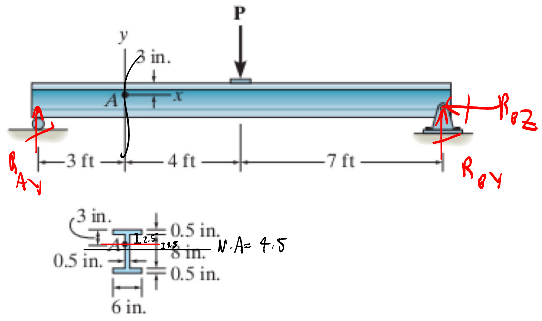
14–98. The strain in the x direction at point A on the steel beam is measured and found to be $\epsilon_x = -100(10^{-6})$. Determine the applied load P. What is the shear strain γ_{xy} at point A? $E_{st} = 29(10^3)$ ksi, $\nu_{st} = 0.3$.



Prob. 14–98

σ_y and $\sigma_z = 0$

$\circlearrowleft + \Sigma M_{A,z} = 0 \Rightarrow -P(7) + R_{By}(14) = 0$

$7P = R_{By}(14)$ — Unit Cancel กัน

$P = 2R_{By}$

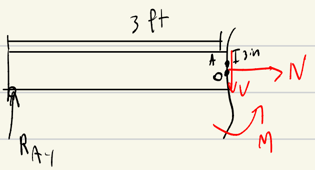
$\downarrow$

$R_{By} = \dfrac{P}{2}$

$+\uparrow \Sigma F_y = 0 \Rightarrow R_{Ay} = \dfrac{P}{2}$

$\tau_{xy} = G\gamma_{xy}$

from Cut 1.



$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow N = 0$

$+\uparrow \Sigma F_y = 0 \Rightarrow -V + \dfrac{P}{2} = 0$

$V = \dfrac{P}{2}$

$\circlearrowleft + \Sigma M_{0,x} = 0 \quad M = \dfrac{P}{2}(3\text{ft})$

$M = \dfrac{3\text{ft}}{2}P$ in

$\sigma = -\left|\dfrac{My}{I}\right|$ Compressed

$\sigma \Rightarrow -\left(\dfrac{\left(\frac{3\text{ft}}{2}P\right)\left(\frac{12\text{ in}}{1\text{ft}}\right)(1.5)}{129.83}\right)$

$\sigma_x = -0.2079P$

Sectional Property

$I = 2\left(\dfrac{(6\text{in})(0.5)^3}{12} + (6\times0.5)(4.25)^2\right) + \left(\dfrac{(0.5)(8)^3}{12}\right) \Rightarrow I = 129.83\text{ in}^4$

$Q = \Sigma \bar{y}A \Rightarrow (2.75)(0.5\times2.5) + (4.25)(6\times0.5) \Rightarrow 16.1875\text{ in}^3$

$t = 0.5$ in

$$\tau_{xy} = \frac{VQ}{It} = \frac{\left(\frac{P}{2}\right)(10.9875\ in^3)}{(129.83\ in^4)(0.5\ in)} \Rightarrow 0.12468P$$

from $\varepsilon_x = -100(10^{-6})$

Generalize Hooke law $\rightarrow$ $\varepsilon_x = \frac{1}{E}[\sigma_x - \cancel{\nu[\sigma_y + \sigma_z]}]$ 0

$$-100(10^{-6}) = \frac{1}{(29000 \times 10^3\ lb/in^2)}[-0.2079P]$$

$$\boxed{P = 13.949\ kip}$$

Find γ_{xy} $\rightarrow$ $\tau_{xy} = G\gamma_{xy}$

$$\tau_{xy} = (13.949 \times 10^3)(0.12468)$$

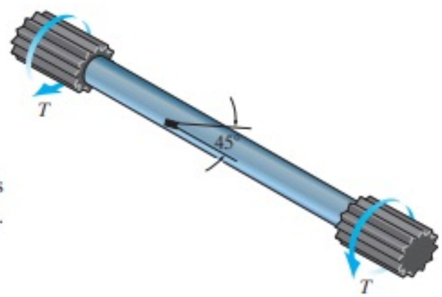
$$\tau_{xy} = 1739.163$$

$$G = \frac{E}{2(1+\nu)}$$

$$= \frac{29000 \times 10^3}{2(1+0.3)} \Rightarrow 11.1538 \times 10^6\ psi$$

$$\frac{(1739)}{(11.1538 \times 10^6)} = \gamma_{xy} \rightarrow \gamma_{xy} = 1.5627 \times 10^{-4}\ rad$$

10–99. A strain gauge forms an angle of 45° with the axis of the 50-mm diameter shaft. If it gives a reading of $\epsilon = -200(10^{-6})$ when the torque **T** is applied to the shaft, determine the magnitude of **T**. The shaft is made from A-36 steel.



d is
)m^4.

No load P apply; Hence $\sigma_x ; \sigma_y ; \sigma_z = 0$

from $\quad \tau = G\gamma$

Find $\gamma_{xy} \Rightarrow \frac{\tau}{G} \longrightarrow \quad \tau = \frac{Tr}{J} \Rightarrow \quad \gamma = \frac{Tr}{GJ}$

$$\boxed{T = \frac{\gamma G J}{r}}$$

Find γ

and $\quad \varepsilon_a = \underbrace{\cancel{\varepsilon_x \cos^2\theta_a}}_{0} + \underbrace{\cancel{\varepsilon_y \sin^2\theta_a}}_{0} + \gamma_{xy} \sin\theta_a \cos\theta_a$

$(-200\times10^{-6}) = \gamma_{xy}\,\frac{1}{2}$ $\qquad G = \frac{E}{2(1+\nu)} = \frac{200\text{ GPa}}{2(1+0.32)}$

$\gamma_{xy} = -4\times10^{-4}$

$$T = \frac{(-4\times10^{-4})(75\times10^{9}\ \text{N/m}^2)\left(\frac{\pi}{2}(0.025)^4\right)}{(0.025)}$$

$T = -736.31$ N·M

10–34. The rod is made of aluminum 2014-T6. If it is subjected to the tensile load of 700 N and has a diameter of 20 mm, determine the absolute maximum shear strain in the rod at a point on its surface.

$$\tau = \gamma G$$

$$\downarrow$$

$$\sigma_x = \frac{P}{A} = \frac{7500}{\frac{\pi}{4}(0.02^2)} = 2.228\ \text{MPa}$$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\cancel{(\sigma_y + \sigma_z)}^{\,0}\right]$$

$$\varepsilon = \frac{1}{73.1(10^9)}\left[2.228(10^6)\right]$$

$$\varepsilon_x = 30.48(10^{-6})$$

$$\varepsilon_y = \frac{1}{E}\left[\cancel{\sigma_y} - \nu(\sigma_x + \cancel{\sigma_z})\right]$$

$$\varepsilon_y = \frac{1}{73.1(10^9)}\left[-0.35(2.228(10^6))\right]$$

$$\varepsilon_y = -10.67(10^{-6}) = \varepsilon_z$$

Hence

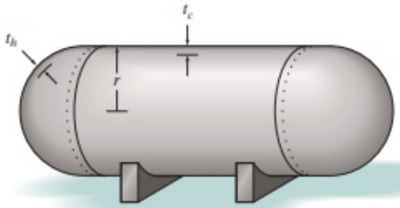
$$\varepsilon_{max} \Rightarrow 30.48(10^{-6}) \quad ; \quad \varepsilon_{min} = -10.67(10^{-6})$$

Absolute max Shear Strain $\rightarrow$ $\gamma_{max} = \varepsilon_{max} - \varepsilon_{min}$

$$= 30.48(10^{-6}) - (-10.67(10^{-6}))$$

$$= 41.1(10^{-6})$$

10–45. The cylindrical pressure vessel is fabricated using hemispherical end caps in order to reduce the bending stress that would occur if flat ends were used. The bending stresses at the seam where the caps are attached can be eliminated by proper choice of the thickness t_h and t_c of the caps and cylinder, respectively. This requires the radial expansion to be the same for both the hemispheres and cylinder. Show that this ratio is $t_c/t_h = (2-\nu)/(1-\nu)$. Assume that the vessel is made of the same material and both the cylinder and hemispheres have the same inner radius. If the cylinder is to have a thickness of 0.5 in., what is the required thickness of the hemispheres? Take $\nu = 0.3$.



for Cylinder Outside $\rightarrow$ $\sigma_x = \sigma_y = \frac{Pr}{2t}$

for Midde Part $\rightarrow$ $\sigma_x = \frac{Pr}{t}$; $\sigma_y = \frac{Pr}{2t}$

from $\quad T = \frac{VQ}{It} = \gamma G$

$\frac{VQ}{IG\gamma} = t$

$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu(\sigma_y + \sigma_z)\right]$

For Outside $\rightarrow$ $\varepsilon_x = \frac{1}{E}\left(\frac{Pr}{2t_h} - \frac{Pr}{2t_h}\nu\right) \rightarrow eq_1$

for inside $\rightarrow$ $\varepsilon_x = \frac{1}{E}\left[\frac{Pr}{t_c} - \nu\left(\frac{Pr}{2t_c}\right)\right] \rightarrow eq_2$

$eq_1 \rightarrow$ $\varepsilon_x = \frac{Pr}{E2t_h}(1-\nu)$

$t_h = \frac{Pr}{E\varepsilon_x 2}(1-\nu)$

$eq_2 \rightarrow$ $\varepsilon_x = \frac{1}{E}\frac{Pr}{t_c}\left[1 - \frac{\nu}{2}\right]$

$t_c = \frac{Pr}{E\varepsilon_x}\left[1 - \frac{\nu}{2}\right]$

&

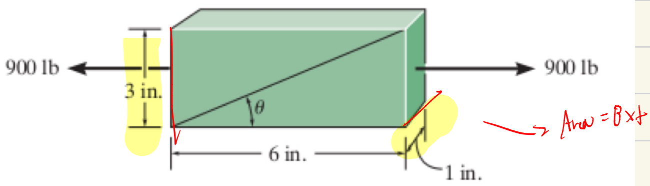
Hence

$$\frac{t_c}{t_h} = \frac{\cancel{\frac{Pr}{E\varepsilon_x}}\left[1 - \frac{\nu}{2}\right]}{\cancel{\frac{Pr}{E\varepsilon_x}}\left(\frac{1}{2} - \frac{\nu}{2}\right)} = \boxed{\frac{t_c}{t_h} \rightarrow \frac{2-\nu}{1-\nu}}$$

#

***14–92.** The polyvinyl chloride bar is subjected to an axial force of 900 lb. If it has the original dimensions shown determine the *change* in the angle θ after the load is applies. $E_{pvc} = 800(10^3)$ psi, $\nu_{pvc} = 0.20$.

14–93. The polyvinyl chloride bar is subjected to an axial force of 900 lb. If it has the original dimensions shown determine the value of Poisson's ratio if the angle θ decreases by $\Delta\theta = 0.01°$ after the load is applied. $E_{pvc} = 800(10^3)$ psi.



Probs. 14–92/93

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x \cancel{- \nu(\sigma_y + \sigma_z)}\right]$$

$$\varepsilon_x = \frac{1}{E}\left(\frac{900\,lb}{3(1)}\right) = \frac{1}{800(10^3)}\left(300\,lb/in^2\right) = 3.75\times10^{-4}$$

from $\varepsilon_x = \dfrac{\Delta L}{L}$

$$\varepsilon_x L = \Delta L$$

$$(3.75\times10^{-4})(6\,in) = \Delta L$$

$$= 2.25\times10^{-3} = \Delta L$$

$$L'(new) = 6.00225$$

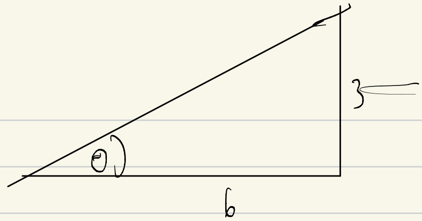
$$\varepsilon_y = \frac{1}{E}\left[\cancel{\sigma_y} - \nu(\sigma_x + \cancel{\sigma_z})\right]$$

$$\varepsilon_y = \frac{1}{E}\left(-0.20\left(300\,\frac{lb}{in^2}\right)\right) = \frac{1}{800(10^3)}\left(-0.20(300)\right) = -7.5\times10^{-5}$$

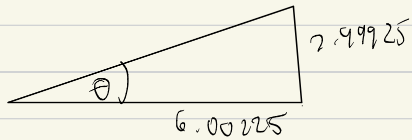
$$\varepsilon_y = \frac{\Delta L}{L} \Rightarrow \quad \Delta L = (-7.5\times10^{-5})(3) = -2.25\times10^{-4}$$

$$B' = 3-(2.25\times10^{-4}) \Rightarrow 2.99977$$

θ old



$\tan\theta = \frac{3}{6} = 26.565^\circ$

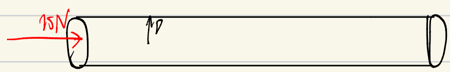


$\tan\theta = \frac{2.99977}{6.00225}$

$\theta \Rightarrow 26.55470$

$\boxed{\theta \text{ change by } 0.0103 \text{ degree}}$

14–91. A rod has a radius of 10 mm. If it is subjected to an axial load of 15 N such that the axial strain in the rod is $\epsilon_x = 2.75(10^{-6})$, determine the modulus of elasticity E and the change in its diameter. $\nu = 0.23$.



$\sigma_y = \sigma_z = 0$

$\sigma_x = \frac{15}{\pi(0.01)^2} \Rightarrow 47.746$ kPa

$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\cancel{(\sigma_y + \sigma_z)}\right) \rightarrow 0$

$E = \frac{47.746\times10^3}{(2.75(10^{-6})} = 17.362$ GPa

Change in diameter $\varepsilon_y = \frac{1}{E}[\cancel{\sigma_y} - \nu(\sigma_x + \cancel{\sigma_z})]$

$$\varepsilon_y = \frac{1}{E}[-0.23(47.746 \times 10^3)]$$

$$\varepsilon_y = -6.325 \times 10^{-7}$$

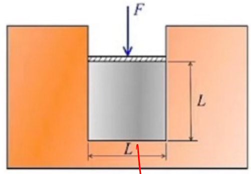
$$\varepsilon_y = \frac{\Delta L}{L}$$

$$\Delta L = \varepsilon_y L$$

$$\Delta L = (-6.325 \times 10^{-7})(0.02)$$

$$\Delta L = -1.265 \times 10^{-8} \text{ m}$$

A 120×120×120 mm rubber block [E = 820 MPa, v = 0.4] is compressed inside a rigid container by a force of F = 120 kN in the vertical direction (y-direction).



Calculate:

a. The stress in the rubber block in the vertical direction (y-direction)

b. The stress in the rubber block in the horizontal direction (x-direction)

c. The magnitude of deformation in the rubber block in the vertical direction (y-direction) ΔL

มีแรง σ_x และ σ_z

เพราะ มี reaction force จาก wall

a.) $\sigma_y = \dfrac{120 \times 10^3}{(0.12 \times 0.12)} = 8.\overline{33}$ MPa

b.) ε_x ; $\varepsilon_z = 0$ (มี wall ด้าน ขวา ซ้าย)

$\cancel{\varepsilon_x}^{\,0} = \frac{1}{E}[\sigma_x - v(\sigma_y + \sigma_z)] \longrightarrow \sigma_x = v(\sigma_y + \sigma_z) \rightarrow$ eq1

$\cancel{\varepsilon_z}^{\,0} = \frac{1}{E}[\sigma_z - v(\sigma_x + \sigma_y)] \qquad \sigma_z = v(\sigma_x + \sigma_y) \rightarrow$ eq2

=> Plug σ_z to eq1 => $\sigma_x = v(\sigma_y + v(\sigma_x + \sigma_y))$

$\sigma_x = v\sigma_y + v^2\sigma_x + v^2\sigma_y$

$\sigma_x(1 - v^2) = \sigma_y(v(1+v))$

$\sigma_x = \dfrac{v\cancel{(1+v)}}{(1-v)\cancel{(1+v)}}\sigma_y$

$\sigma_x = \dfrac{v}{1-v}\sigma_y$

$\sigma_x = \dfrac{0.4}{1-0.4}(8.33 \times 10^6) \Rightarrow 5.556$ MPa

c.) $\varepsilon_y = \frac{1}{E}[\sigma_y - v(\sigma_x + \sigma_z)] \rightarrow$ due to symmetry $\sigma_x = \sigma_z$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\frac{\nu}{1-\nu}\sigma_y + \frac{\nu}{1-\nu}\sigma_y\right)\right]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \frac{2\nu^2}{1-y}\sigma_y\right]$$

$$\varepsilon_y = \frac{\sigma_y}{E}\left[1 - \frac{2\nu^2}{1-\nu}\right] \rightarrow \nu = 0.4$$

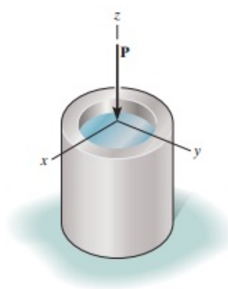
8.333

820

$$\varepsilon_y = 0.00474$$

$$\varepsilon_y = \frac{\Delta L}{L} \rightarrow \Delta L = \varepsilon_y L \Rightarrow (0.00474)(120) = 0.569 \text{ mm}$$

10–58. A soft material is placed within the confines of a rigid cylinder which rests on a rigid support. Assuming that $\epsilon_x = 0$ and $\epsilon_y = 0$, determine the factor by which the modulus of elasticity will be increased when a load is applied if $\nu = 0.3$ for the material.



Normal Strain: Since the material is confined in a rigid cylinder. $\varepsilon_x = \varepsilon_y = 0$. Applying the generalized Hooke's Law,

$$\varepsilon_x = 0 \; ; \; \varepsilon_y = 0$$

$$0 = \cancel{\varepsilon_x} = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] \qquad \sigma_z = E\varepsilon$$

$$0 = \cancel{\varepsilon_y} = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

$$\sigma_x = \nu(\sigma_y + \sigma_z) \rightarrow \text{eq.1}$$

$$\downarrow$$

$$\sigma_y = \nu(\sigma_x + \sigma_z) \rightarrow \text{eq.2}$$

<u>Plug to σ_y to eq1.</u>

$$\sigma_x = \nu(\nu(\sigma_x + \sigma_z) + \sigma_z)$$

$$\downarrow$$

$$\sigma_x = \nu^2\sigma_x + \nu^2\sigma_z + \nu\sigma_z$$

$$\sigma_x(1-\nu^2) = \sigma_z(\nu(\nu+1))$$

$$\sigma_x = \frac{\nu(\nu+1)\sigma_z}{1-\nu^2}$$

$$\sigma_x = \frac{\nu\cancel{(\nu+1)}}{\cancel{(\nu+1)}(1-\nu)}\sigma_z$$

$$\sigma_x = \frac{\nu}{(1-\nu)}\sigma_z$$

load in z-direction

from $\sigma_z = E'\varepsilon_z \longrightarrow$ E' (new) $= \dfrac{\sigma_z}{\varepsilon}$

$$\varepsilon_z \Rightarrow \frac{1}{E}\left[\sigma_z - \nu\left(\frac{\nu}{(1-\nu)}\sigma_z + \frac{\nu}{(1-\nu)}\sigma_z\right)\right.$$

due to symmetry $\sigma_x = \sigma_y$

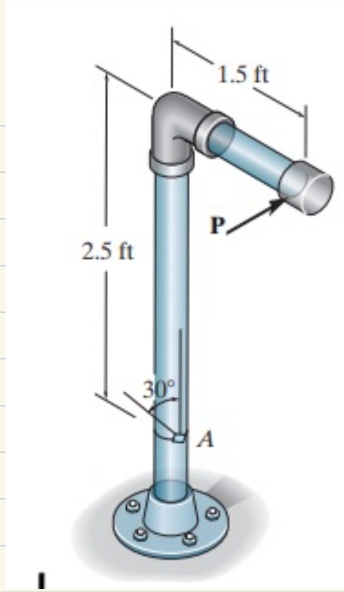
$$\varepsilon_z \Rightarrow \frac{1}{E}\left[\sigma_z - \frac{2\nu^2}{(1-\nu)}\sigma_z\right]$$

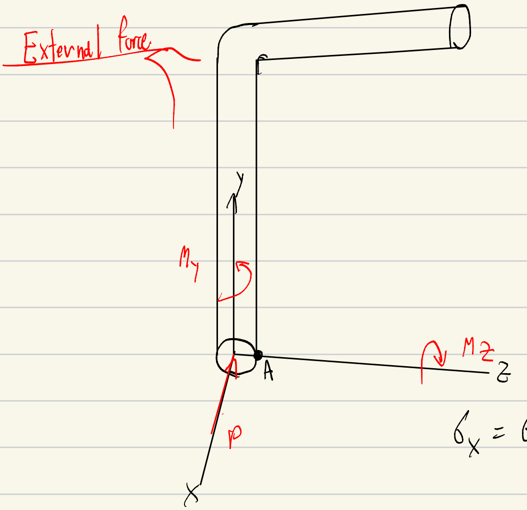
$$\varepsilon_z \Rightarrow \frac{\sigma_z}{E}\left[1 - \frac{2\nu^2}{1-\nu}\right]$$

Hence $E' = \dfrac{\cancel{\sigma_z}}{\dfrac{\cancel{\sigma_z}}{E}\left[1-\dfrac{2\nu^2}{1-\nu}\right]} \longrightarrow \dfrac{E'}{E} = \dfrac{1-\nu}{(1-2\nu)(1+\nu)}$

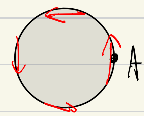
$$\boxed{\text{Plug in } \nu \text{ give } k \text{ or } \frac{E'}{E} = 1.35}$$

10–43. A single strain gauge, placed on the outer surface and at an angle of 30° to the axis of the pipe, gives a reading at point A of $\epsilon_a = -200(10^{-6})$. Determine the horizontal force P if the pipe has an outer diameter of 2 in. and an inner diameter of 1 in. The pipe is made of A-36 steel.





$\sigma_x = \sigma_z = 0$



Internal Force $\Rightarrow$ $\swarrow + \sum F_x = 0 \;;\; V_x = P$

$\xrightarrow{+} \sum F_z = 0 \;;\; V_z = 0$

$+\uparrow \sum F_y = 0 \;;\; V_y = 0$

External Moment $\rightarrow$

$M_z \Rightarrow P(2.5\text{ ft})$ (Negative direct)

Torsion $\rightarrow$ $M_y \Rightarrow P(1.5\text{ ft})$ (Positive direct)

$M_x = 0$

Plane X-Z.

$$\sigma = \cancel{\frac{P}{A}} \pm \cancel{\frac{M_x z}{I_x}}^{\,\rightarrow 0} \pm \cancel{\frac{M_z x}{I_z}}$$

$\rightarrow M_z$ does not produce stress at A.

no normal force

$\sigma = 0$

$$\tau_{xz} = + \frac{Tr}{J} + \frac{V_x Q}{I_z t} \pm \cancel{\frac{V_z Q}{I_x t}} \rightarrow V_z = 0$$

Sectional Property. $\rightarrow$ $I = \frac{\pi}{4}(1^4 - 0.5^4)$

$$I = 0.736 \ In^4$$

$$J = 2I \Rightarrow 1.4726 \ in^4$$

$$Q = \bar{y}A \Rightarrow \left(\frac{4(1)}{3\pi}\right)\left(\frac{\pi}{2}(1)^2\right) - \left(\frac{4(0.5)}{3\pi}\right)\left(\frac{\pi}{2}(0.5)^2\right)$$

$$Q = 0.58\bar{3} \ in^3$$

$$\tau = \frac{18P}{1.4726} + \frac{P(0.583\bar{3})}{(0.736)(1)}$$

$$\tau = 13.0158 \ P$$

since $\sigma = 0$ hence $\varepsilon_x = \varepsilon_z = 0$

$$\sigma_x = 0 \quad \sigma_z = 0$$

$$\varepsilon_a = \cancel{\varepsilon_x \cos^2\theta_a} + \cancel{\varepsilon_z \sin^2\theta_a} + \gamma_{xz}(\sin(150)\cos(150))$$

$$0 \quad 0$$

$$-200(10^{-6}) = \gamma_{xz} \sin 150 \cos 150$$

$$\gamma_{xz} \Rightarrow 461.88(10^{-6}) \ \#$$

$$G = \frac{E}{2(1+\nu)} = \frac{29000 \ KSI}{2(1+0.32)}$$

from $\tau_{xz} = G\gamma$

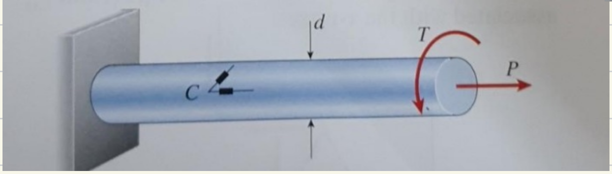
$$13.0158P = (11000 \times 10^3)(461.884(10^{-6}))$$

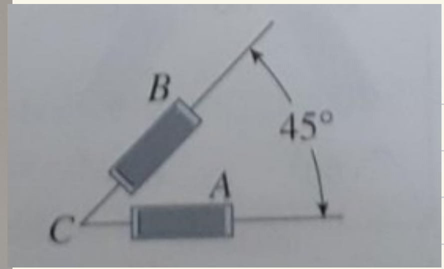
$$P = 0.3904 \ \text{kip}$$

$d = 32$ mm is subjected to an axial force P and a torque T (see figure). Strain gages A and B mounted on the surface of the bar give readings $\varepsilon_A = 140 \times 10^{-6}$ and $\varepsilon_B = -60 \times 10^{-6}$. The bar is made of steel having $E = 210$ GPa and $\nu = 0.29$.

(a) Determine the axial force P and the torque T.

(b) Determine the maximum shear strain γ_{max} and the maximum shear stress τ_{max} in the bar.





We know $\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a - \gamma_{xy}\sin\theta_a\cos\theta_a$

$\varepsilon_b = \varepsilon_x \cos^2\theta_b + \varepsilon_y \sin^2\theta_b + \gamma_{xy}\sin\theta_b\cos\theta_b$

$\varepsilon_a \Rightarrow \theta = 0 \rightarrow \quad \varepsilon_a = \varepsilon_x + 0 - 0$

$\varepsilon_a = \varepsilon_x = 140 \times 10^{-6}$

$\sigma_y = \sigma_z = 0$

$\sigma_x = \frac{P}{A} = \frac{P}{\pi(0.016^2)} = 1243.397\,P \cdot \frac{1}{m^2}$

$\tau_{xy} = \frac{Tr}{J} = \frac{T(0.016)}{\frac{\pi}{2}(0.016^4)} \Rightarrow (155.424 \times 10^3 \frac{1}{m^3})\,T$

$\varepsilon_x = \frac{1}{E}\sigma_x \Rightarrow \varepsilon_x = \frac{1}{(210 \times 10^9)}(1243.397P)$

$\frac{(140 \times 10^{-6})(210 \times 10^9)}{(1243.397)} = P$

$\boxed{P = 23.644 \times 10^3 \text{ N}}$

$\downarrow$

Find torque T $\longrightarrow$ $(-60\times10^{-6}) = \frac{1}{2}(140\times10^{-6}) - (0.29)(140\times10^{-6})\times\frac{1}{2}$

$+ \gamma_{xy}(\frac{1}{2}$

$$\varepsilon_y = \frac{1}{E}[-\nu\sigma_x)$$

$$\varepsilon_y \Rightarrow \frac{\sigma_x}{E}(-\nu)$$

$$\gamma_{xy} = 2(-60\times10^{-6}) - (140\times10^{-6}) + (0.29(140\times10^{-6}))$$

$$\gamma_{xy} = -2.194\times10^{-4}\ \text{rad}$$

$$\varepsilon_x \Rightarrow \frac{\sigma x}{E}$$

Hence $\varepsilon_y = -\nu\varepsilon_x$

$$\tau_{xy} = G\gamma_{xy}$$

$$\frac{Tr}{J} = \gamma_{xy}G$$

$$\boxed{T = \frac{\gamma_{xy}GJ}{r}}$$

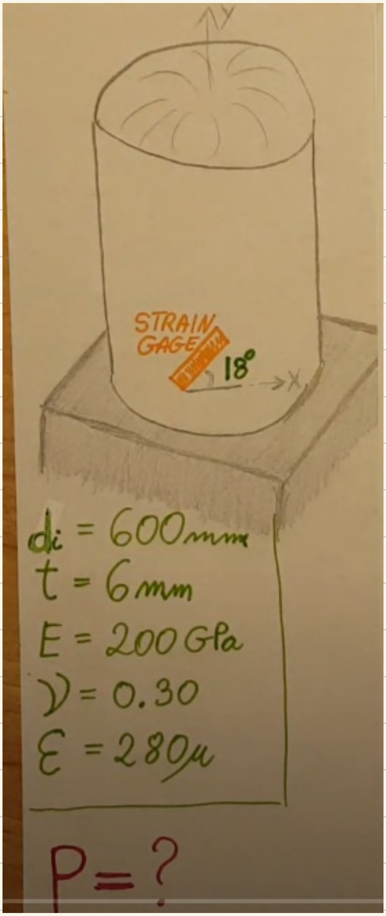
find T

$$T = \frac{(2.194\times10^{-4})(8.139\times10^{10})(\frac{\pi}{2}(0.016)^4)}{(0.016)}$$

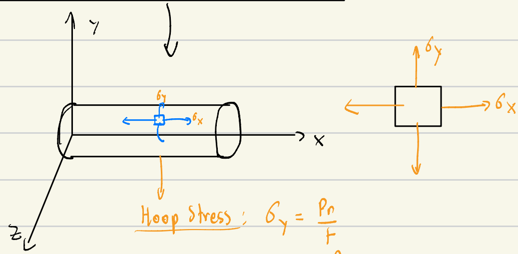
$$G = \frac{E}{2(1+\nu)} = \frac{(210\times10^{9})}{2(1+0.29)} = (8.139\times10^{10})$$

$$T = 114.89\ \text{N.M}$$

for Mohr Circle $\longrightarrow$ Same as shear stress



In thin Wall Pressured Vessell.



Hoop Stress: $\sigma_y = \frac{Pr}{t}$

Longitudinal Stress: $\sigma_x = \frac{Pr}{2t}$

$$\sigma_x = \frac{Pr}{t} \; ; \quad \sigma_y = \frac{1}{2}\sigma_x$$

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu(\sigma_y + \cancel{\sigma_z}^{\,0})\right)$$

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\left(\frac{1}{2}\sigma_x\right)\right)$$

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x\left(1 - \frac{\nu}{2}\right)\right)$$

$$\varepsilon_x = 0.85\,\frac{\sigma_x}{E}$$

$$\varepsilon_y = \frac{1}{E}\left(-\nu\sigma_x + \sigma_y - \cancel{\nu\sigma_z}^{\,0}\right)$$

$$\varepsilon_y = \frac{1}{E}\left(\sigma_x\left(-\nu + \frac{1}{2}\right)\right) = 0.20\,\frac{\sigma_x}{E}$$

String Gauge => $\varepsilon = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy}\sin\theta_a\cos\theta_a$

$$(280 \times 10^{-6}) = \frac{0.85\sigma_x}{(200\times10^9)}\cos^2(18) + \frac{0.20\sigma_x}{(200\times10^9)}\sin^2(18) + 0$$

no shear; $\gamma_{xy} = 0$

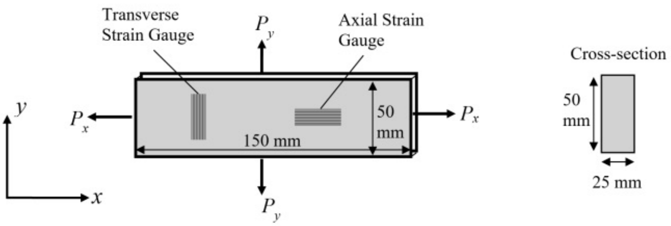
$$\sigma_x = 71.072 \text{ MPa}$$

$$\sigma_x = \frac{Pr}{t} \rightarrow \frac{(71.072\times10^6)(0.006)}{0.3} = P$$

$$\boxed{P = 1{,}421 \text{ MPa}}$$

Q1 (10 points): A 150 mm × 50 mm × 25 mm specimen is subjected to the forces $P_x = 100$ kN and $P_y = 150$ kN, as shown below. The strain gauges are used to measure strains in the specified directions. The axial strain gauge appears to be uncalibrated, leaving only the transverse strain measurement $\varepsilon_y = 0.0002$. The Young's modulus of the material is $E = 100$ GPa.

(a) What is the Poisson's ratio ν of the material?
(b) What measurement should be expected from the axial strain gauge?
(c) If a decrease in temperature of 50 K brings the transverse strain ε_y to zero, what is the coefficient of thermal expansion α of this material?



(a.) From Generalized hooke's law

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \cancel{\sigma_z}^{\,0})\right]$$

$$\sigma_y \Rightarrow \frac{(150 \times 10^3)\,N}{(0.15)(0.025)} = 40\,MPa$$

$$\sigma_x \Rightarrow \frac{(100 \times 10^3)\,N}{(0.025)(0.05)} \Rightarrow 80\,MPa$$

Poison Ratio $\Rightarrow$ $$0.0002 = \frac{1}{(100 \times 10^9 \frac{N}{m^2})}\left[(40 \times 10^6) - \nu(80 \times 10^6)\right]$$

$$20 \times 10^6 = (40 \times 10^6) - \nu(80 \times 10^6)$$

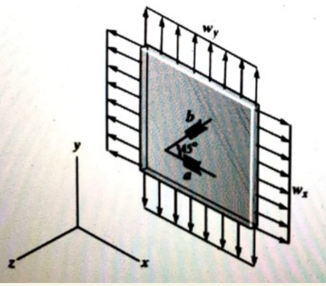
$$\nu$$

$$\boxed{\nu = 0{,}25}$$

2. (a) Two strain gages a and b are attached to the surface of the plate which is subjected to the uniform distributed load of $w_x = 750$ KN/m and $w_y = -175$ KN/m. If the gages give a reading of $\varepsilon_a = 450(10^{-6})$ and $\varepsilon_b = 100(10^{-6})$, determine the modulus of elasticity E, and Poisson's ratio, ν for the material. The thickness of the plate is 25 mm. (25 points)

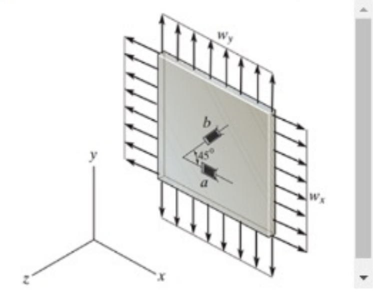
Note: Normal stress equals force per unit length (w) divided by thickness.

(b) What should be the thickness so that the plate has a factor of safety of 3 based on Tresca yield criterion. Take Sy = 80 MPa. (5 points)



Two strain gauges a and b are attached to the surface of the plate made from a material having a modulus of elasticity of E = 70 GPa and Poisson's ratio ν = 0.35. (Figure 1) The gauges give a reading of $\epsilon_a = 510(10^{-6})$ and $\epsilon_b = 120(10^{-6})$. The thickness of the plate is 25 mm.

Figure 1 of 1



Determine the intensity of the uniform distributed load w_x acting on the plate.

Express your answer to three significant figures and include the appropriate units.

w_x = Value Units

Submit Request Answer

Part B

Determine the intensity of the uniform distributed load w_y acting on the plate.

Express your answer to three significant figures and include the appropriate units.

w_y = Value Units

$\rightarrow W_x = \sigma_x \times t$

$\downarrow$ find σ_x

No shear in this problem hence no γ_{xy}

from Gauge $\Rightarrow$ $\varepsilon_x = \varepsilon_a$

Hence $\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)$

$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x)$

from poisson ratio $\Rightarrow$

eq1/2 $\begin{cases} \varepsilon_x = \frac{1}{E}\sigma_x - \frac{\nu}{E}\sigma_y \\ \varepsilon_y = \frac{1}{E}\sigma_y - \frac{\nu}{E}\sigma_x \end{cases}$

$$\varepsilon_b = \varepsilon_x \cos^2\theta_b + \varepsilon_y \sin^2\theta_b - \overset{=0}{\cancel{\gamma_{xy}\sin\theta_b\cos\theta_b}}$$

no shear stress occur

$$120(10^{-6}) = 510(10^{-6})\cos^2(45°) + \varepsilon_y \sin^2(45)$$

(2.55×10^{-4})

$$-1.35\times10^{-4} = \varepsilon_y\left(\frac{1}{2}\right)$$

$$\varepsilon_y = -2.7\times10^{-4}$$

$$(510\times10^{-6}) = \frac{1}{(70\times10^9)}\sigma_x - \frac{0.35}{(70\times10^9)}\sigma_y$$

$$(-2.7\times10^{-4}) = -\frac{0.35}{(70\times10^9)}\sigma_x + \frac{1}{(70\times10^9)}\sigma_y$$

$\sigma_x = 33{,}145{,}299.15 \ \frac{N}{m^2}$; $\sigma_y = -7{,}299{,}145.3 \ \frac{N}{m^2}$

$$33.145 \times 10^6 \ \frac{N}{m^2} \qquad\qquad -7.299 \times 10^6 \ \frac{N}{m^2}$$

Hence $W_x = \sigma_x \cdot t \Rightarrow (33.145 \times 10^6)(0.025)$

$$W_x \Rightarrow 827.996 \times 10^3 \ \frac{N}{m}$$

$$W_y = \sigma_y \cdot t \Rightarrow (-7.299 \times 10^6)(0.025)$$

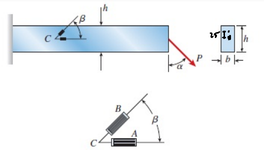
$$W_y \Rightarrow 182.487 \times 10^3 \ \frac{N}{m}$$

Question: A cantilever beam of rectangular cross section (width b=25mm, height h=100mm) is loaded by a force P that acts at the mid-height of the bea...

A cantilever beam of rectangular cross section (width b=25mm, height h=100mm) is loaded by a force P that acts at the mid-height of the beam and is inclined at an angle ? to the vertical, (see Figure 2). Two strain gauges are placed at point C, which also is at the mid-height of the beam. Gauge a measures the strain in the horizontal direction and gauge B measures the strain at angle ? =60° to the horizontal. The measure strains are ?a=125x10^{-6} and ?b=-375x10^{-6}.

Determine the force P and the angle ?, assuming the material is steel with E = 200 GPa and ? =1/3.

Figure 2



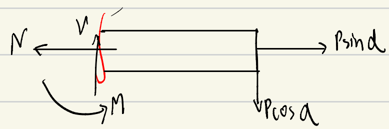
$\sigma_y = \varepsilon_{lat}$ (lateral)

strain that occur perpendicular to the direction of applied load or stress

$\varepsilon_x = \varepsilon_{long} \Rightarrow$ longitudinal occurs along the direction of the applied load.

$\rightarrow \sum F_x = 0 \Rightarrow -N + P\sin\alpha = 0$

$N = P\sin\alpha$



C at Mid point; Hence no Moment Create a stress.

$$\sigma_x = \frac{P}{A} \quad ; \quad \tau = \frac{VQ}{It}$$

$$\sigma_x = \frac{P\sin\alpha}{(0.025)(0.1m)} = 400P\sin\alpha \quad ; \quad \tau = \frac{(P\cos\alpha)(0.025(0.025\times 0.05))}{\left(\frac{(0.025)(0.1)^3}{12}\right)(0.025)}$$

$$\tau = 600P\cos\alpha$$

From $\boxed{\nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}} \Rightarrow -\frac{\varepsilon_y}{\varepsilon_x}} \Rightarrow \frac{1}{3} = -\frac{\varepsilon_y}{\varepsilon_x}$

$\varepsilon_x = -3\varepsilon_y$

$\theta_a = 0$

from $\varepsilon_A = \varepsilon_x\cos^2\theta_a + \cancel{\varepsilon_y\sin^2\theta_a} + \cancel{\gamma_{xy}\sin\theta_a\cos\theta_a}$

$\varepsilon_A = \varepsilon_x$

$$\varepsilon_y = -\frac{\varepsilon_x}{3} \Rightarrow -\frac{(125 \times 10^{-6})}{3}$$

$$\varepsilon_y = -4.16\overline{6} \times 10^{-5}$$

from $\tau_{xy} = G\gamma_{xy} \rightarrow 600 P\cos\alpha = G\gamma_{xy}$

$$\gamma_{xy} = \frac{600 P\cos\alpha}{G}$$

$$G = \frac{E}{2(1+\nu)} = \frac{200 \times 10^9}{2(1+\frac{1}{3})}$$

$$G = 7.5 \times 10^{10}$$

$$\gamma_{xy} = \frac{600 P\cos\alpha}{(7.5 \times 10^{10} \frac{N}{m^2})}$$

$$\gamma_{xy} = (8 \times 10^{-9}) P\cos\alpha$$

from $\varepsilon_b = \varepsilon_x \cos^2\theta_b + \varepsilon_y \sin^2\theta_b + \gamma_{xy} \sin\theta_b \cos\theta_b$

$$(-375 \times 10^{-6}) = (125 \times 10^{-6}) \cos^2(60) + (-4.166 \times 10^{-5}) \sin^2(60) + \gamma_{xy} \frac{\sqrt{3}}{4}$$

$$\gamma_{xy} = 8.66 \times 10^{-4} \text{ rad}$$

5×10^{-4}

(-375×10^{-6})

$$-\frac{(8.66\times10^{-4})}{(8\times10^{-9})} = P\cos\alpha$$

$$\frac{-108250}{\cos\alpha} = P$$

<u>Hence</u> from $\sigma_x = E\,\varepsilon_x$ $E = 200$ GPa ; $\varepsilon_x = 125\times10^{-6}$

$$400\,P\sin\alpha = E\,\varepsilon_x$$

$$= 400\,(108250)(\tan\alpha) = (200\times10^{9})\,(125\times10^{-6})$$

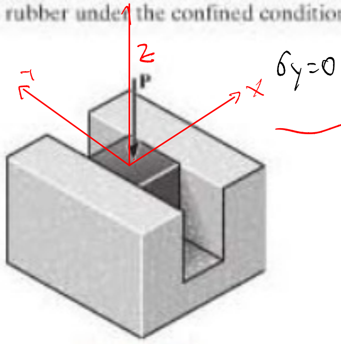
$$\tan\alpha = 0.5774.$$

$$\alpha = -30^\circ$$

Hence $P = \dfrac{108250}{\cos(30^\circ)}$

$$P \approx 125\times10^{3}\ \text{N}.$$

10–57. The rubber block is confined in the U-shape smooth rigid block. If the rubber has a modulus of elasticity E and Poisson's ratio ν, determine the effective modulus of elasticity of the rubber under the confined condition.



from FBD ; We can see that

$$\varepsilon_x = 0$$

$$\sigma_z = \frac{P}{A}$$

from $\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$

$$0 = \frac{1}{E}\left[\sigma_x - \nu(0 + \sigma_z)\right]$$

$$0 = \frac{\sigma_x}{\cancel{E}} - \frac{\nu\sigma_z}{\cancel{E}}$$

$$\sigma_x = \nu\sigma_z$$

In confined condition $\varepsilon_z \neq 0$

$$\varepsilon_z = \frac{1}{E}\left(\sigma_z - \nu(\sigma_x + 0)\right)$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\sigma_x\right] \rightarrow \varepsilon_z = \frac{\sigma_z}{E}\left[1 - \nu^2\right]$$

note that $\sigma_z = E_{eff}\,\varepsilon_z$ Hooke's law

$$E_{eff} = \frac{\sigma_z}{\varepsilon_z} \Rightarrow \frac{\cancel{\sigma_z}}{\frac{\cancel{\sigma_z}}{E}(1-\nu^2)}$$

$$E_{eff} = \frac{E}{1-v^2}$$