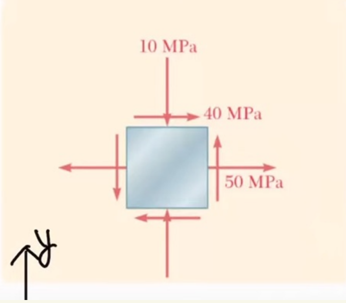
(*a*) construct Mohr's circle, (*b*) determine the principal stresses, (*c*) determine the maximum shearing stress and the corresponding normal stress. (d) determine the state of stress if rotate the elememet 40° clockwise.



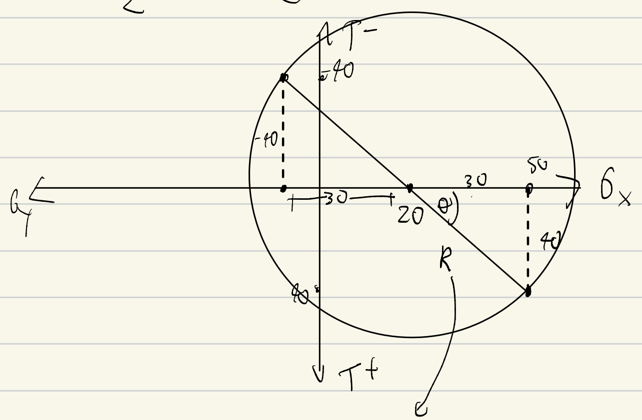
$\sigma_x = 50\,MPa$; $\sigma_y = -10\,MPa$; $\tau_{xy} = 40\,MPa$

σ τ_{xy}

$x\,(50\,MPa,\ 40\,MPa)$

$y\,(-10\,MPa,\ -40\,MPa)$

$\sigma_{Avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{50-10}{2} = 20\,MPa$; $C \Rightarrow \frac{\sigma_x - \sigma_y}{2} = \frac{50-(-10)}{2} = 30$



$R = \sqrt{30^2 + 40^2} \Rightarrow \boxed{50\,MPa}$

τ_{max}

Avrgge normal Stress $\Rightarrow$ 20 MPa

Principal Stress $\Rightarrow \sigma_{avg} \pm R \Rightarrow 20 \pm 50$

$\sigma_1 = 70$; $\sigma_2 = -30$ //

max min

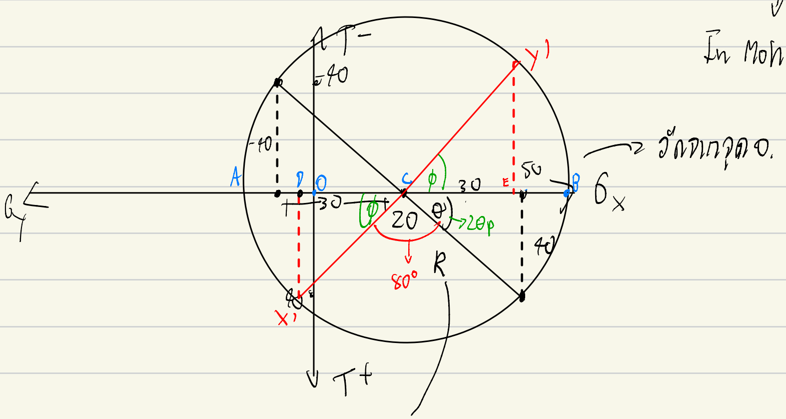
<u>Find Angle</u>

$$\tan 2\theta_p = \frac{40}{30}$$

$$2\theta_p = 53.1 \rightarrow \theta_p = 26.6^\circ$$

(d.) determine the state of stress if rotate the element 40 degree clockwise

In Mohr Circle = 80 degree



$\phi \Rightarrow 180 - 80 - 53.1 = 46.9^\circ$

$OD = DC - OC = R\cos\phi - OC$

$= 50\cos 46.9 - 20$

$= 14.16$

$Dx' \Rightarrow R\sin\phi = 50\sin(46.9^\circ) = 36.5$

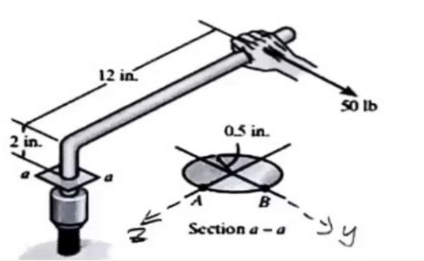
$x' \Rightarrow (-14.16, 36.5)$

find y'

$DE = OC + CE \Rightarrow 20 + 50\cos 46.9^\circ$

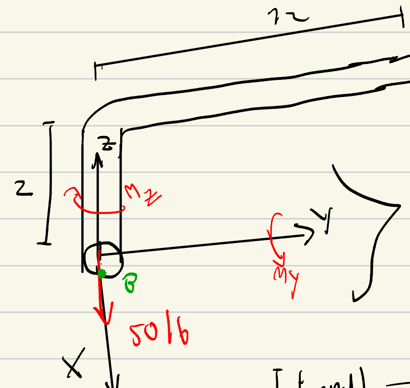
$y' = (54.16, -36.5)$

5. If the box wrench is subjected to the 50 lb force, determine the principal stress and maximum in-plane shear stress at point **B** on the cross section of the wrench at section *a–a*. Specify the orientation of these states of stress and indicate the results on elements at the point.



$\sigma_{1,2}$ and τ_{max} (in-plane) + their associated orientations/ elements

3D rigid body ; circular cross section



External force act on the cross section

Internal →
$\Sigma F_x = 0 \Rightarrow V_x = 50\ lb$
$\Sigma F_y = 0 \Rightarrow V_y = 0\ lb$
$\Sigma F_z = 0 \Rightarrow V_z = 0\ lb$

External Moment → $M = r \times F \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 12 & 2 \\ 50 & 0 & 0 \end{vmatrix}$ $-(-50 \times 2)$

$= 0\hat{i} + 100\hat{j} - 600\hat{k}$

External ← $M_x = 0\ lb \cdot in$; $M_y = \pm 100\ lb \cdot in$; $M_z = -600\ lb \cdot in$ → (negative direct)

Torsion

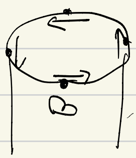
Internal = $-r \times F$

Plane x-y. σ_{zx} ; $\sigma_{yz} = 0$

$$\sigma_{yx} = \cancel{\frac{P}{A}} \pm \frac{M_y x}{I_y} \pm \cancel{\frac{M_x y}{I_x}} \quad ; \quad \tau_{yx} = \frac{Tr}{J} \pm \cancel{\frac{V_x Q}{I_y t}} \pm \cancel{\frac{V_y Q}{I_x t}}$$

$M_x = 0$; $V_y = 0$

No normal force



Section Property.

$$I = \frac{\pi}{4}(0.5)^4 \Rightarrow 0.049\ in^4$$

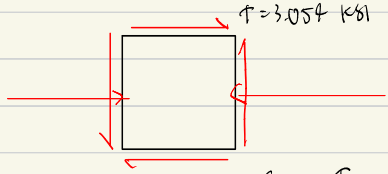
$$J = 2I \Rightarrow 0.0982\ in^4$$

Compressed

$$\sigma_{yx} \Rightarrow -\frac{(100\,lb\cdot in)(0.5\,in)}{0.049\ in^4} \Rightarrow -1.020\ ksi$$

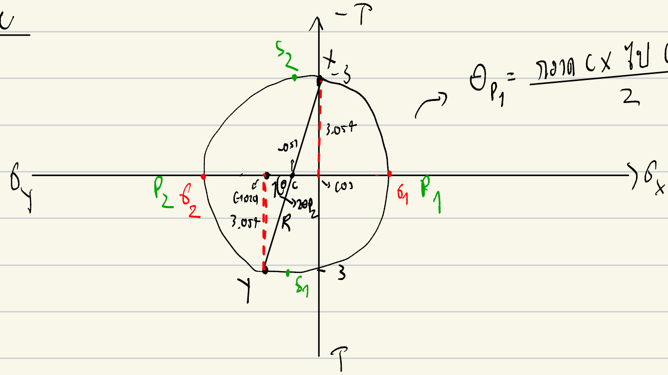
$$\tau_{yx} = \frac{-600\,(0.5\,in)}{0.0982\,in^4} \Rightarrow 3.054\ ksi$$

Stress element.



	σ	τ
x	(−1.020 ,	3.054)
y	(0 ,	−3.054)

Mohr's Circle



$$\theta_{p_1} = \frac{\text{กวาด } CX \text{ ไป } CP_1}{2}$$

(กวาดตามติดลบ

$$C = \frac{\sigma_x - \sigma_y}{2} = \frac{-1.020 - 0}{2} \Rightarrow -0.51 \rightarrow C \; ; \quad \sigma_{avg} = \frac{\sigma_x + \sigma_x}{2} = -0.51$$

$$R = \sqrt{(0.51)^2 + (3.054)^2} \Rightarrow 3.096 \rightarrow \text{Max Shear Stress}$$

Principal Stress $\rightarrow$ $= \sigma_{avg} \pm R \Rightarrow -0.51 \pm 3.096$

$\sigma_1 \Rightarrow 2.586$, $\sigma_2 = -3.606$

Max min

Orientation $\rightarrow$ $\tan 2\theta_p \Rightarrow \frac{3.054}{0.51}$

$2\theta_{p_1} = 80.52$

$\theta_{p_2} = +40.259$

$\theta_{p_1} = \theta_{p_2} - 90$

$\theta_{p_1} = +49.741$

for Max inplane - Shear angle $\rightarrow$ we know

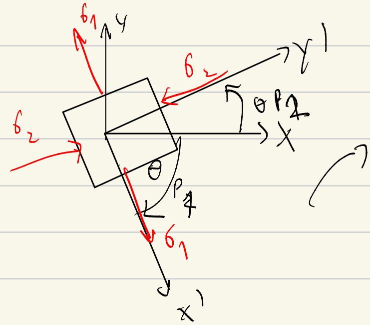
$\phi_s = \phi_p + 45$

$\phi_{s1} \Rightarrow \tan 2\theta_s \Rightarrow \frac{0.51}{3.054}$

$\theta_{s_2} = -4.741$

$\phi_{s_1} = \phi_{s_2} + 90 \Rightarrow \theta_{s_1} = 85.27^\circ$

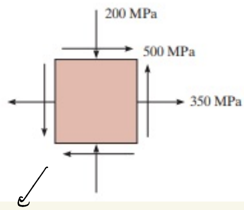
Stress element $\rightarrow$ (Principal)



X YU $x' = P_1$

X YU $y' = P_2$

9–67. Determine the principal stress, the maximum in-plane shear stress, and average normal stress. Specify the orientation of the element in each case.

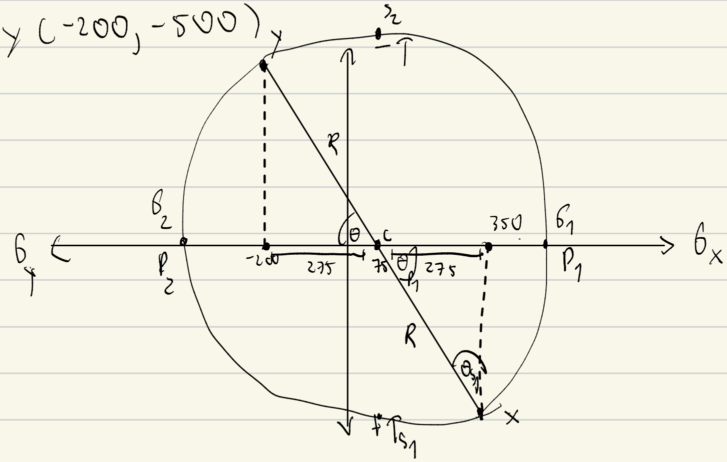


$\sigma_x = 350$; $\sigma_y = -200$; $\tau_{xy} = 500$ MPa

σ τ

x (350 , 500)

y (-200, -500)



$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{350 - 200}{2} = 75$ MPa ; $C = \frac{\sigma_x - \sigma_y}{2} \Rightarrow \frac{350-(-200)}{2} \Rightarrow 275$

$R = \sqrt{(275)^2 + (500)^2} \Rightarrow$ 570.636 MPa → Max Shear Stress.

Principal Stress = $\sigma_{avg} \pm R \Rightarrow 75 \pm 570.636$

$\boxed{\sigma_1 = 646 \text{ MPa} \quad ; \quad \sigma_2 = -496 \text{ MPa}}$

max min

Orientation.

$\tan 2\theta_p = \frac{500}{275} \rightarrow 2\theta_p = 61.189$

$\theta_{p_1} = 30.59$ → Counter clockwise

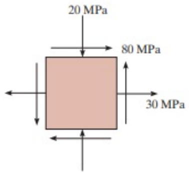
$\theta_{p_2} \Rightarrow -59.41$

for Shear Stress

$\tan 2\theta_s = \frac{275}{500}$ $\quad 2\theta_s = 28.81$

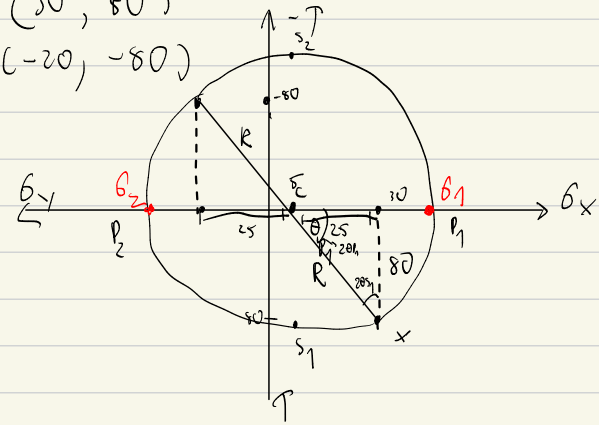
$\theta_{s_1} = 14.40$ (clockwise

*9–64. Determine the principal stress, the maximum in-plane shear stress, and average normal stress. Specify the orientation of the element in each case.



$\sigma_x = 30\,MPa$; $\sigma_y = -20\,MPa$; $\tau_{xy} = 80\,MPa$

$\sigma \quad \tau$
$x\,(30, 80)$
$y\,(-20, -80)$



$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{30-20}{2} = 5\,MPa$; $C = \frac{\sigma_x - \sigma_y}{2} = \frac{30-(-20)}{2} = 25\,MPa$

Average Normal Stress

$R = \sqrt{25^2 + 80^2} = 83.815\,MPa$ τ_{max} (Max in plane shear stress)

Principal Stress => $\sigma_{avg} \pm R \Rightarrow 5 \pm 83.815 \Rightarrow$

$\boxed{\sigma_1 = 88.815 \; ; \; \sigma_2 = -78.815}$

σ_{Max} $\quad \sigma_{min}$

<u>Orientation</u>

$\tan 2\theta_p = \frac{80}{25}$

<u>for Principal</u>

$2\theta_p = 72.646$

$\theta_{p_1} = 36.32 \rightarrow$ (Counterclockwise)

$\theta_{p_2} = -53.677$ (Clockwise

<u>for shear stress.</u>

$\tan 2\theta_s = \frac{25}{80}$

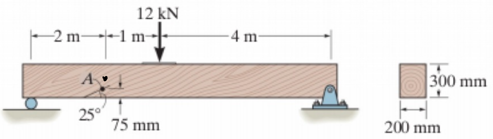
$2\theta_s = 17.35$

$\theta_{s_1} = -8.677$ (Clockwise)

$\theta_{s_2} = 81.323$ (Counter)

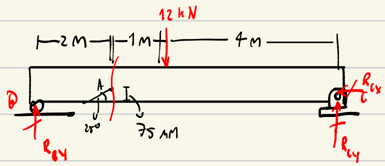
14–17. The wood beam is subjected to a load of 12 kN. If a grain of wood in the beam at point A makes an angle of 25° with the horizontal as shown, determine the normal and shear stress that act perpendicular and parallel to the grain due to the loading.

14–18. The wood beam is subjected to a load of 12 kN. Determine the principal stress at point A and specify the orientation of the element.



Probs. 14–17/18

14-18



For force

$$\circlearrowleft + \Sigma M_{B,z} = 0 \Rightarrow R_{Cy}(7m) - 12kN(3m) = 0$$

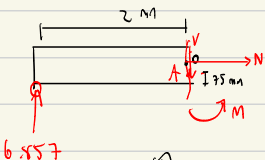
$$R_{Cy}(7m) = 36\ kN$$

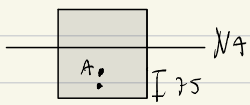
$$R_{Cy} = 5.143\ kN$$

$$+\uparrow \Sigma F_y = 0 \Rightarrow R_{By} - 12kN + 5.143\ kN = 0$$

$$R_{By} = 6.857\ kN$$

Cut 1 At the cross section.





$$\circlearrowleft + \Sigma M_{0,z} = 0 \Rightarrow 6.86 \times 2 = M = 13.72\ kN \cdot m$$

$$+\uparrow \Sigma F_y = 0 \Rightarrow V = 6.86\ kN$$

$$+\rightarrow \Sigma F_x = 0 \Rightarrow N = 0.$$

$$\sigma_A = -\frac{My}{I}$$ Compressed at A

$$I = \frac{(0.2)(0.3)^3}{12} = 4.5 \times 10^{-4}$$

$$\sigma_A = +\frac{(13.72 \times 10^3)(0.075 m)}{(4.5 \times 10^{-4} m^4)} \rightarrow \sigma_A = 2.286\ MPa$$

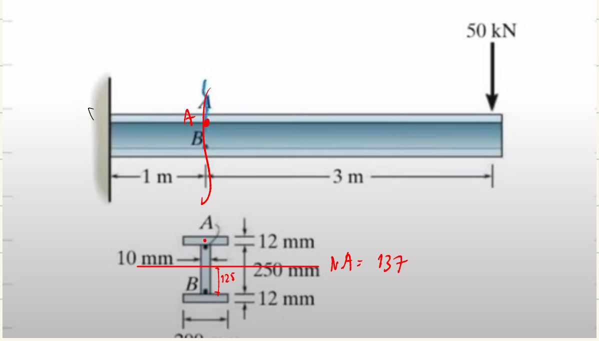
$$\tau = \frac{VQ}{It} \Rightarrow \frac{(6.86 \times 10^3)(1.6875 \times 10^{-3})}{(4.5 \times 10^{-4})(0.2m)}$$

$$Q = (112.5)(200 \times 75) = 1.6875 \times 10^{-3} m^3$$

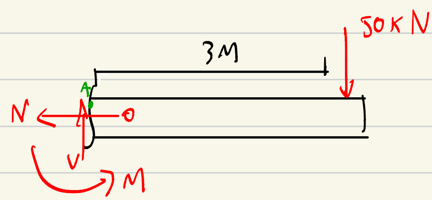
$\tau = 0.1286$ MPa

***9–40.** The wide-flange beam is subjected to the 50-kN force. Determine the principal stresses in the beam at point A located on the *web* at the bottom of the upper flange. Although it is not very accurate, use the shear formula to calculate the shear stress.

9–41. Solve Prob. 9–40 for point B located on the *web* at the top of the bottom flange.



from Cut:



$$\overset{+}{\rightarrow} \sum F_x = 0 \;=\; N = 0$$

$$+\uparrow \sum F_y = 0 \;=\; V - 50 = 0 \quad V = 50 \text{ kN}$$

$$\circlearrowleft \sum M_{0,z} = 0 \Rightarrow M = 150 \text{ kN}\cdot\text{m}$$

$$\sigma = + \left| \frac{My}{I} \right|$$

$$\tau = \frac{VQ}{It}$$

Section Property $\Rightarrow$ $$I = \frac{(200)(274)^3}{12} - 2\left(\frac{(95)(250)^3}{12}\right)$$

$$I = 9.545 \times 10^{-5} \text{ m}^4$$

$$\sigma = \frac{(150 \times 10^3 \text{ N}\cdot\text{m})(0.125 \text{ m})}{(9.545 \times 10^{-5}) \text{ m}^4}$$

$$\sigma = 196.438 \text{ MPa}$$

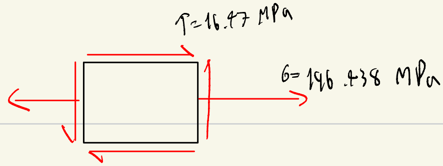
$$Q = \bar{y}A = (131)(200 \times 12)$$
$$Q = 3.144 \times 10^{-4} \text{ m}^3$$

$$t = 0.01 \text{ m}$$

$$\tau = \frac{(50 \times 10^3)(3.144 \times 10^{-4})}{(9.545 \times 10^{-5})(0.01)} \Rightarrow 16.47 \text{ MPa}$$

$\downarrow$

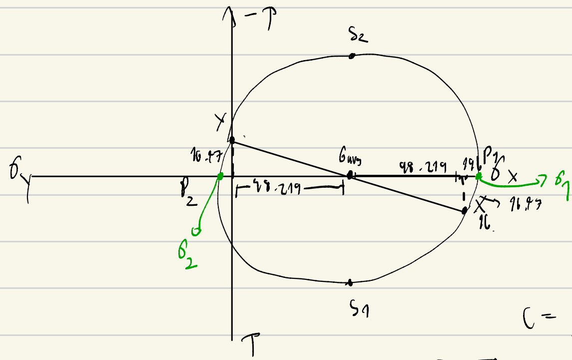
Stress element ->



$\sigma \quad \tau$

$X(196.438,\ 16.47)$

$Y(0,\ -16.47)$



$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = 98.219$$

$$C = \frac{\sigma_x - \sigma_y}{2} = 98.219$$

$$R = \sqrt{(98.219)^2 + (16.47)^2}$$

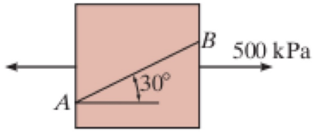
$R = 99.59$ MPa -> Max Shear Stress

Principal Stress $= \sigma_{avg} \pm R \Rightarrow 98.219 \pm 99.59$

$\sigma_1 = 197.809$ MPa ; $\sigma_2 = -1.371$ MPa

Principal Stress

F14–7. Determine the normal stress and shear stress acting on the inclined plane AB. Sketch the result on the sectioned element.



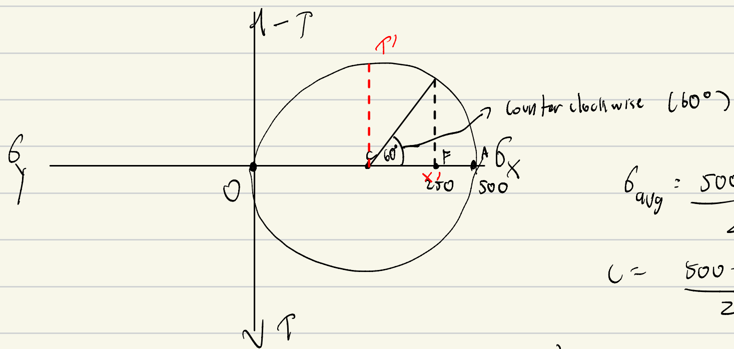
F14–7

$\sigma_x = 500$ kPa ; $\sigma_y = 0$

$\sigma \quad \tau$

$X = (500, 0)$

$Y = (0, 0)$



$\sigma_{avg} = \dfrac{500 + 0}{2} = 250$

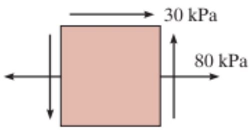
$C = \dfrac{500 - 0}{2} = 250$

$X' \Rightarrow 250 - 250 \cos 60$

$X' \Rightarrow 125$ kPa

$\tau' = 250 \sin 60 \Rightarrow 216.5063$

F14–8. Determine the equivalent state of stress on an element at the same point that represents the principal stresses at the point. Also, find the corresponding orientation of the element with respect to the element shown. Sketch the results on the element.



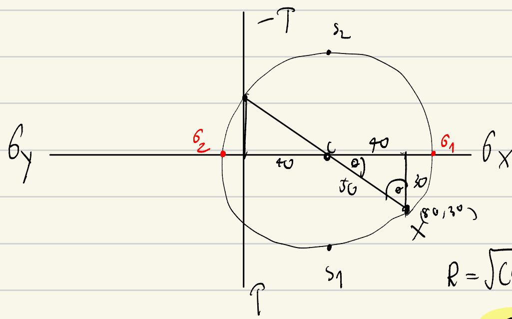
F14–8

$\sigma_x = 80\ kPa\ ;\ \sigma_y = 0\ ;\ \tau_{xy} = 30\ kPa$

$\sigma \quad \tau$

$X\ (80,\ 30\ kPa)$

$Y\ (0,\ -30)$



$\sigma_{avg} = C = \dfrac{\sigma_x + \sigma_y}{2}$

$\dfrac{80}{2} = 40$

$C = \dfrac{\sigma_x - \sigma_y}{2} = \dfrac{80}{2} = 40$

$R = \sqrt{(40)^2 + (30)^2}$

$= 50\ kPa \rightarrow \tau_{max}$

Principal Stress $\rightarrow$ $\sigma_{avg} \pm R \Rightarrow 40 \pm 50$

$\boxed{\sigma_1 = 90\ kPa\ ;\ \sigma_2 = -10\ kPa}$

Orientation $\rightarrow$ $\tan 2\theta_{p_1} = \dfrac{30}{40}$

$2\theta_{p_1} = 36.87 \rightarrow \theta_{p_1} = 18.43$ (Counter Clockwise)

$\theta_{p_2} = -71.565$ (Clockwise)

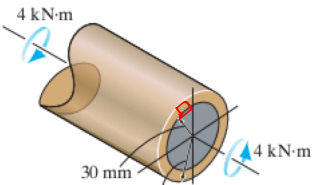
$\downarrow$

$$\tan 2\theta_{s_1} = \frac{40}{30}$$

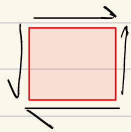
$$2\theta_{s_1} = 53.13$$

$$\theta_{s_1} = 21.56 \; ; \; \theta_{s_2} = -63.434$$

F14–9. The hollow circular shaft is subjected to the torque of 4 kN·m. Determine the principal stress developed at a point on the surface of the shaft.



F14–9



$\sigma_x = 0 \; ; \; \sigma_y = 0$

$$\tau = \frac{Tc}{J} \Rightarrow \frac{(4\times10^3 \text{ N·m})(0.04\text{ m})}{(2.748\times10^{-6}\text{ m}^4)}$$

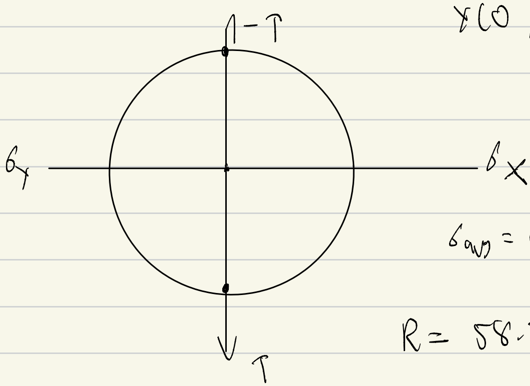
$$J = \frac{\pi}{2}\left((0.04)^4 - (0.03)^4\right)$$

$$J = 2.748\times10^{-6}\text{ m}^4$$

$\tau = 58.22$ MPa

$X(0, 58.22)$

$Y(0, -58.22)$



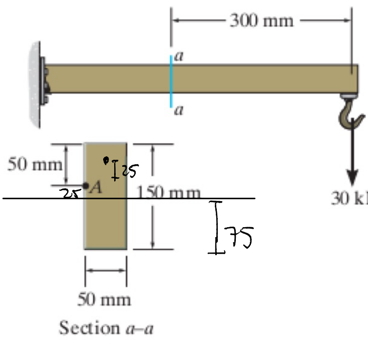
$\sigma_{avg} = 0$

$R = 58.22$ MPa

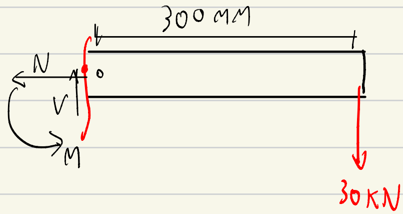
Principal stress $\boxed{\sigma_1 = 58.22\text{ MPa} \; ; \; \sigma_2 = -58.22\text{ MPa}}$

Principal Stress

F14–10. Determine the principal stress developed at point A on the cross section of the beam at section a–a.



F14–10



$\xrightarrow{+} \sum F_x = 0 \Rightarrow N = 0$

$+\uparrow \sum F_y = 0 \Rightarrow V - 30\,kN = 0$

$V = 30\,kN$

$\circlearrowleft \sum M_{0,z} = 0 \Rightarrow M = 9\,kN \cdot M$

30×0.3

$I = \dfrac{(0.05)(0.15)^3}{12}$

$I = 1.40625 \times 10^{-5}\,M^4$

$\sigma = + \dfrac{(9 \times 10^3)(0.025M)}{1.40625 \times 10^{-5}\,M^4}$

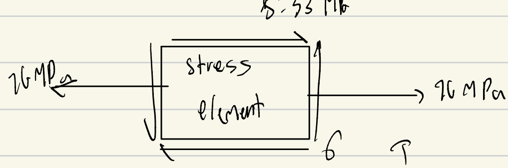
$\sigma_x = 16\,MPa$

$\tau = \dfrac{(30 \times 10^3)(1.25 \times 10^{-4})}{(1.40625 \times 10^{-5})(0.05)}$

$\tau = 5.\overline{33}\,MPa$

$Q = \bar{Y}A = 0.05\,(0.05 \times 0.05)$

$Q = 1.25 \times 10^{-4}\,M^3$



$\sigma \quad \tau$

$X\,(16,\ 5.33)$

$Y\,(0,\ -5.33)$

Principal Stress $\rightarrow \sigma_{avg} \pm R$

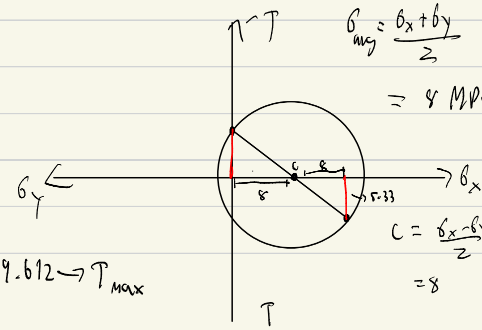
$8\,MPa \pm 9.612$

$\boxed{\sigma_1 = 17.612\,MPa;\ \sigma_2 = -1.612\,MPa}$

Principal Stress

$\sigma_{avg} = \dfrac{\sigma_x + \sigma_y}{2}$

$= 8\,MPa$

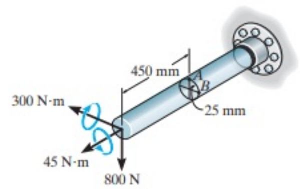


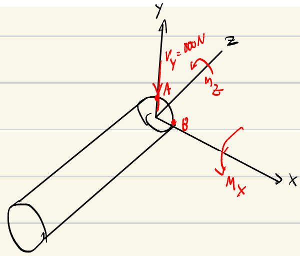
$C = \dfrac{\sigma_x - \sigma_y}{2}$

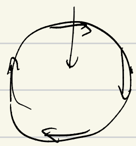
$= 8$

$R = \sqrt{(8)^2 + (5.33)^2} \rightarrow R = 9.612 \rightarrow \tau_{max}$

*9–92. The solid shaft is subjected to a torque, bending moment, and shear force as shown. Determine the principal stress acting at points A and B and the absolute maximum shear stress.







Plane -Y-X

At Point A

$$\sigma = \cancel{\frac{P}{A}} \pm \cancel{\frac{M_y x}{I_y}} \pm \frac{M_x y}{I_x}$$

$$M_x \Rightarrow -300 N\cdot M + 360 N\cdot M$$

$$M_x = 60 N\cdot M$$

$$\sigma_A = \frac{60 N\cdot M (0.025 m)}{(3.068 \times 10^{-7})}$$

$$I = \frac{\pi}{4}(0.025)^4$$

$$I = 3.068 \times 10^{-7} M^4$$

$$\sigma = 4.89 MPa$$

$$\tau = -\frac{Tr}{J} \pm \cancel{\frac{V_y Q}{I_x t}} \pm \cancel{\frac{V_x Q}{I_y t}}$$

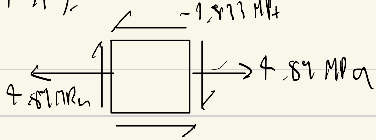
$$\tau = -\frac{(45 N\cdot M)(0.025)}{2(3.068 \times 10^{-7})} \Rightarrow \tau = -1.833 MPa$$

At Point B.

$$\sigma_B = 0 \; ; \; \tau_B \Rightarrow -1.833 + \frac{800\left(\frac{2}{3}(0.025)^3\right)}{(3.068 \times 10^{-7})(0.05)}$$

$$\tau_B = -1.24 MPa$$

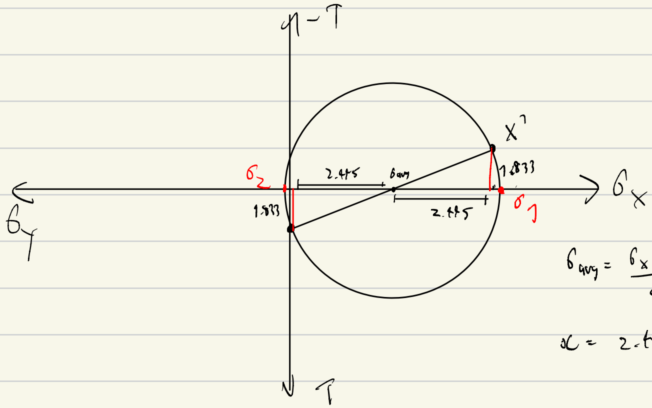
Mohr Circles (At point A).



$\sigma \quad \tau$

$X\,(4.89, -1.833)$

$Y\,(0, 1.833)$



$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{4.89}{2} \Rightarrow 2.445\ MPa$

$c = 2.445$

$R = \sqrt{(2.445)^2 + (1.833)^2}$

Absolute Max $= \frac{\sigma_1}{2}$ as σ_1 and σ_2 got the same sign

$R = 3.056\ MPa \longrightarrow$ Max shear stress

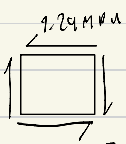
<u>Principal shear stress</u>

$\sigma_{avg} \pm R$

$= 2.445 \pm 3.056 = 5.501 \rightarrow \sigma_1$

$= 2.445 - 3.056 \Rightarrow -0.611 \rightarrow \sigma_2$

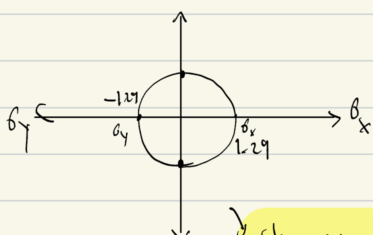
For Point B.



$\sigma \quad \tau$

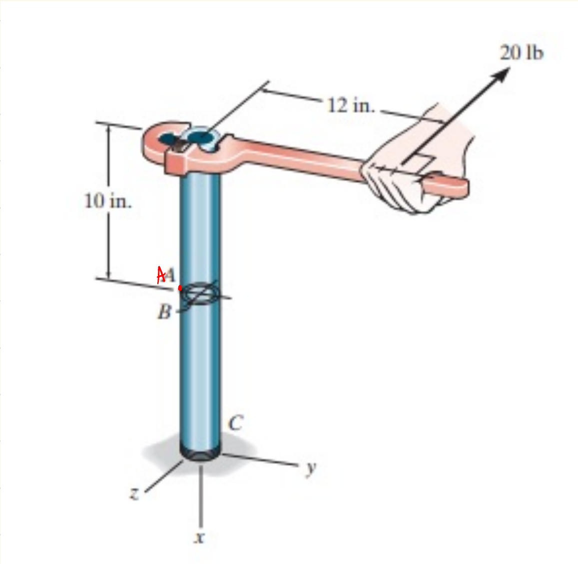
$X = (0, -1.29)$

$Y = (0, 1.29)$

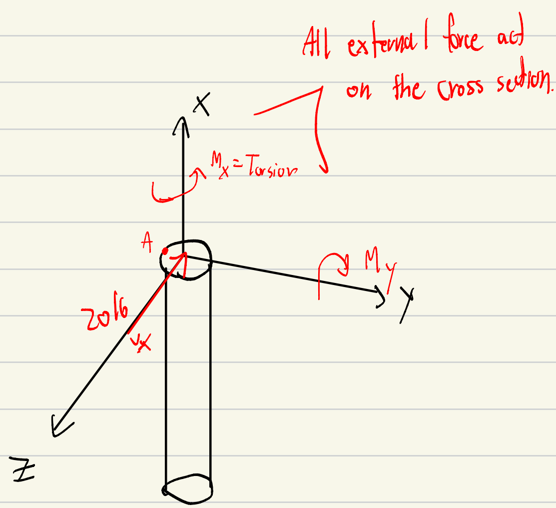


Shear max $= 1.29\ MPa$

9–98. The steel pipe has an inner diameter of 2.75 in. and an outer diameter of 3 in. If it is fixed at C and subjected to the horizontal 20-lb force acting on the handle of the pipe wrench at its end, determine the principal stresses in the pipe at point A, which is located on the surface of the pipe.



FBD At Cross Section



Internal Force.

$$\uparrow \sum F_x = 0 = V_z = 0$$

$$\xrightarrow{+} \sum F_y = 0 = V_y = 0$$

$$\swarrow + \sum F_z = 0 = V_z = 20\,\text{lb}$$

External Moment.

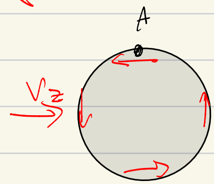
$$M = \vec{r} \times \vec{F} \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 10 & 12 & 0 \\ 0 & 0 & 20 \end{vmatrix}$$

$$= 240\hat{i} - 200\hat{j} + 0\hat{k}$$

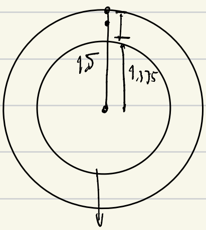
$M_x = 240$ lb·in ; $M_y = -200$ lb·in (Negative direct) ; $M_z = 0$ lb·in for plane zy => $\sigma_{zy} = \cancel{\frac{P}{A}} \pm \cancel{\frac{M_z y}{I_z}} \pm \cancel{\frac{M_y z}{I_y}}$

$$\tau_{zy} = \frac{Tr}{J} \pm \frac{V_z Q}{I_y t} \pm \cancel{\frac{V_y Q}{I_z t}}$$

No $\sigma_{zy} = 0$



Section Property.



$t = 0.125 \times 2 = 0.25$

$$J = \frac{\pi}{2}(1.5^4 - 1.375^4)$$

$$J = 2.337 \text{ in}^4$$

$$I = \frac{\pi}{4}(1.5^4 - 1.375^4) \Rightarrow 1.1687 \text{ in}^4$$

$$Q = \left(\frac{4(1.5)}{3\pi}\left(\frac{\pi}{2}(1.5)^2\right)\right) - \left(\frac{4(1.375)}{3\pi}\left(\frac{\pi}{2}(1.375^2)\right)\right)$$

$$Q = 0.51673 \text{ in}^3$$

$$\tau_{zy} = \frac{(240)(1.5)}{(2.337)} - \frac{(20)(0.51693)}{(1.1687)(0.25)}$$

$\tau_{zy} = 118.66$ → if internal use => -118.66

Principal Stress: $\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$$\sigma_{1,2} = \pm\sqrt{(-118.66)^2}$$

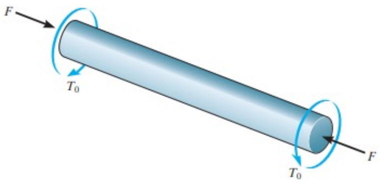
$$= \pm 118.66$$

$$\boxed{\sigma_1 = 118.66 \qquad \sigma_2 = -118.66}$$

↓

Principal

9–101. The shaft has a diameter d and is subjected to the loadings shown. Determine the principal stress and the maximum in-plane shear stress that is developed anywhere on the surface of the shaft.



$$\sigma = \frac{N}{A} = \frac{-F}{\frac{\pi}{4}d^2} = -\frac{4F}{\pi d^2}$$

$$\tau = \frac{Tc}{J} = \frac{T_0\left(\frac{d}{2}\right)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4} \Rightarrow \frac{16T_0}{\pi d^3}$$

In - Plane Principal Stress: $\sigma_x = -\frac{4F}{\pi d^2}$, $\sigma_y = 0$, and $\tau_{xy} = -\frac{16T_0}{\pi d^3}$ for any point on the shaft's surface. Applying Eq. 9-5,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-\frac{4F}{\pi d^2} + 0}{2} \pm \sqrt{\left(\frac{-\frac{4F}{\pi d^2} - 0}{2}\right)^2 + \left(-\frac{16T_0}{\pi d^3}\right)^2}$$

$$= \frac{2}{\pi d^2}\left(-F \pm \sqrt{F^2 + \frac{64T_0^2}{d^2}}\right)$$

$$\sigma_1 = \frac{2}{\pi d^2}\left(-F + \sqrt{F^2 + \frac{64T_0^2}{d^2}}\right) \qquad \textbf{Ans.}$$

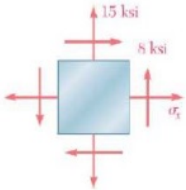
$$\sigma_2 = -\frac{2}{\pi d^2}\left(F + \sqrt{F^2 + \frac{64T_0^2}{d^2}}\right) \qquad \textbf{Ans.}$$

Maximum In - Plane Shear Stress: Applying Eq. 9-7,

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-\frac{4F}{\pi d^2} - 0}{2}\right)^2 + \left(-\frac{16T_0}{\pi d^3}\right)^2}$$

$$= \frac{2}{\pi d^2}\sqrt{F^2 + \frac{64T_0^2}{d^2}} \qquad \textbf{Ans.}$$



PROBLEM 7.52

Solve Prob. 7.30, using Mohr's circle.

PROBLEM 7.30 Determine the range of values of σ_x for which the maximum in-plane shearing stress is equal to or less than 10 ksi.

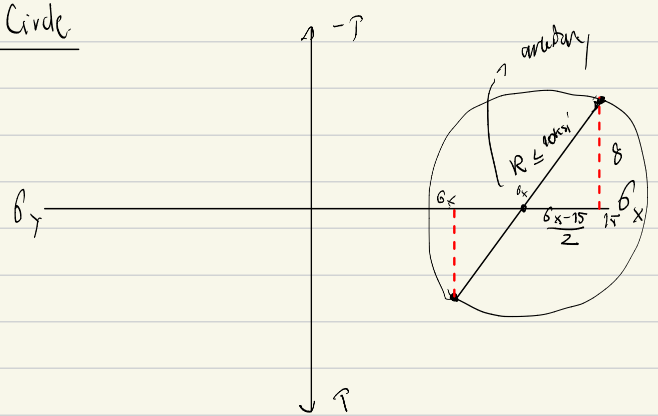
R

$\sigma \quad \tau$

$X(\sigma_x, 8)$

$Y(15, -8)$

Apply Mohr'r Circle



$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_x + 15}{2}$

$C = \frac{\sigma_x - 15}{2}$

Max Shear Stress is the radius.

Hence:

$$10 = \sqrt{\left(\frac{\sigma_x - 15}{2}\right)^2 + 8^2}$$

$$100 = \left(\frac{\sigma_x - 15}{2}\right)^2 + 64$$

case

$$36 = \left(\frac{\sigma_x - 15}{2}\right)^2$$

$$6 = \frac{\sigma_x - 15}{2}$$

$$12 = \sigma_x - 15$$

$12 + 15 = \sigma_x \rightarrow \sigma_x = 27$ KSI

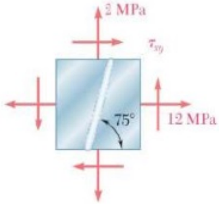
Case −

$$-6 \le \frac{\sigma_x - 15}{2}$$

$$-12 + 15 \le \sigma_x$$

$$\sigma_x \ge 3$$

Ans ⇒ Range $3 \text{ ksi} \le \sigma_x \le 27 \text{ ksi}$



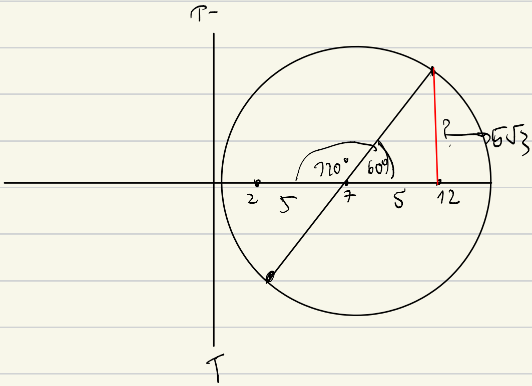
PROBLEM 7.53

Solve Problem 7.29, using Mohr's circle and assuming that the weld forms an angle of 60° with the horizontal.

PROBLEM 7.29 For the state of plane stress shown, determine (*a*) the value of τ_{xy} for which the in-plane shearing stress parallel to the weld is zero, (*b*) the corresponding principal stresses.

$X(12, \tau)$

$Y(2, \tau)$



$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{14}{2} = 7$$

$$C = \frac{\sigma_x - \sigma_y}{2} = \frac{12-2}{2} = 5$$

$$\tan 60 = \frac{?}{5}$$

$$5 \tan 60 = ?$$

$$5\sqrt{3}$$

Hence $\tau_{xy} = \pm 5\sqrt{3}$ MPa

$$R = \sqrt{5^2 + (5\sqrt{3})^2}$$

$R = 10$ MPa

Principal Stress $\Rightarrow \sigma_{avg} \pm R \Rightarrow 7 \pm 10$

$$= \boxed{\sigma_1 = 17 \; ; \; \sigma_2 - 3}$$

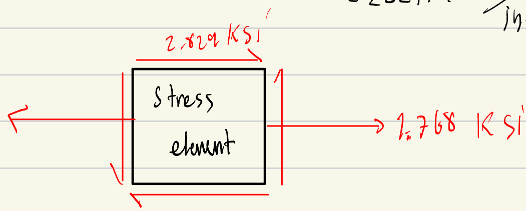
Max Min

14–50. The rotor shaft of the helicopter is subjected to the tensile force and torque shown when the rotor blades provide the lifting force to suspend the helicopter at midair. If the shaft has a diameter of 6 in., determine the principal stress and maximum in-plane shear stress at a point located on the surface of the shaft.

Prob. 14–50

$$\sigma_x = \frac{50\times10^3}{\pi(3)^2} = 1768.38\ lb/in^2$$

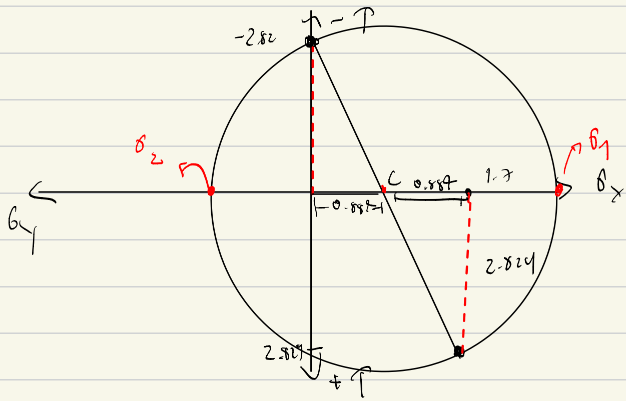
$$\tau_{xy} = \frac{Tc}{J} = \frac{(10\times10^3\times12)(3\ in)}{\frac{\pi}{2}(3^4)} \Rightarrow = 2829.42\ \frac{lb}{in^2}$$



σ τ

X(1.768 , 2.829 ksi)

Y(0, −2.829 ksi)



$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

$$= \frac{1.768}{2} = 0.884$$

$$C = \frac{\sigma_x - \sigma_y}{2} = 0.884$$

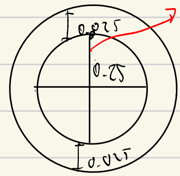
Max Shear Stress = $R = \sqrt{(0.884)^2 + (2.829)^2} \Rightarrow$ 2.964 ksi $\rightarrow \tau_{max}$

Principal Stress $\Rightarrow \sigma_{avg} \pm R \Rightarrow 0.884 \pm 2.964$

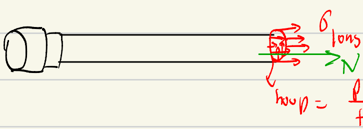
$\sigma_1 = 3.848$ ksi ; σ_2 −2.08 ksi

max min

14–47. The thin-walled pipe has an inner diameter of 0.5 in. and a thickness of 0.025 in. If it is subjected to an internal pressure of 500 psi and the axial tension and torsional loadings shown, determine the principal stress at a point on the surface of the pipe.

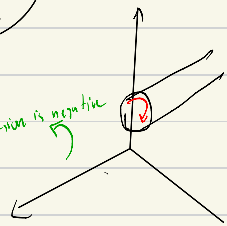


Internal Pressure need to use r inside



$\sigma_{long} = \frac{Pr}{2t}$

hoop $= \frac{Pr}{t}$



Torsion is negative

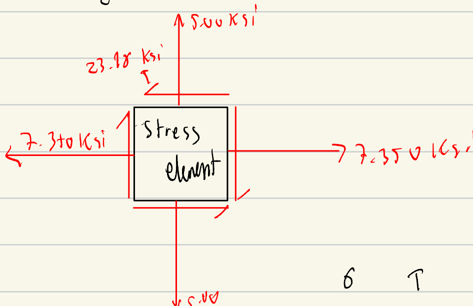
$$\sigma_x = \frac{Pr}{2t} + \frac{P}{A} \Rightarrow \frac{(500)(0.25)}{2(0.025)} + \frac{(200)}{\pi(0.275^2 - 0.25^2)} \Rightarrow 7.350 \text{ ksi}$$

$$\sigma_y = \frac{Pr}{t} \Rightarrow \frac{500(0.25)}{0.025} \Rightarrow 5.00 \text{ ksi}$$

The most outerside.

$$\tau = \frac{Tr}{J} = \frac{(20 \text{ lb.ft} \times 12)(0.275)}{\frac{\pi}{2}(0.275^4 - 0.25^4)} \Rightarrow 23.18 \text{ ksi}$$

Inside Pipe = Pressure จะทำแรงกับพื้นที่ตรงแค่ inner diameter.



σ τ

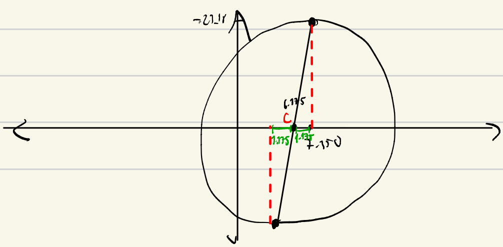
X (7.350, −23.18)

Y (5.00, 23.18)

ksi

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{7.350 + 5}{2} \Rightarrow 6.175$$

$$C = \frac{\sigma_x - \sigma_y}{2} \Rightarrow \frac{7.350 - 5}{2} \Rightarrow 1.175$$

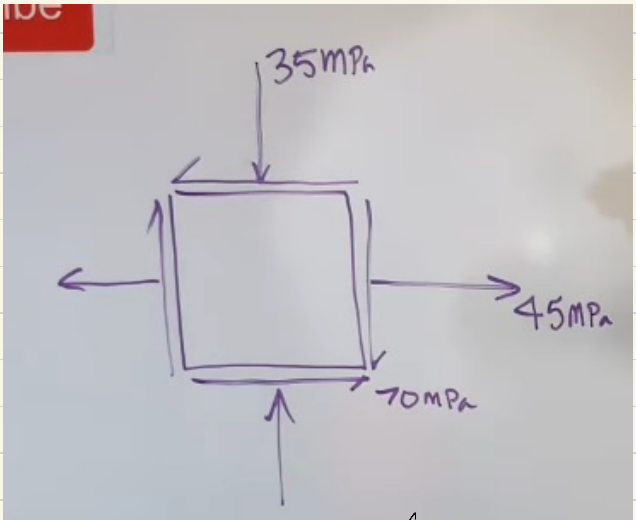


R (Max Shear Stress) $\Rightarrow \sqrt{(1.175)^2 + (23.18)^2}$

$R = 23.2097$

<u>Principal Stress</u> $\Rightarrow$ $\sigma_{avg} \pm R \Rightarrow 6.175$ ksi $+ 23.2097$ ksi

$\boxed{\sigma_1 = 29.3847 \text{ ksi}; \ \sigma_2 = -17.0347 \text{ ksi}}$



Find:
1) AVG Normal STRESS
2) Max τ STRESS
3) Principal Stresses
4) $\theta_{p_1}, \theta_{p_2}$
5) STATE of STRESS if Rotated 25° Clockwise

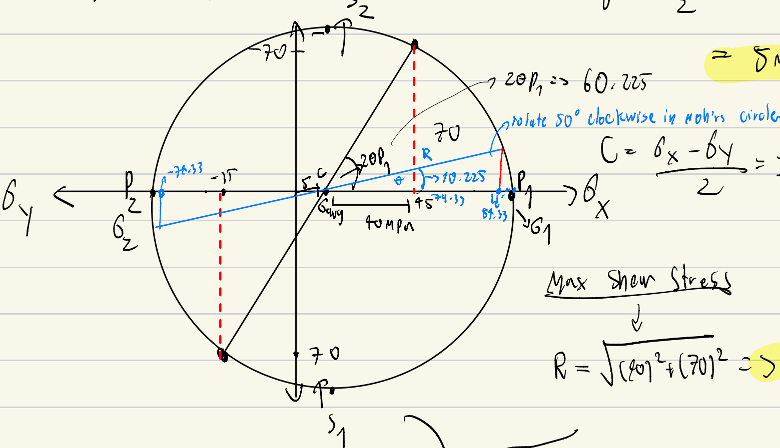
$\qquad\ \sigma \qquad\quad \tau$

X (45 MPa, −70 MPa)

Y (−35, 70)

$\sigma_{avg} = \dfrac{\sigma_x + \sigma_y}{2} \Rightarrow \dfrac{45 - 35}{2}$

$= 5$ MPa



$C = \dfrac{\sigma_x - \sigma_y}{2} \Rightarrow 40$ MPa

<u>Max Shear Stress</u>

$R = \sqrt{(40)^2 + (70)^2} \Rightarrow 80.622$ MPa

3.) Principal Stress $\rightsquigarrow \sigma_{avg} \pm R \Rightarrow 5 \pm 80.622$

$$\boxed{\sigma_1 = 85.622 \text{ MPa} ; \sigma_2 = -75.622 \text{ MPa}}$$

max

Principal Stress

Min

4.) $\tan 2\theta_{p_1} = \frac{70}{40}$

$2\theta_{p_1} \Rightarrow 60.255 \rightarrow \theta_{p_1} = 30.127^{\circ}$

$\theta_{p_2} = \theta_{p_1} - 90$

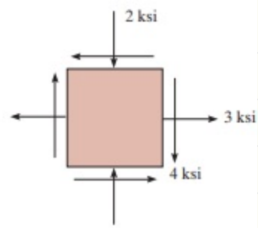
$\theta_{p_2} = -59.872^{\circ}$

5.) If rotate 25° clockwise

= rotate 50° in Moh's circle

$$\boxed{\begin{aligned} \tau'_{xy} &= R\sin(10.255) = 14.35 \\ \sigma'_x &\Rightarrow R\cos 60 + 5 \Rightarrow 84.33 \\ \sigma'_y &\Rightarrow -74.33 \end{aligned}}$$

9–59. Determine the equivalent state of stress if an element is oriented 20° clockwise from the element shown.



Construction of the Circle: In accordance with the sign convention, $\sigma_x = 3$ ksi, $\sigma_y = -2$ ksi, and $\tau_{x'y'} = -4$ ksi. Hence,

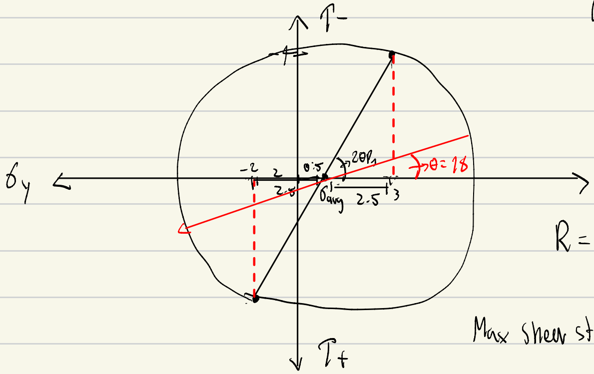
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{3 + (-2)}{2} = 0.500 \text{ ksi}$$

The coordinates for reference points A and C are

$\sigma \quad \tau$

$x\ (3, -4)$

$y\ (-2, 4)$



$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} \Rightarrow \frac{3-2}{2} = \frac{1}{2} \text{ ksi}$$

$$C = \frac{\sigma_x - \sigma_y}{2} \Rightarrow \frac{5}{2} = 2.5 \text{ ksi}$$

$$R = \sqrt{(2.5)^2 + (4)^2}$$

Max shear stress $R = 4.717$ ksi

$$\tan 2\theta_{P_1} = \frac{4}{2.5}$$

$$2\theta_{P_1} \approx 58$$

$$\theta_{P_1} = 29$$

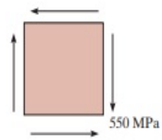
If rotate 20° clockwise then in Mohr's circle = 40° clockwise

$$\tau'_{xy} = R\sin(18°) \Rightarrow (4.717)\sin 18° \Rightarrow \pm 1.457 \text{ ksi}$$

$$\sigma'_x \Rightarrow R\cos(18°) + 0.5 \Rightarrow 4.986 \text{ ksi}$$

$$\sigma'_y \Rightarrow -3.986 \text{ ksi}$$

9–58. Determine the equivalent state of stress if an element is oriented 25° counterclockwise from the element shown.



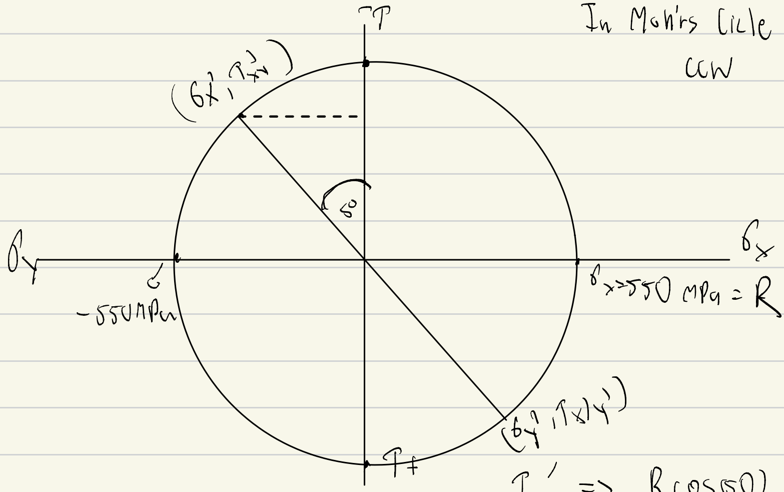
$\sigma_x = 0$; $\sigma_y = 0$

$\quad \sigma \quad \tau$

$X(0, -550)$

$Y(0, 550)$

In Mohr's Cicle = rotate 50°
CCW



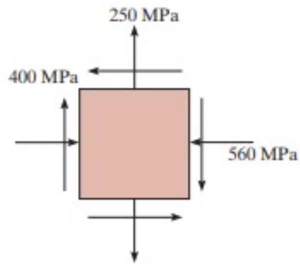
$\tau_{x'y'}' \Rightarrow R\cos(50)$

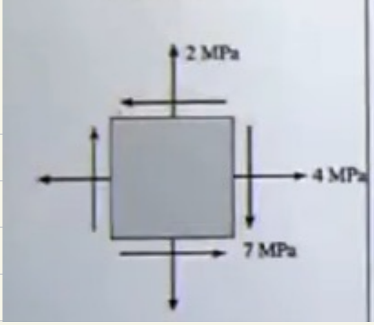
$= 550\cos(50) \Rightarrow -353.53\ MPa$

$\sigma_x' = \sigma_y' \Rightarrow 550\sin(50) \Rightarrow 421.324$

$\sigma_x' = -421.324\ MPa$; $\sigma_y' = +421.324\ MPa$

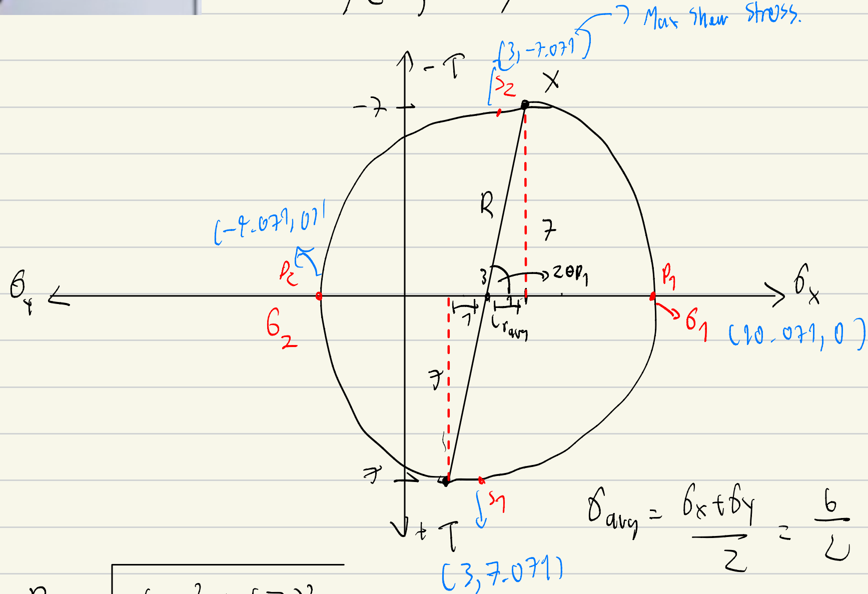
•**9–61.** Determine the equivalent state of stress for an element oriented 60° counterclockwise from the element shown. Show the result on the element.





$\sigma_x = +4\,MPa$; $\sigma_y = +2\,MPa$; $\tau_{xy} = -7\,MPa$

$$\begin{array}{lcc} & \sigma & \tau \\ X(& 4, & -7\ ) \\ Y(& 2, & +7\ ) \end{array}$$



$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{6}{2} = 3\,MPa$$

$$R = \sqrt{(1)^2 + (7)^2}$$

$$C = \frac{\sigma_x - \sigma_y}{2} = \frac{4-2}{2} = 1\,MPa$$

$$R = \sqrt{1 + 49}$$

$$R = \sqrt{50} \Rightarrow 7.071\,MPa$$

Max shear stress

Principal Stress $\Rightarrow$ $\sigma_{avg} \pm R \Rightarrow 3\,MPa \pm \sqrt{50}$

$$\boxed{\sigma_1 = 10.071\,MPa\ ;\ \sigma_2 = -4.071\,MPa}$$

Orientation

$$\tan 2\theta P_1 \Rightarrow \frac{7}{1} \rightarrow 2\theta P_1 = 81.87$$

Principal stress

$$\boxed{\begin{array}{l} \theta_{p_1} = -40.935 \text{ (clockwise)} \\ \theta_{p_2} = +49.065 \text{ (counter clockwise)} \end{array}}$$

Orientation for shear stress. $\longrightarrow$ $\tan 2\theta_s = \frac{1}{7}$

$2\theta_{s_1} = 8.13$

$$\boxed{\begin{array}{l} \theta_{s_2} = +4.065 \text{ (CCW)} \\ \theta_{s_1} = -85.935 \text{ (clockwise)} \end{array}}$$