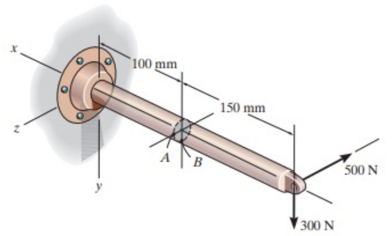
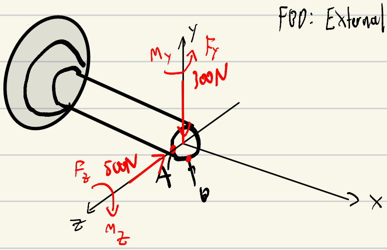
8–55. The bar has a diameter of 40 mm. If it is subjected to the two force components at its end as shown, determine the state of stress at point A and show the results on a differential volume element located at this point.



FBD: External force on the crosscetion.



$\rightarrow \sum F_x = 0 = V_x = 0$

$\nearrow + \sum F_z = 0 \Rightarrow V_z = 500\ N$

$+\uparrow \sum F_y = 0 \Rightarrow V_y = 300\ N$

Find External Moment.

$$M = r \times F \Rightarrow \begin{vmatrix} i & j & k \\ 150 & 0 & 0 \\ 0 & -300 & -500 \end{vmatrix}$$

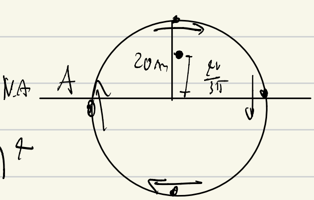
$$M = 0\hat{i} - (-75000\ N\cdot mm)\hat{j} + (-45000)\hat{k}$$

Hence external Moment on the cross section → is

$$M_x = 0\ ;\ M_y = 75{,}000;\ M_z = -45{,}000$$

For plane Y-Z at A → $\sigma = \cancel{\frac{P}{A}} \pm \left|\frac{M_y z}{I_y}\right| \pm \cancel{\left|\frac{M_z y}{I_z}\right|}$ → No normal face act on the cross section and M_z produce no stress at point A.

$$\tau = \cancel{\frac{T}{J}} \pm \frac{V_y Q}{I_z t} \pm \cancel{\frac{V_z Q}{I_y t}}$$



Section Property.

$$I = \frac{\pi}{4}(20)^4 \Rightarrow 1.256 \times 10^{-7}\ m^4$$

or $40{,}000\pi\ mm^4$

$$Q \Rightarrow \sum \tilde{y}A \Rightarrow \left(\frac{4(20)}{3\pi}\right)\left(\frac{\pi}{2}(20)^2\right)$$

$$Q \Rightarrow 5333.33 \ mm^3$$

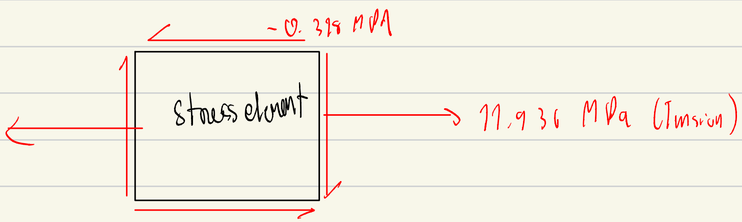
$$\sigma = \pm \frac{(75{,}000 \ N\cdot mm)(20 \ mm)}{40{,}000\pi \ mm^4}$$

$$\boxed{\sigma = 11.936 \ MPa} \rightarrow MPa \ (\text{Tension})$$

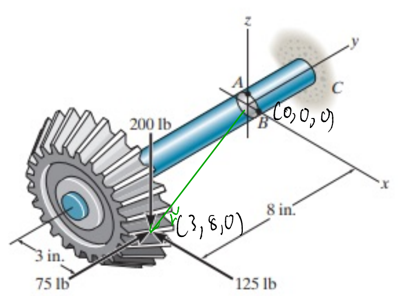
σ_x

$$\tau \Rightarrow \frac{(300 \ N)(5333.33 \ m^3)}{(40000\pi \ mm^4)(40 \ mm)}$$

$$\boxed{\tau = -0.318 \ MPa}$$

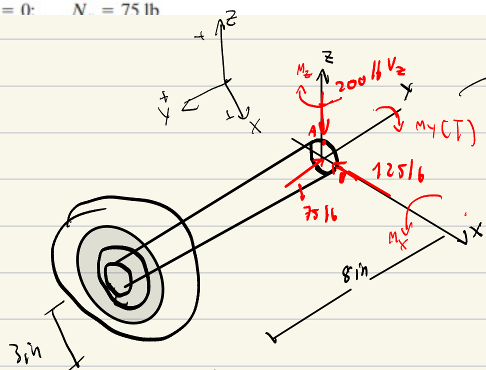


•8–61. The beveled gear is subjected to the loads shown. Determine the stress components acting on the shaft at point *A*, and show the results on a volume element located at this point. The shaft has a diameter of 1 in. and is fixed to the wall at *C*.



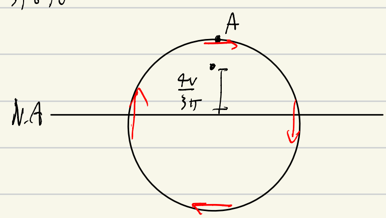
$\Sigma F_x = 0; \quad V_x - 125 = 0; \quad V_x = 125 \text{ lb}$

$\Sigma F_y = 0; \quad 75 - N_y = 0; \quad N_y = 75 \text{ lb}$



External Force act on the Cross section.

r = 3, 8, 0



$$+\searrow \Sigma F_x = 0 \rightarrow V_x = 125 \text{ lb}$$

$$\swarrow + \Sigma F_y = 0 \rightarrow V_y = 75 \text{ lb}$$

$$+\uparrow \Sigma F_z = 0 \rightarrow V_z = 200 \text{ lb}$$

External Moment.

(negative direct)

$$M_x \Rightarrow 200(8) = 1600; \quad M_y = 200(3) = 600 \text{ lb}\cdot\text{in}$$

$$M_z \Rightarrow -125(8) + 75(3) = -775 \text{ lb}$$

External $\Rightarrow$ $M_x \Rightarrow 1600 \text{ lb}\cdot\text{in}$; $M_y = -600 \text{ lb}\cdot\text{in}$; $M_z = -775 \text{ lb}\cdot\text{in}$

Plane Z-X

Point A.

$$\sigma_{yz} = \tau_{xy} = 0$$

$$\sigma_{zx} = -\frac{P}{A} \pm \cancel{\frac{M_z x}{I_z}} \pm \frac{M_x z}{I_x}$$

$$\tau_{zx} = \frac{Tc}{J} \pm \cancel{\frac{V_z Q}{I_y t}} \pm \frac{V_x Q}{I_z t}$$

Section Property.

$$I = \frac{\pi}{4}(0.5)^4 \Rightarrow 0.049 \text{ in}^4$$

$$A = \pi(0.5)^2 \Rightarrow \frac{\pi}{4} \text{ in}^2$$

$$J = \frac{\pi}{2}(r)^4 = 2I \Rightarrow 0.098 \text{ in}^4$$

$$Q = \Sigma \bar{y}A = \frac{2}{3}r^3 = \frac{1}{12}$$

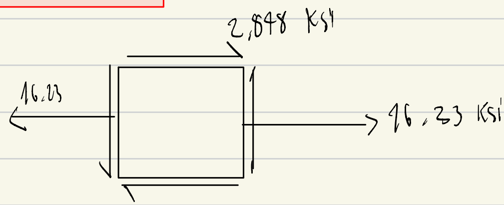
$$\sigma_{zx} = \frac{-75 \text{ lb}}{\frac{\pi}{4} \text{ in}^2} + \left| \frac{(1600 \overset{\text{lb}\cdot\text{in}}{})(0.5 \text{ in})}{0.049 \text{ in}^4} \right|$$

$$\boxed{\sigma_{zx} \Rightarrow 16.23 \text{ ksi}} \rightarrow \text{Tension}$$

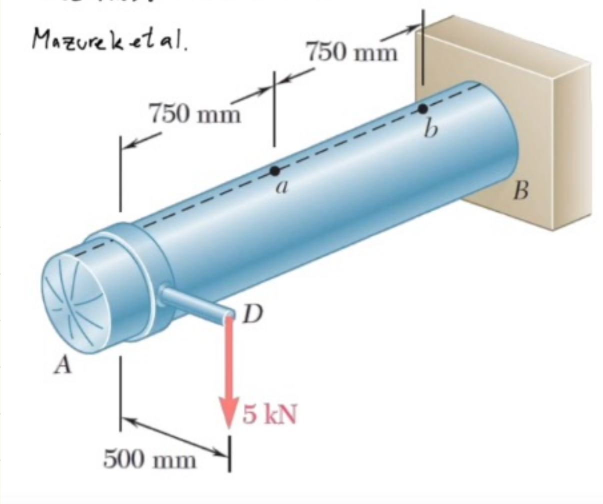
$$\tau_{zx} = +\frac{600(0.5)}{0.098 \text{ in}^4} - \frac{125 \text{ lb}\left(\frac{1}{12}\right)}{(0.049)(1 \text{ in})}$$

$$\boxed{\tau_{zx} = 2.848 \text{ ksi}}$$

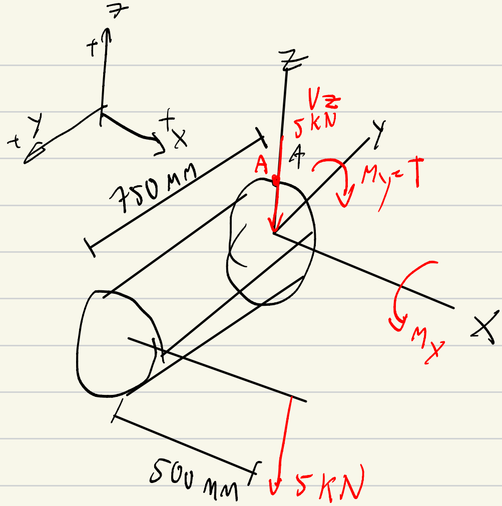
Stress element.



Mazurek et al.



$r_i = 225$ mm ; $p = 1.2$ MPa ; $t = 6$ mm



$\sum F_x = 0 \Rightarrow V_x = 0$

$\sum F_y = 0 \rightarrow V_y = 0$

$+\uparrow \sum F_z = 0 \rightarrow V_z = 5$ kN

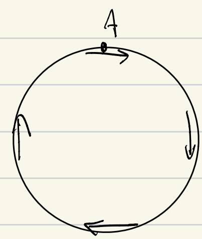
External Moment

$M = r \times F$

External moment: $M_x = 3750$ kN·mm (5×750)

$M_y = -2500$ kN·mm $(-5(500))$

$M_z = 0$ kN m



Plane zx σ_{zy} ; $\sigma_{xy} = 0$

$$\sigma_{xz} \Rightarrow \cancel{\frac{P}{A}} \overset{+}{\pm} \frac{M_x z}{I_x} \pm \cancel{\frac{M_z x}{I_z}}$$

$$\tau_{xz} \Rightarrow + \frac{Tr}{J} \pm \cancel{\frac{V_x Q}{I_z t}} \pm \cancel{\frac{V_z Q}{I_x t}}$$ → ตำแหน่ง A

Section Property $\Rightarrow$ $r_i = 225$; thickness $\Rightarrow$ 6

$$I = \frac{\pi}{4}(231^4 - 225^4)$$

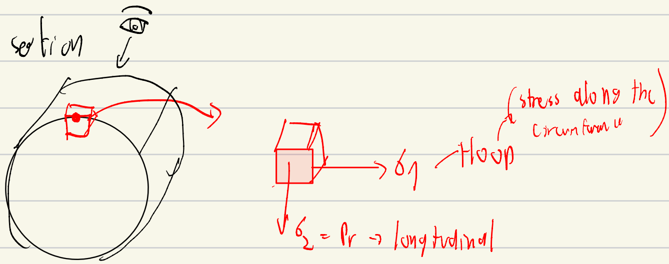
$$\sigma_{xz} = \frac{(3750 \times 10^3\ N\ mm)(231\ mm)}{\frac{\pi}{4}(231^4 - 225^4)}$$

$$\sigma_{xz} = 3.88\ MPa$$

$$\tau_{xz} \Rightarrow \frac{(2500 \times 10^3)(231)}{\frac{\pi}{2}(231^4 - 225^4)} \Rightarrow 1.29\ MPa$$

Also if we look at the Crossection and the pressure is given Hence there is $\sigma_1 = \frac{Pr}{t}$; $\sigma_2 = \frac{Pr}{2t}$

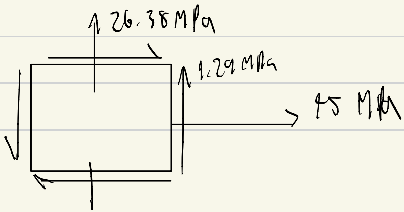
If we take a cut at the cross section



$$\sigma_1 = \frac{Pr}{t} = \frac{1.2\ MPa\ (225)}{6} = 45\ MPa$$

$$\sigma_2 = \frac{45}{2}\ MPa = 22.5\ Mp$$

Hence for longitudinal axis $\Rightarrow$ $22.5 + 3.88 = 26.38\ MPa$



Principal Stress $\Rightarrow \sigma_{1,2} \Rightarrow \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$$= \frac{45 + 26.38}{2} \pm \sqrt{\left(\frac{45 - 26.38}{2}\right)^2 + (1.29)^2}$$

$$\sigma_1 \Rightarrow 45.088 \quad ; \quad \sigma_2 \Rightarrow 26.291$$

Principal

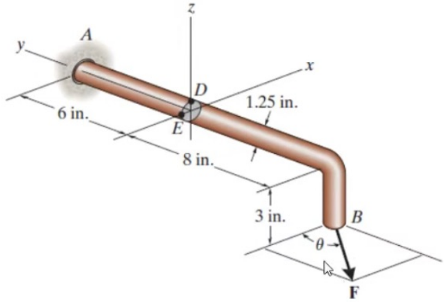
τ_{max} in plane $\Rightarrow \sqrt{\left(\frac{45 - 26.38}{2}\right)^2 + (1.29)^2}$

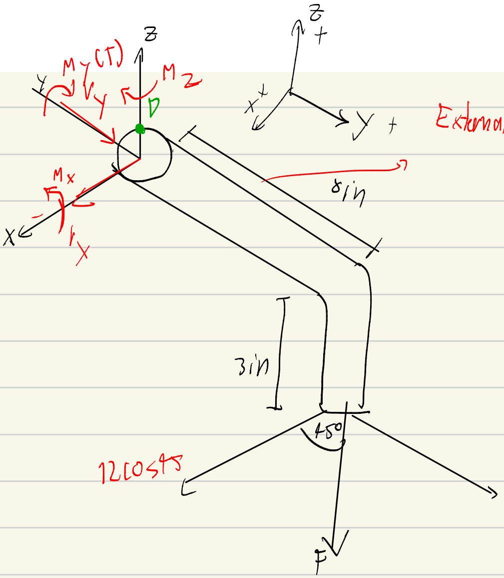
$$= \tau_{max} = 9.398$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} \Rightarrow 35.69$$

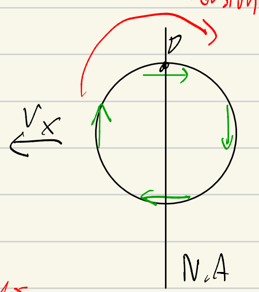
Combined Loading Example

- Determine the stress components at point D. Show the results on a volume element. Take F = 12 lb and θ = 45°.





External force on the area
cross sec



$$\sum F_x = 0 \Rightarrow V_x = 12\cos 45 \rightarrow 6\sqrt{2}\ \text{lb}$$

$$\sum F_y = 0 \Rightarrow V_y \Rightarrow 12\sin 45 \rightarrow 6\sqrt{2}\ \text{lb}$$

$$\sum F_z = 0 \Rightarrow V_z = 0$$

Find External Moment. $\longrightarrow$ $M = r \times F \rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 8 & -3 \\ 6\sqrt{2} & 6\sqrt{2} & 0 \end{vmatrix}$

$$\Rightarrow 18\sqrt{2}\,\hat{i} - 18\sqrt{2}\,\hat{j} - 48\sqrt{2}\,\hat{z}$$

external M $\rightarrow$ $M_x = 18\sqrt{2}$ lb·in ; $M_y = -18\sqrt{2}$ lb·in ; $M_z = -48\sqrt{2}$ lb·in

Plane zx $\quad \sigma_{yx} = \sigma_{yz} = 0 \quad$ (Point D)

$$\sigma_{zx} \Rightarrow \overset{+\ \to \text{Tension}}{\frac{P}{A}} \pm \cancel{\frac{M_z x}{I_z}} \pm \overset{- \longrightarrow \text{Compress}}{\frac{M_x z}{I_x}}$$

$$\tau_{zx} \Rightarrow + \frac{Tr}{J} \pm \cancel{\frac{V_z Q}{I_x t}} \pm \overset{-}{\frac{V_x Q}{I_z t}}$$

Sectional Property $\Rightarrow$

$D = 1.25\ in \to r = \frac{5}{8}\ in$

$A \Rightarrow \pi\left(\frac{5}{8}\right)^2 \Rightarrow 1.227\ in^2$

$I \Rightarrow \frac{\pi}{4}\left(\frac{5}{8}\right)^4 \Rightarrow 0.1198\ in^4$

$J = 2I \Rightarrow 0.2396\ in^4$

$Q = \frac{2}{3} r^3 \Rightarrow \frac{2}{3}\left(\frac{5}{8}\right)^3 \Rightarrow 0.1628\ in^3$

Hence

$$\sigma_{zx} \Rightarrow \frac{6\sqrt{2}\ 16}{1.227\ in^2} - \frac{18\sqrt{2}\left(\frac{5}{8}\right)}{0.1198}$$

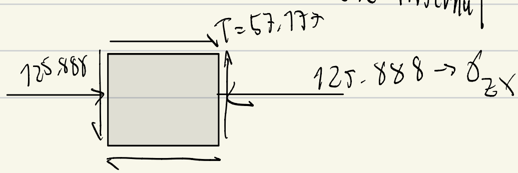
$$\boxed{\sigma_{zx} = -125.888\ \text{psi}} \to \text{Compressed.}$$

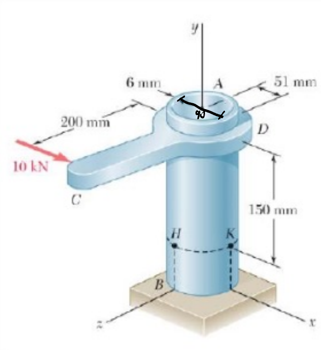
$$\tau_{zx} \Rightarrow \frac{18\sqrt{2}\left(\frac{5}{8}\right)}{0.2396} - \frac{6\sqrt{2}\,(0.1628)}{(0.1198)(1.25)}$$

$$\boxed{\tau_{zx} \Rightarrow 57.177\ \text{psi}}$$

or -57.177 if use internal

Stress element $\Rightarrow$

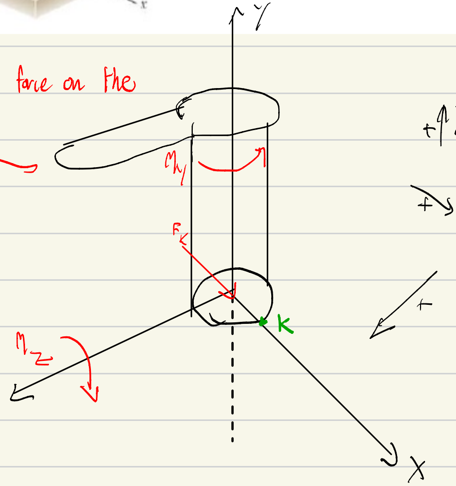




PROBLEM 7.26

The steel pipe AB has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm CD is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point K.

FBD of External force on the cross section



$+\uparrow \sum F_y = 0 \Rightarrow V_y = 0$

$\xrightarrow{+} \sum F_x = 0 \Rightarrow V_x = 10\ kN$

$\swarrow + \ \sum F_z = 0 \Rightarrow V_z = 0$

External Moment: $\sum M_z \Rightarrow (10 \times 0.15) = 1.5\ kN\text{-}m$ (To the right.)

Internal $\Rightarrow -1.5\ kN\cdot m$ (left)

$\uparrow \sum M_y = 0 \Rightarrow (10\ kN \times 0.2) = 2\ kN\cdot m$ positive

Torsion

Internal $\Rightarrow -2\ kN\cdot m$ (down)

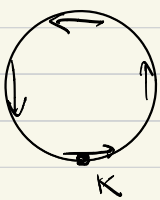
Plane x-z

$\sigma_{yz} = \sigma_{xy} = 0$

no normal force 0

$$\sigma_{x\text{-}z} \Rightarrow \cancel{\frac{P}{A}} \pm \cancel{\frac{M_x z}{I_x}} \pm \frac{M_z x}{I_z}$$

$-$



$$\tau_{x\text{-}z} \Rightarrow +\frac{Tr}{J} \pm \cancel{\frac{V_x Q}{I_z t}} \pm \cancel{\frac{V_z Q}{I_x t}} \rightarrow V_z = 0$$

0

Section Property

$$I = \frac{\pi}{4}(51^4 - 45^4) \Rightarrow 2.043 \times 10^{-6}\ m^4$$

$$J = 2I \Rightarrow 4.185 \times 10^{-6}$$

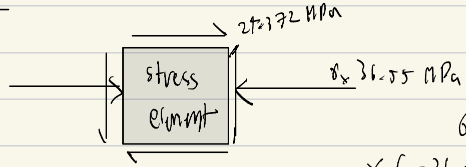
Hence →

$$\sigma_{x-z} \Rightarrow -\left| \frac{(1.5 \times 10^3)(0.051 m)}{(2.043 \times 10^{-6})\ m^4} \right|$$

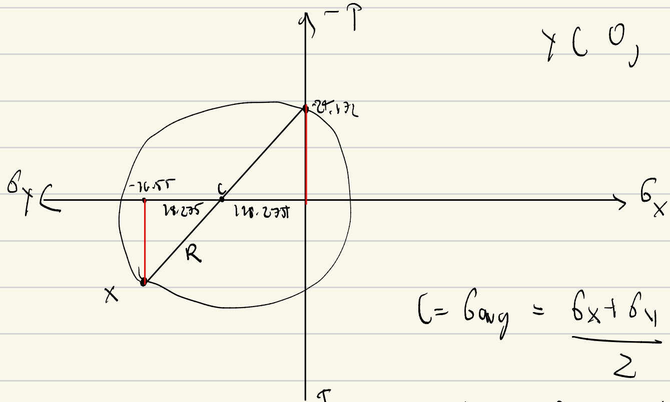
$$\sigma_{x-z} \Rightarrow -36.55\ MPa \quad (\text{Compressed})$$

$$\tau_{x-z} \Rightarrow \frac{(2 \times 10^3 N.m)(0.051 m)}{(4.185 \times 10^{-6}\ m^4)} \Rightarrow 24.372\ MPa$$

Principal Stress / Max Shear



	σ	τ
X	(-36.55,	24.372)
Y	(0,	-24.372)



$$C = \sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-36.55}{2} \Rightarrow -18.275$$

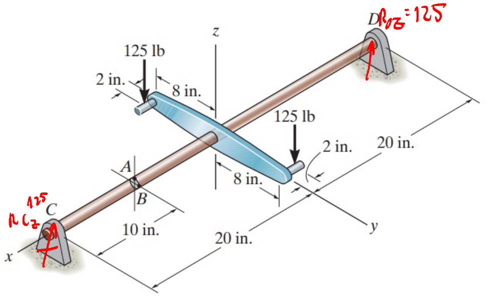
$$\text{Distance } C \Rightarrow \frac{\sigma_x - \sigma_y}{2} \Rightarrow \frac{-36.55}{2} \Rightarrow |-18.275|$$

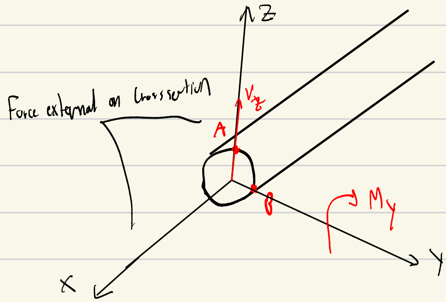
$$R = \sqrt{(18.275)^2 + (24.372)^2}$$

$$R \Rightarrow 30.463\ MPa$$ → Max Shear Stress

Principal Stress → $\sigma_{avg} \pm R \Rightarrow -18.275 \pm 30.463 \rightarrow \boxed{\begin{array}{l}\sigma_1 \Rightarrow 12.188\ MPa \\ \sigma_2 \Rightarrow -48.738\ MPa\end{array}}$

8–70. The $\frac{3}{4}$-in.-diameter shaft is subjected to the loading shown. Determine the stress components at point A. Sketch the results on a volume element located at this point. The journal bearing at C can exert only force components $\mathbf{C}_y$ and $\mathbf{C}_z$ on the shaft, and the thrust bearing at D can exert force components $\mathbf{D}_x$, $\mathbf{D}_y$, and $\mathbf{D}_z$ on the shaft.





Sum of Force.

$+\uparrow \sum F_z = 0 \Rightarrow V_z = 125$ lb

$\xrightarrow{+} \sum F_y = 0 \Rightarrow V_y = 0$

$\swarrow + \sum F_x = 0 \Rightarrow V_x = 0$

External Moment $\rightarrow$ $M_x = M_z = 0$; $M_y = 125(10) = 1250$ lb·in

Plane y-z

$\sigma_{zx} = \sigma_{xy} = 0$

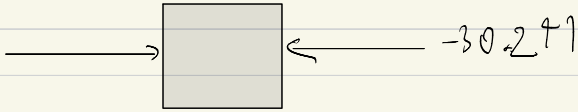
$$\sigma_{y-z} \Rightarrow \cancel{\frac{P}{A}} \pm \frac{M_y z}{I_y} \pm \cancel{\frac{M_z y}{I_z}}$$

$$\tau_{y-z} = \cancel{\frac{T\rho}{J}} \pm \cancel{\frac{V_y Q}{I_z t}} \pm \cancel{\frac{V_z Q}{I_y t}} \Big\} \ \tau_{y-z} = 0$$

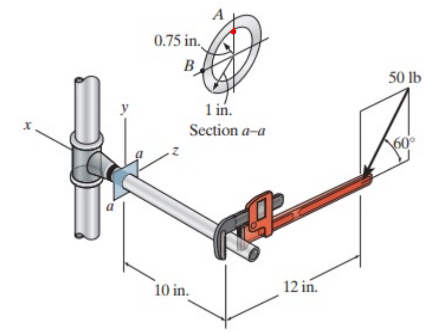
$I = \frac{\pi}{4}\left(\frac{3}{8}\right)^4 \Rightarrow 0.0155 \text{ in}^4$

$$\sigma_{y-z} \Rightarrow -\left|\frac{(1250 \text{ lb·in})(\frac{3}{8} \text{ in})}{0.0155 \text{ in}^4}\right| \Rightarrow -30.241 \text{ KPa}$$

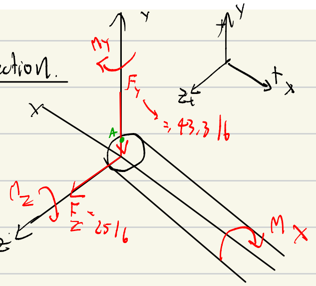
$\tau_{y-z} = 0$

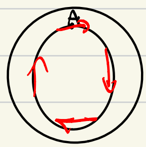


•8–65. Determine the state of stress at point A on the cross section of the pipe at section a–a.



External Force on the cross section.





Internal Force.

$\swarrow^{+} \sum F_z = 0 \Rightarrow V_z = 25\ \text{lb} \rightarrow (50 \cos 60)$

$+\uparrow \sum F_y = 0 \Rightarrow V_y = 43.3\ \text{lb} \quad (50 \sin 60)$

$\xrightarrow{+} \sum F_x = 0 \Rightarrow V_x = 0$

External Moment. $M = r \times F \rightarrow$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 10 & 0 & -12 \\ 0 & -43.3 & 25 \end{vmatrix}$$

$$-519.6\,\hat{i} - 250\,\hat{j} - 433\,\hat{k}$$

Torsion

$M_x = 519.6$ lb·in (left) ; $M_y = 250$ lb·in (left) ; $M_z = 433$ lb·in (left)

Plane zy. $\sigma_{xy} = \sigma_{xz} = 0$

$$\sigma_{zy} = \cancel{\frac{P}{A}} \overset{+}{\pm} \frac{M_z y}{I_z} + \cancel{\frac{M_y z}{I_y}}$$

$$\tau_{zy} = + \frac{Tc}{J} \overset{-}{\pm} \frac{V_z Q}{I_y t} \pm \cancel{\frac{V_y Q}{I_z t}}$$

Section Property.

$$I = \frac{\pi}{4}(1^4 - 0.75^4) \Rightarrow 0.537 \text{ in}^4$$

$$J = 2I \Rightarrow 1.0737 \text{ in}^4$$

$$Q = \frac{2}{3}(1^3 - 0.75^3) \Rightarrow 0.385 \text{ in}^3$$

At point A.

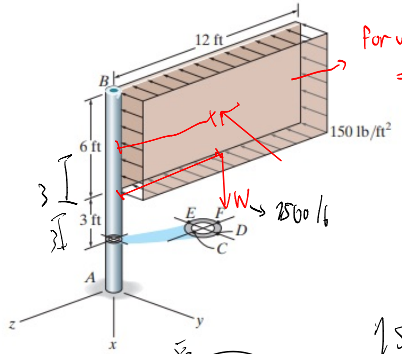
$$\leftarrow \sigma_{zy} = \frac{(433 \text{ lb}\cdot\text{in})(0.75)}{(0.537 \text{ in}^4)} \Rightarrow 604.74 \text{ psi} \text{ (Tension)}$$

!!! At point A → use 0.75

$$\tau_{zy} \Rightarrow \frac{(579.6 \text{ lb}\cdot\text{in})(0.75 \text{ in})}{1.0737 \text{ in}^4} - \frac{25 \text{ lb}(0.385)}{(0.537)(0.5 \text{ in})}$$

$$\tau_{zy} = 327 \text{ psi}$$

8–63. The uniform sign has a weight of 1500 lb and is supported by the pipe AB, which has an inner radius of 2.75 in. and an outer radius of 3.00 in. If the face of the sign is subjected to a uniform wind pressure of $p = 150\ \text{lb/ft}^2$, determine the state of stress at points C and D. Show the results on a differential volume element located at each of these points. Neglect the thickness of the sign, and assume that it is supported along the outside edge of the pipe.



For uniform load

$= 150\ \text{lb/ft}^2 \times 12\ \text{ft} \times 6\ \text{ft}$

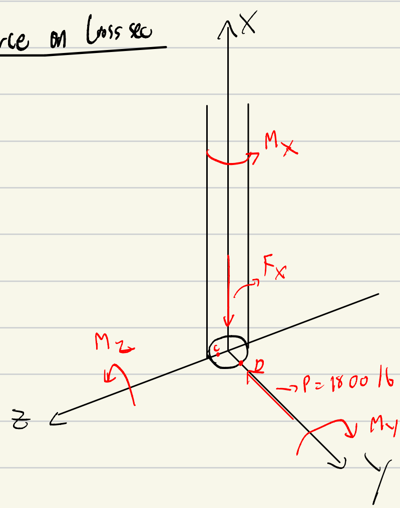
$= 10800\ \text{lb}$

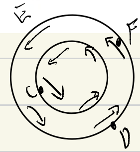
$150\ \text{lb/ft}^2$

$\frac{150}{\text{ft}^2} = 144\ \text{ft}^2$

$150\ \text{lb/ft}^2 \times$

FBD of External force on Cross sec





Internal Force

$+\uparrow \sum F_x = 0 \Rightarrow V_x = 1500\ \text{lb}$

$\xrightarrow{+} \sum F_y = 0 \Rightarrow V_y = 1800\ \text{lb}$

$+\sum F_z = 0 \Rightarrow V_z = 0\ \text{lb}$

External Moment.

$M_x \Rightarrow 10800(6\ \text{ft}) = 64800\ \text{lb}\cdot\text{ft}$ ↑ (up)

Torsion

Internal = -10800

$M_z \Rightarrow 10800\ (6\ \text{ft}) = 64800\ \text{lb}\cdot\text{ft}$ left (Positive)

Internal -10800

$M_y \Rightarrow 1500\ (6\ \text{ft}) = -9000\ \text{lb}\cdot\text{ft}$ left (Negative direct)

Internal => to the right

Plane z-y $\quad \sigma_{yx} = \sigma_{zx} = 0$

At Point C. $\quad \sigma_{zy} = -\frac{P}{A} \pm \cancel{\frac{M_z y}{I_z}} \pm \frac{M_y z}{I_y}$; $\quad \tau_{zy} = \frac{Tr}{J} \pm \cancel{\frac{V_z Q}{I_y t}} \pm \frac{V_y Q}{I_z t}$

$V_z = 0$

Section Property

$I = \frac{\pi}{4}(3^4 - 2.75^4) = 18.7\ in^4$

$A = \pi(3^2 - 2.75^2)$
$= 4.516\ in^2$

$J = 2I \Rightarrow 37.39\ in^4$

$Q = \frac{2}{3}(3^3 - 2.75^3) \Rightarrow 4.135\ in^3$

$t = 0.5\ in$

$$\sigma_{zy} \Rightarrow -\frac{1500}{(4.516\ in^2)} + \frac{(9000\ lb\text{-}ft \times 12\frac{in}{ft})(2.75)}{(18.7)}$$

$\sigma_{zy} \Rightarrow 15.55\ ksi$ (Tension)

$$\tau_{zy} \Rightarrow \frac{(64800\ lb\text{-}ft \times 12\frac{in}{ft})(2.75)}{37.39\ in^4} - \frac{(20800)(4.135)}{(18.7)(0.5)}$$

$\tau_{zy} \Rightarrow 52.195\ ksi$

If use internal = negative (opposite side)

At Point D

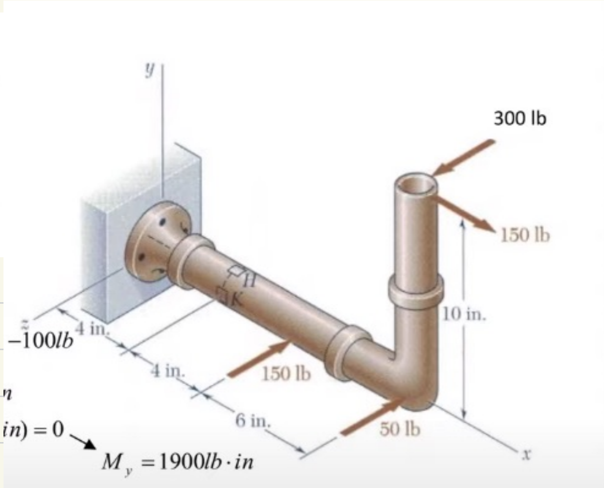
$$\sigma = \frac{-P}{A} + \frac{M_z y}{I_z} \quad ; \quad \tau = \frac{Tr}{J}$$

$$= \frac{-1500}{(4.516)} + \frac{(64800\ lb\text{-}ft \times 12\frac{in}{ft})(3\ in)}{18.7}$$

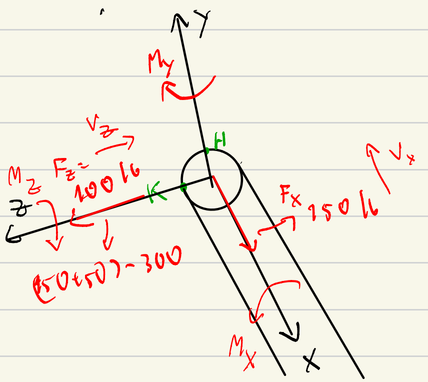
$\sigma = 124.41$ ksi T ~~A~~

$$\tau = \frac{(64800\ lb\text{-}ft \times 12\frac{in}{ft})(3)}{37.39\ in^4} \Rightarrow 62.39\ ksi$$

2. (40 points) Calculate the normal stress and shear stresses acting at point H. Be sure to clarify the direction(s) of the shear stresses and whether the normal stress is in tension or compression. Assume the pipe is a solid, circular cross section, with a diameter of 1.5 in.

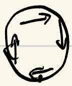


FBD At the Cross section (external).



$-300(10) + 150(4) + 50(10)$

-1900



Internal Force → $\sum F_x = 0 = V_x = 150\,lb \rightarrow P$

$\sum F_z = 0 \Rightarrow V_z = 100\,lb$

both opposite from F_x & F_z direction.

$+\uparrow \sum F_y = 0 \Rightarrow V_y = 0$

External Moment act on the plane → $\sum M_y = -300(10) + 150(4) + 50(10)$

$\Rightarrow M_y = -1900$ (down - negative direct)

$\Rightarrow M_z = 150 \times 10 = 1500\,lb \cdot in$ (to the negative)

$\Rightarrow M_x \Rightarrow 3000\,lb \cdot in$ (positive)

$\boxed{\text{At Point H}}$

Plane Z-Y. $\sigma_{xy} = \sigma_{xz} = 0$

$$\sigma_{zy} \Rightarrow +\frac{P}{A} \pm +\frac{M_z y}{I_z} \pm \cancel{\frac{M_y z}{I_y}}$$

$$\tau_{zy} \Rightarrow -\frac{Tr}{J} \pm -\frac{V_z Q}{I_y t} \pm \cancel{\frac{V_y Q}{I_z t}}$$

$\Downarrow$

same direction

Section Property. $\longrightarrow$

$$I = \frac{\pi}{4}(0.75^4) = 0.248 \text{ in}^4$$

$$J = 2I \Rightarrow 0.248(2) \Rightarrow 0.497 \text{ in}^4$$

$$Q = \frac{2}{3}(r^3) \Rightarrow \frac{2}{3}(0.75^3) \Rightarrow 0.281 \text{ in}^3$$

$$A = \pi(0.75^2) \Rightarrow 1.767 \text{ in}^2$$

$t = 1.5$ in

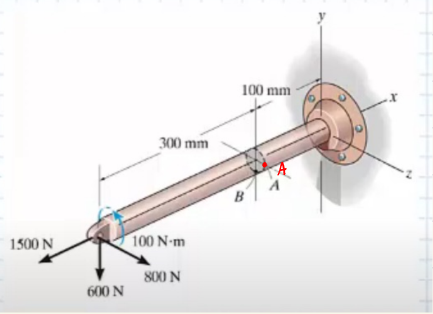
$$\sigma_{zy} \Rightarrow \frac{(150)(6)}{(1.767)} + \frac{(1500)(0.75)}{0.248}$$

$$\boxed{\sigma_{zy} \Rightarrow 4621.18 \text{ psi or } 4.621 \text{ ksi (Tension)}}$$

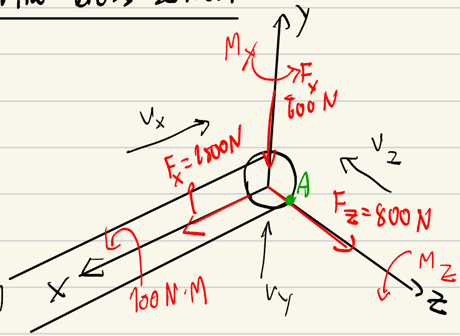
$$\tau_{zy} \Rightarrow -\frac{Tr}{J} - \frac{V_z Q}{I_y t} \Rightarrow -\frac{(3000)(0.75)}{0.497} - \frac{100(0.281)}{(0.248)(1.5)}$$

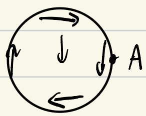
$$\boxed{\tau_{zy} = -4.602 \text{ ksi}}$$

For the loads shown, find the stress tensor at point *A* for the 40-mm diameter rod.



FBD for External force act on the cross section





Internal Force

$\swarrow + \sum F_x = 0 \Rightarrow V_x + 1500 = 0$

$V_x = -1500\,N$

$+\searrow \sum F_z = 0 \Rightarrow V_z + 800 =$

$V_z = -800\,N$

$\uparrow + \sum F_y = 0 \Rightarrow V_y - 600N = 0$

$V_y = 600N$

External force act on the Cross section

$M_z \Rightarrow 600N(0.3) = 180\,N\cdot M$

& Positive z axis

$M_y \Rightarrow 800(0.3) = 240\,N\cdot M$

Positive y-axis

$M_x = 0$

Plane Y-Z = $\sigma_{xz} = \sigma_{xy} = 0$

$$\sigma_{yz} \Rightarrow +\frac{P}{A} \pm -\frac{M_y z}{I_y} \pm \cancel{\frac{M_z y}{I_z}}$$

$$\tau_{yz} = -\frac{Tr}{J} \pm + \frac{V_y Q}{I_z t} \pm \cancel{\frac{V_z Q}{I_y t}}$$

Section Property

$I = \frac{\pi}{4}(20)^4 = 1.2566 \times 10^{-7}\,m^4$

$Q = \frac{2}{3}(20^3) \Rightarrow 5.33 \times 10^{-6}\,m^3$

$J = 2I = 2.5132 \times 10^{-7}\,m^4$

$A = \pi(0.02^2) \Rightarrow 1.2567 \times 10^{-3}\,m^2$

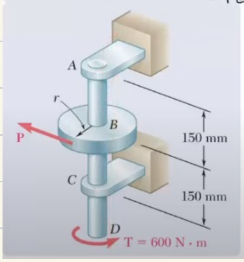
$$\sigma_{yz} \Rightarrow \frac{1500\,N}{(1.2567\times10^{-3})} - \frac{240\,N\cdot m\,(0.02\,m)}{(1.2566\times10^{-7})} \Rightarrow -37.00\ MPa$$

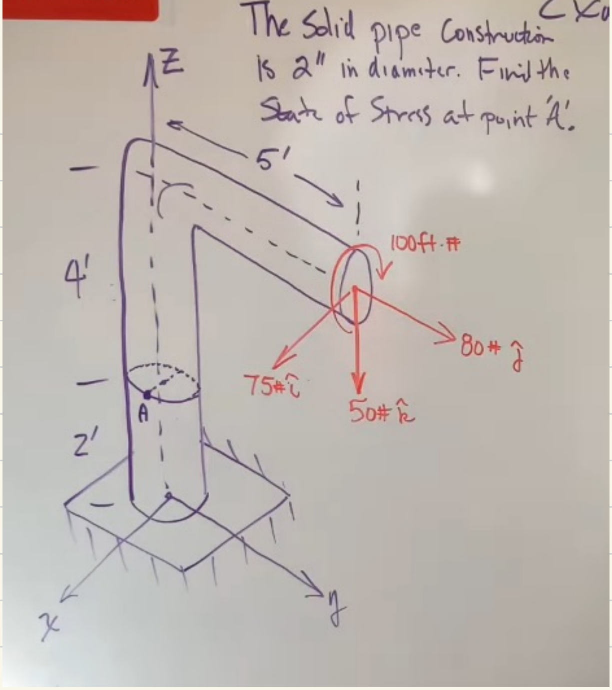
(compressed)

$$\tau_{yz} = \frac{(600)(5.33\times10^{-6})}{(1.2566\times10^{-7})(0.04\,m)} - \frac{(100\,N\cdot m)(0.02\,m)}{(2.5132\times10^{-7}\,m^4)}$$

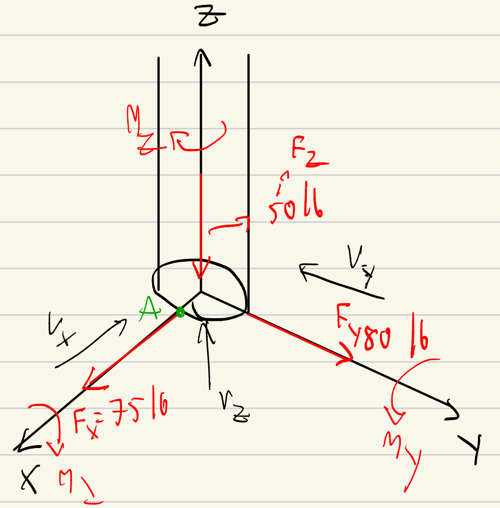
$$\tau_{yz} = -7.322\ MPa$$

Determine the smallest allowable diameter of the solid shaft $ABCD$, knowing that $\tau_{all} = 60$ MPa and that the radius of disk B is $r = 80$ mm. Free to rotate at A and C, ignore transverse shear stress.





FBD of force at the cross section



Sum of Internal Force

$$+\swarrow \sum F_x = 0 = V_x + 75 = 0 \rightarrow V_x = -75 \text{ lb}$$

$$\xrightarrow{+} \sum F_y = 0 = -V_y + 80 = 0 \rightarrow V_y = 80 \text{ lb}$$

$$+\uparrow \sum F_z = 0 \Rightarrow +V_z - 50 = 0 \rightarrow V_z = 50 \text{ lb}$$

External Moment

$$M_y = 75(4) - 100$$

$$M_y = 200 \text{ lb-ft} \quad \text{(Positive y-axis)}$$

for Plane x-y $\Rightarrow \sigma_{yz} = \sigma_{xz} = 0$

$$\sigma_{xy} = -\frac{P}{A} \pm \cancel{\frac{M_x y}{I_x}} \pm -\frac{M_y x}{I_y}$$

$$M_x = 50(5) + 80(4)$$

$$M_x = 570 \text{ lb-ft} \quad \text{(toward -x axis)}$$

$$\tau_{xy} = -\frac{Tr}{J} \pm \cancel{\frac{V_x Q}{I_y t}} \pm + \frac{V_y Q}{I_x t}$$

$$M_z = 75(5) = 375 \text{ lb-ft}$$

Sectional Property $\rightarrow I = \frac{\pi}{4}(1)^4 \Rightarrow \frac{\pi}{4}\,in^4$

$$J = 2I = \frac{\pi}{2}\,in^4$$

$$Q = \frac{2}{3}(1)^3 = \frac{2}{3}\,in^3$$

$$A = \pi(1)^2 = \pi$$

$$\sigma_{xy} = -\frac{50}{\pi} - \frac{(200 \times 12)(1)}{\frac{\pi}{4}}$$

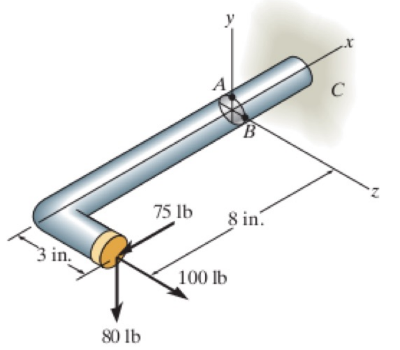
$$\boxed{\sigma_{xy} = -3.071\ ksi}$$ → Compressive.

$$\tau_{xy} \Rightarrow -\frac{(375 \times 12)(1)}{\frac{\pi}{2}} + \frac{(80)(\frac{2}{3})}{\frac{\pi}{4}(2)}$$

$$\boxed{\tau_{xy} \Rightarrow -2.83\ ksi}$$

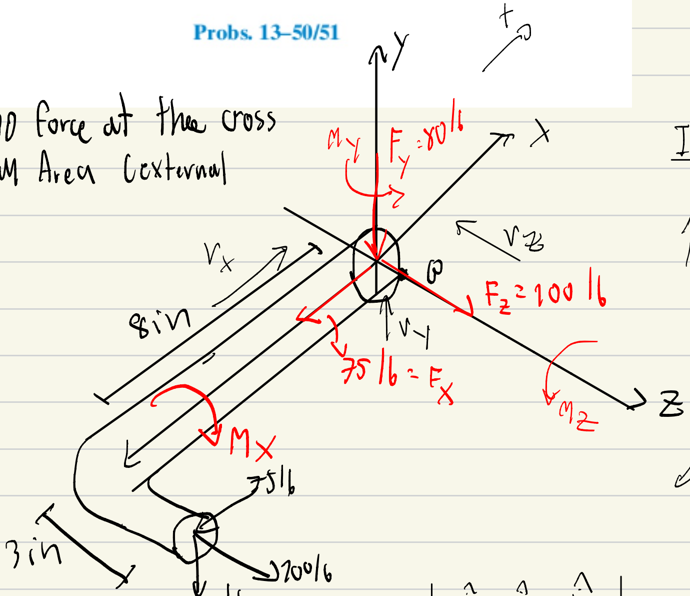
13–50. The 1-in.-diameter rod is subjected to the loads shown. Determine the state of stress at point A, and show the results on a differential element located at this point.

13–51. The 1-in.-diameter rod is subjected to the loads shown. Determine the state of stress at point B, and show the results on a differential element located at this point.



Probs. 13–50/51

FBD force at the cross sectional Area (external)



Internal Force

$\uparrow + \sum F_y = 0 \Rightarrow -80 + V_y = 0$

$V_y = 80$ lb

$\nearrow + \sum F_z = 0 \Rightarrow +100 + V_z = 0$

$V_z = -100$ lb

$\swarrow + \sum F_x = 0 \Rightarrow +75 \text{ lb} - V_x = 0$

$V_x = 75$ lb

External Moment

$M = |r \times F|$ → Magnitude

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -8 & 0 & 3 \\ -75 & -80 & 100 \end{vmatrix} = +240\hat{i} - (800 + 225)\hat{j} + (+640)\hat{k} \quad \checkmark \text{ Internal}$$

$M_x = 240$ lb-in (to the right) ; $M_y = 575$ lb·in up (Positive Y axis)

$M_z = 640$ lb·in (to the right)

Plane zy. $\sigma_{xz} = \sigma_{yx} = 0$

At Point B.

$$\sigma_{yz} = {}^{+}\frac{P}{A} \pm {}^{-}\frac{M_y z}{I_y} \pm \cancel{\frac{M_z y}{I_z}}$$

$$\tau_{yz} = {}^{+}\frac{Tr}{J} \pm \cancel{\frac{V_z Q}{I_y t}} \pm {}^{+}\frac{V_y Q}{I_z t}$$

Section Property.

$$A = \pi(0.5)^2 = \frac{\pi}{4}\ in^2$$

$$I = \frac{\pi}{4}(0.5)^4 = 0.049\ in^4$$

$$J = 2I = 0.0981\ in^4$$

$$Q = \frac{2}{3}(0.5)^3 \Rightarrow 0.083\overline{3}\ in^3$$

Hence

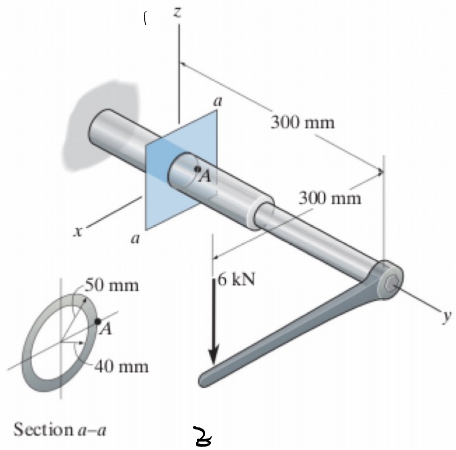
$$\sigma_{yz} = \frac{(75\ lb)}{\frac{\pi}{4}} - \frac{(575\ lb\cdot in)(0.5)}{0.049}$$

$$\boxed{\sigma_{yz} = -5.771\ ksi\ \text{(Compressed)}}$$

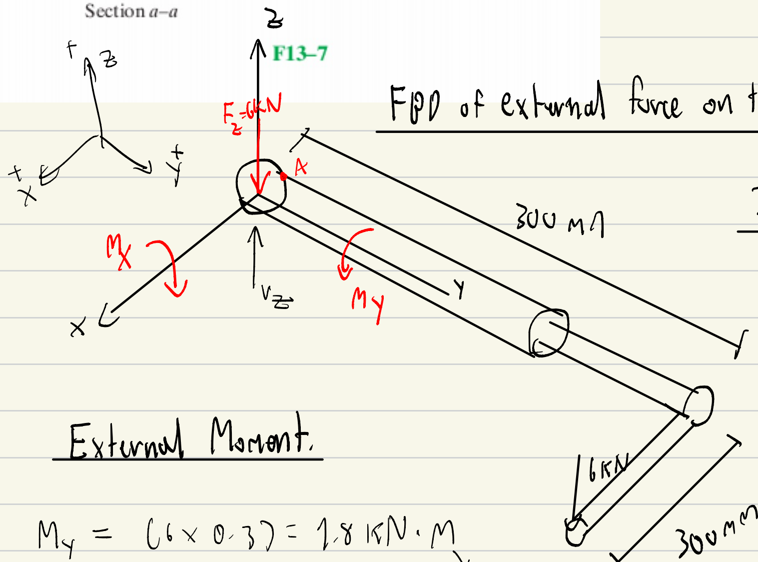
$$\tau_{yz} = \frac{(240)(0.5)}{0.0981} + \frac{(80)(0.0833)}{(0.049)(1\ in)}$$

$$\boxed{\tau_{yz} \Rightarrow 1.36\ ksi}$$

F13–7. Determine the state of stress at point A on the cross section of the pipe at section a–a. Show the results on a differential volume element at the point.



F13–7



FBD of external force on the cross section.

Internal force

$+\uparrow \Sigma F_z = 0 \Rightarrow -6\text{ kN} + V_z = 0$

$V_z = 6\text{ kN}$

$\xrightarrow{+} \Sigma F_y = 0 \Rightarrow V_y = 0$

$\swarrow + \Sigma F_x = 0 \Rightarrow V_x = 0$

External Moment.

$M_y = (6 \times 0.3) = 1.8\text{ kN}\cdot\text{m}$ + y direct

$M_x = (6 \times 0.3) = 1.8\text{ kN}\cdot\text{m} \rightarrow$ − x direct

$M_z = 0$

Plane x-z. $\sigma_{yz} = \sigma_{xy} = 0$

$\sigma_{x-z} = 0 \Rightarrow$

$$\tau_{xz} = -\frac{Tr}{J} \pm \cancel{\frac{V_x Q}{I_z t}} \overset{+}{\pm} \frac{V_z Q}{I_x t}$$

Section Property.

$$I = \frac{\pi}{4}(50^4 - 40^4) = 2.898 \times 10^{-6}\ m^4$$

$$J = 2I \Rightarrow 5.796 \times 10^{-6}\ m^4$$

$$Q = \frac{2}{3}(50^3 - 40^3) \Rightarrow 4.0666 \times 10^{-5}\ m^3$$

Hence

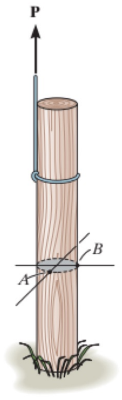
$$\tau_{xz} = -\frac{(1.8 \times 10^3)(0.05)}{(5.796 \times 10^{-6}\ m^4)} + \frac{(6 \times 10^3)(4.0666 \times 10^{-5})}{(2.898 \times 10^{-6})(0.02)}$$

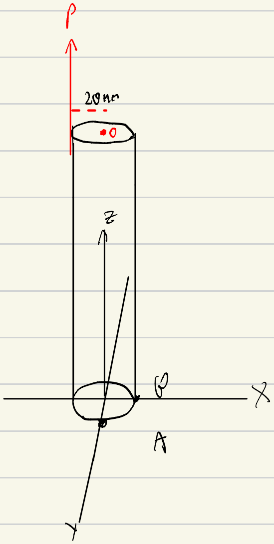
$$\boxed{\tau_{xz} \Rightarrow -11.318\ MPa}$$

***13–28.** The cylindrical post, having a diameter of 40 mm, is being pulled from the ground using a sling of negligible thickness. If the rope is subjected to a vertical force of $P = 500$ N, determine the normal stress at points A and B. Show the results on a volume element located at each of these points.

13–29. Determine the maximum load P that can be applied to the sling having a negligible thickness so that the normal stress in the post does not exceed $\sigma_{allow} = 30$ MPa. The post has a diameter of 50 mm.

13

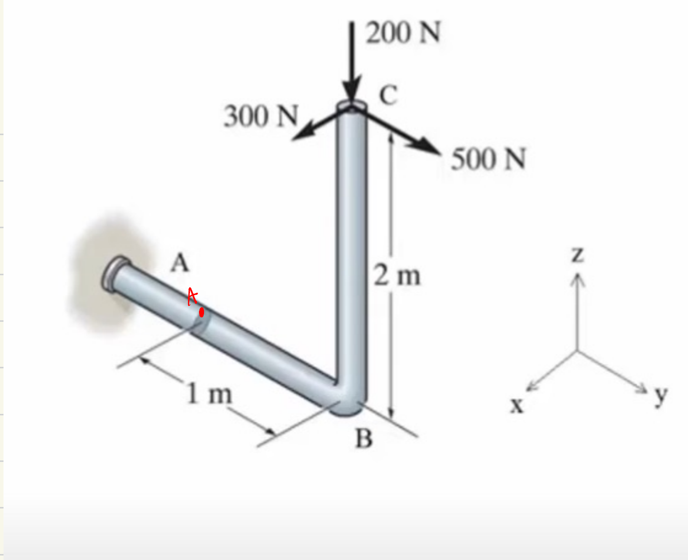




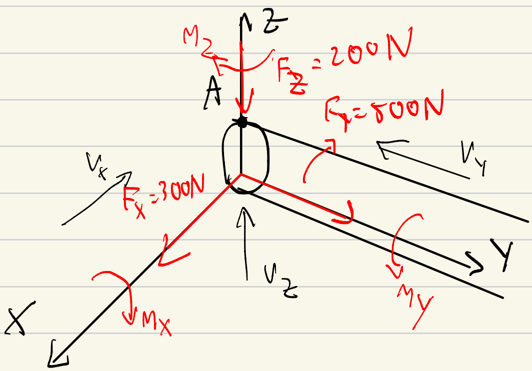
Normal Stress at A. $\sigma = \frac{P}{A} = \frac{500N}{\pi(20)^2} \Rightarrow 0.397$ MPa

Normal Stress at B. $\sigma = \frac{P}{A} - \frac{M_y X}{I_y} = (0.397) - \frac{(500 \times 20)(20)}{\frac{\pi}{4}(20)^4}$

$\sigma_B \Rightarrow -1.19$ MPa Compressed



FBD for force at the crosssection area



Internal force.

$\overset{+}{\rightarrow} \sum F_y = 0 \Rightarrow V_y = 500N$

$\uparrow + \sum F_z = 0 \Rightarrow V_z = 200N$

$\swarrow + \sum F_x = 0 \Rightarrow V_x = 300N$

External Moment act on the cross section

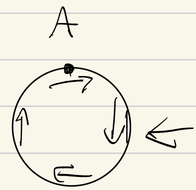
$$M = r \times F \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 2 \\ 300 & 500 & -200 \end{vmatrix} \Rightarrow (-200-1000)\hat{i} - (-600)\hat{j} + (-300)\hat{k}$$

$$= -1200\hat{i} + 600\hat{j} - 300\hat{k}$$

$M_x = 1200 N \cdot M$ (to the negative x-axis) ; $M_y = 600 N \cdot M$ (positive y-axis)

$M_z \Rightarrow 300$ N·M (negative z axis)

<u>Plane z-x</u>



$$\sigma_{yz} = \sigma_{xy} = 0$$

$$\sigma_{z-x} = +\frac{P}{A} \pm \cancel{\frac{M_z x}{I_z}} \pm +\frac{M_x z}{I_x}$$

$$\tau_{z-x} = -\frac{T_r}{J} \pm \cancel{\frac{V_z Q}{I_x t}} \pm -\frac{V_x Q}{I_z t}$$

<u>Sectional Property</u> $\rightarrow$ $I = \frac{\pi}{4}(20)^4 = 1.256 \times 10^{-7}\ M^4$

$\downarrow$ $d = 40$ mm

$J = 2I \Rightarrow 2.51327 \times 10^{-7}\ M^4$

$A = \pi(0.02)^2$
$= 1.2566 \times 10^{-3}\ M^2$

$Q = \frac{2}{3}(r)^3$ or $\left(\frac{4(20)}{3\pi}\right)\left(\frac{\pi}{2}(20)^2\right)$

$\downarrow$

$Q = (5.333 \times 10^{-6})$

$t = 0.04$ M

$$\sigma_{zx} = \frac{500\text{N}}{(1.256 \times 10^{-3})} + \frac{(1200\ \text{N·M})(0.02)}{(1.256 \times 10^{-7})}$$

$\sigma_{zx} \Rightarrow 191.48$ MPa

$$\tau_{z-x} \Rightarrow -\frac{(600\ \text{N·m})(0.02)}{(2.51327 \times 10^{-7})} - \frac{(300\text{N})(5.333 \times 10^{-6})}{(1.256 \times 10^{-7})(0.04)}$$

$\tau_{zx} \Rightarrow -48.065$ MPa

$\rightarrow$ same direction