$F_{BD} = \dfrac{-(5.359\times10^{-5})}{\cos(36.86)}$

$\xrightarrow{+}\sum F_x = 0$

$F_{BC} + 30kN + (-6.69\times10^{-5})\sin(36.86) = 0$

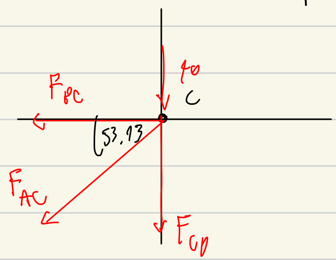
$F_{BD} = -6.69\times10^{-5}$ kN

compression

$F_{BC} = -29.9\bar{9}$ kN

compress

Joint C.



$\xrightarrow{+}\sum F_x = 0 \Rightarrow -F_{BC} - F_{AC}\cos(53.13) = 0$

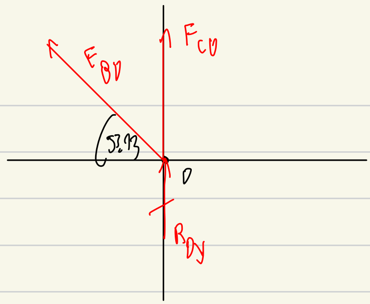
$\dfrac{29.99}{\cos(53.13)} = F_{AC}$

$+\uparrow\sum F_y = 0 \Rightarrow -40 - 49.98\sin(53.13) - F_{CD} = 0$

$F_{CD} = -79.98$ kN (C)

$F_{AC} = 49.98$ kN (T)

Joint D.



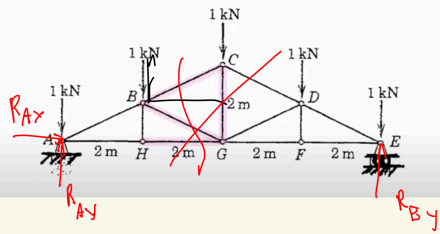
$$= -79.98 + 80 + F_{BD}\sin 53.13 = 0$$

$$F_{BD} = \frac{-0.02}{\sin(53.13)}$$

$$F_{BD} = -0.025 \text{ kN}$$

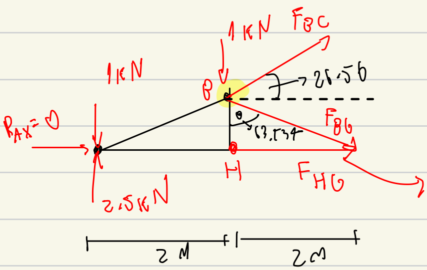
(C)

5. จงคำนวณหาแรงในชิ้นส่วน BC BG CG และ HG ของโครงข้อหมุน (Truss) รับแรงดังแสดงในรูป (ปีการศึกษา 59)



$$\circlearrowleft + \sum M_{B,z} = 0 \Rightarrow -R_{Ay}(8) + 1(8) + 1(6) + 1(4) + 1(2) = 0$$

$$R_{Ay} = 2.5 \text{ kN}$$



หาตรง G ได้ด้วยแต่ต้องแตกแรงเป็น cos / sin

$$\circlearrowleft + \sum M_{B,z} = 0 \Rightarrow 1\text{ kN}(2) - 2.5(2) + F_{HG}(1) = 0$$

$$F_{HG} = 3 \text{ kN}$$

$$\xrightarrow{+} \sum F_x = 0 \Rightarrow 3\text{ kN} + F_{BG}\sin(63.434) + F_{BC}\cos(26.56) = 0$$

$$0.894 F_{BG} + \frac{2}{\sqrt{5}} F_{BC} = -3 \rightarrow (1)$$

$$+\uparrow \sum F_y = 0 \Rightarrow 2.5 - 1 - 1 + F_{BC}\sin(26.56) - F_{BG}\cos(63.434) = 0$$

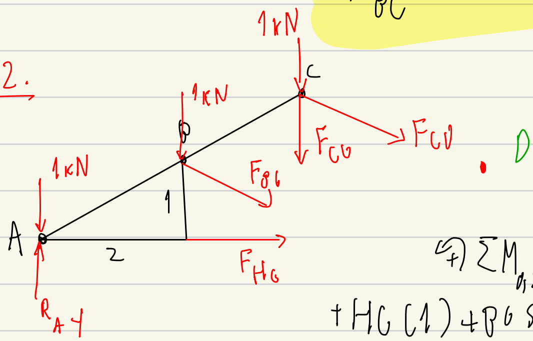
$$-0.447 F_{BG} + 0.447 F_{BC} = -0.5 \rightarrow eq\,2$$

eq 1 + eq 2 $\Rightarrow$

$$\boxed{F_{BG} = -1.118 \text{ kN}} \rightarrow C$$

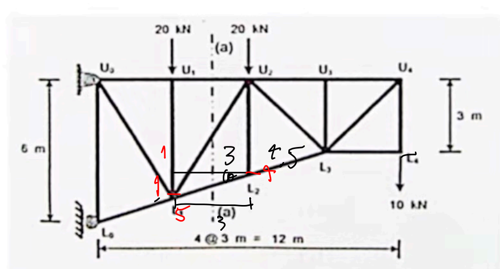
$$\boxed{F_{BC} = -2.236 \text{ kN}} \rightarrow C$$

Cut 2.

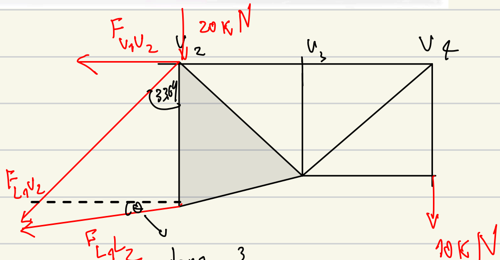


$$\circlearrowleft + \sum M_{D} = 0; \quad -R_{Ay}(6) + 1(6) + 1(4) + 1(2)$$
$$+ HG(1) + BG\sin\theta(4) + CG(2) = 0$$
$$\downarrow$$
$$CG = 0$$

การคำนวณหาแรงในชิ้นส่วน U_1U_2 , L_1L_2 และL_1U_2 โครงถักยื่นและรับน้ำหนักดังรูป (โดยวิธีวิเคราะห์ตรงหน้าตัด Method of section)



$$\frac{3}{H} = \frac{12}{6}$$
$$H = \frac{3}{2}$$



$$\tan\theta = \frac{\frac{3}{2}}{4} = \theta = 20.556$$

$$\circlearrowleft + \sum M_{L_1,2} = 0$$

$$\Rightarrow F_{U_1U_2}(5) - 20(3) - 10(9) = 0$$

$$\boxed{F_{U_1U_2} = 30 \text{ kN}} \rightarrow T$$

$+\uparrow \sum F_y = 0 \quad -F_{L_1V_2}\cos(30.96) - F_{L_1L_2}\sin(18.43) - 20 - 10 = 0$

$$-F_{L_1V_2}\cos(30.96) - F_{L_1L_2}\sin(18.43) = 30$$

eq 1.

$\overset{+}{\rightarrow} \sum F_x = 0 \Rightarrow \quad -30 \quad - F_{L_1V_2}\sin(30.96) - F_{L_1L_2}\cos(18.43) = 0$

$$-F_{L_1V_2}\sin(30.96) - F_{L_1L_2}\cos(18.43) = 30$$

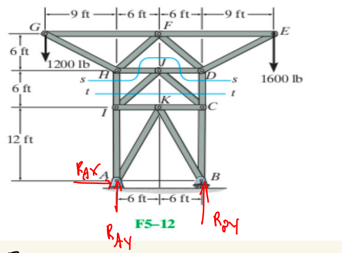
eq 2

eq 1 + eq 2

$$F_{L_1V_2} = -27.15 \rightarrow C,$$

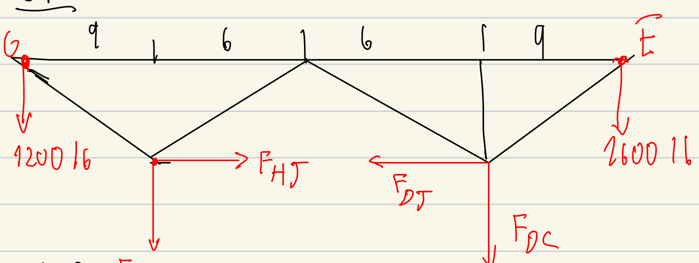
$$F_{L_1L_2} = -15.87 \rightarrow C$$

F5–12. Determine the force in members DC, HI, and JI of the truss. State if the members are in tension or compression.



F5–12

Cut 1.



$\overset{+}{\rightarrow} \sum F_x = 0 \Rightarrow R_{Ax} = 0$

$\circlearrowleft + \sum M_{B,z} = 0 \Rightarrow -R_{Ay}(12) + 21(1200) - 1600(9) = 0$

$$12 R_{Ay} = 10800$$

$$R_{Ay} = 900 \text{ lb/ft}$$

$+\uparrow \sum F_y = 0$

$\circlearrowleft + \sum M_{H,z} = 0 \Rightarrow 1200(9) - 1600(27) - F_{DC}(12) = 0$

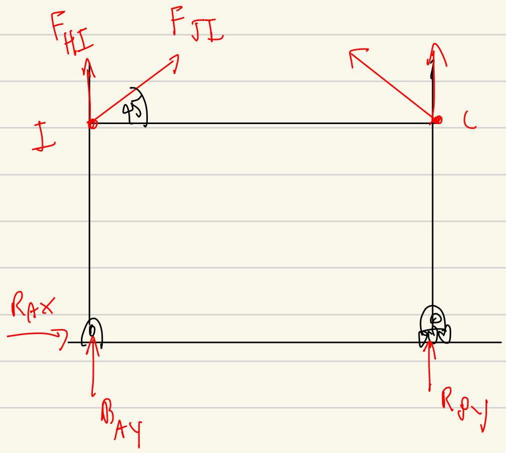
$$-1200 - 1600 + 1900 - F_{HI} = 0$$

$$F_{DC}(12) = -22800$$

$$F_{HI} = -900 \text{ lb/ft} \rightarrow C$$

$$F_{DC} = -1900 \text{ lb/ft} \rightarrow C$$

Cut 2.



$$\circlearrowleft + \sum M_{C,z} = 0$$

$$\downarrow$$

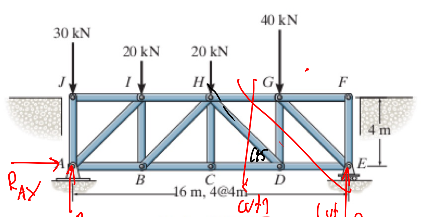
$$= -F_{HI}(12) - F_{JI}\sin(45) - R_{Ay}(12) = 0$$

$$F_{JI} = \underline{900(12) - 900(12)}$$

$$F_{JI} = 0$$

***5–24.** The Howe bridge *truss* is subjected to the loading shown. Determine the force in members *HD*, *CD*, and *GD*, and state if the members are in tension or compression.

5–25. The Howe bridge *truss* is subjected to the loading shown. Determine the force in members *HI*, *HB*, and *BC*, and state if the members are in tension or compression.



Probs. 5–24/25

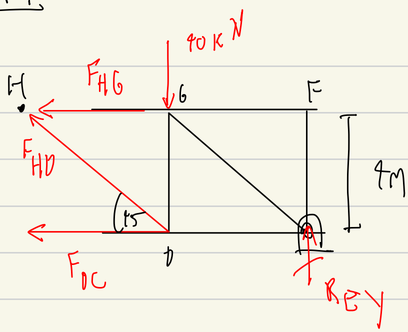
From whole FBD.

$$\circlearrowleft + \sum M_{A,z} = 0 \Rightarrow \quad -20(4) - 20(8) - 40(12) + R_{Ey}(16) = 0$$

$$16\, R_{Ey} = 720$$

$$R_{Ey} = 45$$

From cut 1.



$$\circlearrowleft \sum M_{H,z} = 0 = -F_{DC}(4) - 40(4) + R_{Ey}(8) = 0$$

$$-4F_{DC} = -200$$ โยนถ่อนแล้วก่อนเปลี่ยนเรื่อง

$$F_{DC} = 50 \quad (T)$$

$$+\uparrow \sum F_y = 0 \Rightarrow \quad F_{HD}\sin 45 - 40 + 45 = 0$$

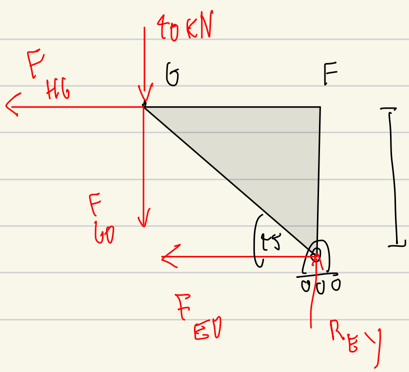
$$F_{HD} = \frac{-5}{\sin 45}$$

$$F_{HD} = -5\sqrt{2} \text{ kN} \quad (C)$$

$$\overset{+}{\rightarrow} \sum F_x = 0 \Rightarrow \quad -F_{HG} - 50 + 5\sqrt{2}\cos 45 = 0$$

$$-F_{HG} = 45$$

$$F_{HG} = -45 \quad C$$



$$+\circlearrowleft \sum M_{E,z} = 0$$

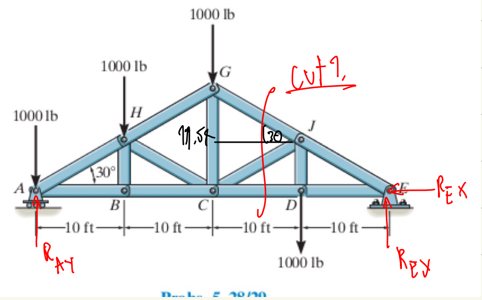
$$F_{GD}(4) + 40(4) - 45(4) = 0$$

$$F_{GD}(4) = 20$$

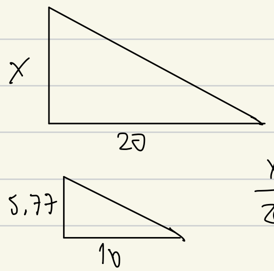
$$F_{GD} = 5 \quad T$$

***5–28.** Determine the force in member *GJ* of the truss and state if this member is in tension or compression.

5–29. Determine the force in member *GC* of the truss and state if this member is in tension or compression.



Probs. 5–28/29

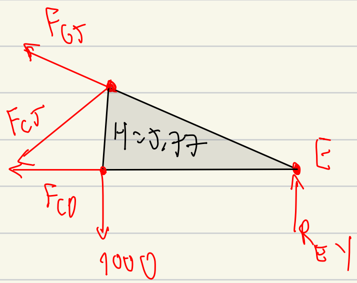


$$\frac{x}{20} = \frac{5.77}{10} = 11.57$$

$$\circlearrowleft + \sum M_{A,z} = 0 \Rightarrow R_{EY}(40) - 1000(30) - 1000(20) - 1000(10) = 0$$

$$40 R_{EY} = 60000$$

$$R_{EY} = 1500 \text{ lb}$$



Solve for H $\quad \tan 30 = \frac{H}{10} \Rightarrow H = 5.77$

$$\circlearrowleft + \sum M_{C,z} = 0$$

$$R_{EY}(20) - 1000(10) + F_{GJ}\cos 30\,(5.77) + F_{GJ}\sin 30\,(10) = 0$$

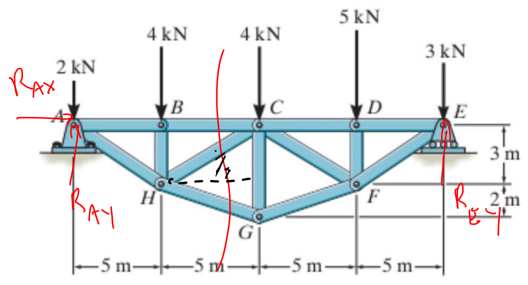
$$F_{GJ}\cos(30)(5.77) + F_{GJ}\sin(30)(10) = -20000$$

$$F_{GJ} = -\frac{20000}{9.997}$$

5–30. Determine the force in members *BC*, *HC*, and *HG*. After the truss is sectioned use a single equation of equilibrium for the calculation of each force. State if these members are in tension or compression.

5–31. Determine the force in members *CD*, *CF*, and *CG* and state if these members are in tension or compression.

$F_{GJ} \approx -2000.6$ C



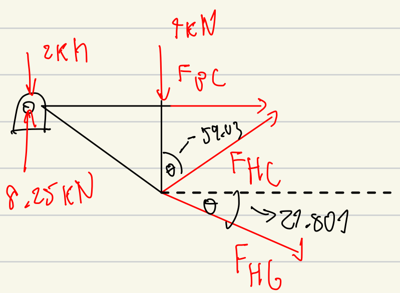
$\circlearrowleft + \Sigma M_{E,z} = 0$

$-R_{Ay}(20) + 2(20) + 4(15) + 4(10) + 5(5)$

$-R_{Ay}(20) = -165$

$R_{Ay} = 8.25$ kN → T

$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow R_{Ax} = 0$



$\circlearrowleft + \Sigma M_{H,z} = 0$

$= -F_{BC}(3) + 2(5) - 8.25(5) = 0$

$-3F_{BC} = 31.25$

$F_{BC} = -10.416$ (C)

$\xrightarrow{+} \Sigma F_x = 0 \quad F_{BC} + F_{HC}\sin(59.03) + F_{HG}\cos(21.801) = 0 \rightarrow (1)$

$+\uparrow \Sigma F_y = 0 \quad -2 + 8.25 - 4 + F_{HC}\cos(59.03) - F_{HG}\sin(21.801) = 0$

solve

$F_{HC}\sin(59.03) + F_{HG}\cos(21.801) = +10.416$

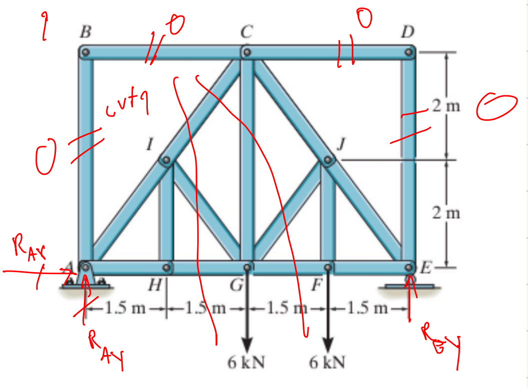
$F_{HC}\cos(59.03) - F_{HG}\sin(21.801) = -2.25$

$F_{HC} = 2.234$ kN (T) , $F_{HG} = 9.154$ kN (T)

*5–32. Determine the force in members *IC* and *CG* of the truss and state if these members are in tension or compression. Also, indicate all zero-force members.

5–33. Determine the force in members *JE* and *GF* of the truss and state if these members are in tension or compression. Also, indicate all zero-force members.

$\xrightarrow{+} \sum F_x = 0 \Rightarrow F_{BC} = 0$, $+\uparrow \sum F_y = 0 \Rightarrow F_{BA} = 0$



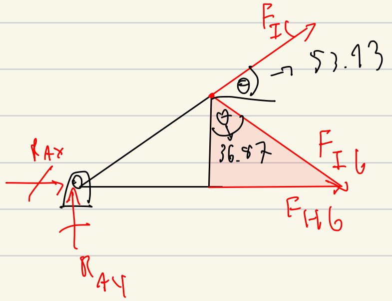
Probs. 5–32/33

$\circlearrowleft + \sum M_{E,z} = 0 \Rightarrow -R_{Ay}(6) + 6(3) + 6(1.5) = 0$

$-6R_{Ay} = -27$

$R_{Ay} = 4.5$ kN

$\xrightarrow{+} \sum F_x = 0 \Rightarrow R_{Ax} = 0$



$\circlearrowleft + \sum M_{I,z} = 0 \Rightarrow F_{HG}(2) - R_{Ay}(1.5) = 0$

$2F_{HG} - 6.75 = 0$

$2F_{HG} = 6.75$

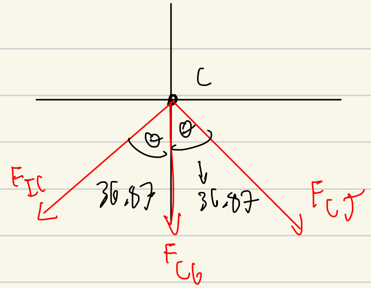
$F_{HG} = 3.375$ kN

$\xrightarrow{+} \sum F_x = 0 \Rightarrow F_{IC}\cos(53.13) + F_{IG}\sin(36.87) = -3.375$

$+\uparrow \sum F_y = 0 \Rightarrow F_{IC}\sin(53.13) - F_{IG}\cos(36.87) = -4.5$

$\boxed{F_{IC} = -5.624 \text{ kN (C)}}$, $F_{IG} = 1.046 \times 10^{-5}$ T

Method of Joint at C.



$$+\uparrow \Sigma F_x = 0 \Rightarrow -F_{IC}\sin(36.87) + F_{CJ}\sin(36.87) = 0$$

$$F_{CJ}\sin(36.87) = -3.374$$

$$F_{CJ} = -5.624 \rightarrow G$$

$$+\uparrow \Sigma F_y = 0 \Rightarrow -F_{IC}\cos(36.87) - F_{CG} - F_{CJ}\cos(36.87) = 0$$

$$\boxed{F_{CG} = 8.998 \text{ kN T}}$$