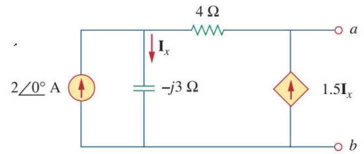
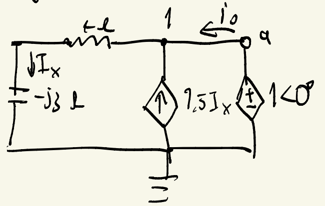
**Exercise 5.3** (30%)

(a) (15%) Find the Thevenin equivalent at terminal a-b of the circuit below. And also draw the phasor diagram of the Thevenin equivalent impedance.




Find $Z_{TH}$ → close all the independent source and apply $V = 1\angle 0°$

$$Z_{TH} = \frac{V}{I_o}$$




$$\frac{1}{4-j3} - 1.5I_x - i_o = 0$$

also → $I_x = \dfrac{1-0}{4-j3}$

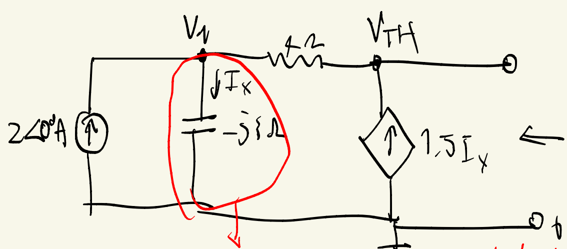
$$\frac{1}{4-j3} - 1.5\left(\frac{1}{4-j3}\right) = i_o$$

$$i_o = -0.08 - 0.06j$$

$$i_o = 0.1\angle -143.1301$$

$$Z_{TH} = \frac{V}{I_o} = \frac{1}{(0.1\angle -143.1301)} \approx -8 + 6j$$

Find $V_{TH}$ →




← $V_{TH} = V_{oc} = V_a - V_b$

current $I_x$ นับเป็นทางเส้นเดียว เส้นอื่นไม่เกี่ยว เพราะฉะนั้น current $2\angle 0°$ ไม่หาย

KCL at $V_1$

$$\left(\frac{-2\angle 0}{} + \frac{V_1}{-3j} + \frac{V_1 - V_{TH}}{4} = 0\right) - 12j$$

$$24j + 4V_1 - 3jV_1 + 3jV_{TH} = 0$$

$$(4-3j)V_1 + 3jV_{TH} = -24j \rightarrow (1)$$

KCL at $V_{TH}$ $\rightarrow$ $\left(\frac{V_{TH} - V_1}{4} - 1.5 I_x = 0\right) 4$

$$V_{TH} - V_1 - 6I_x = 0$$

$$I_x = \frac{V_1}{-3j}$$

$$V_{TH} - V_1 - 6\left(\frac{V_1}{-3j}\right) = 0$$

$$V_{TH} - V_1 + \frac{2V_1}{j} = 0$$

$$\left(\frac{2}{j} - 1\right)V_1 + V_{TH} = 0 \rightarrow (2)$$

use 1/2 $\rightarrow$ $\begin{vmatrix} (4-3j) & (3j) \\ \frac{2}{j} - 1 & 1 \end{vmatrix} \begin{pmatrix} V_1 \\ V_{TH} \end{pmatrix} = \begin{vmatrix} -24j \\ 0 \end{vmatrix}$

$\downarrow$

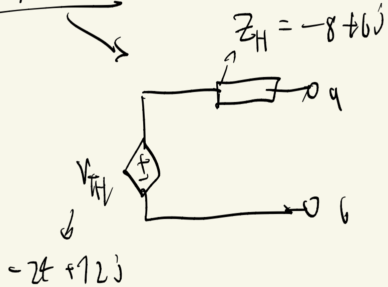
$$\Delta = -2$$

$$\Delta_2 \Rightarrow \begin{vmatrix} 4-3j & -24j \\ \frac{2}{j}-1 & 0 \end{vmatrix}$$

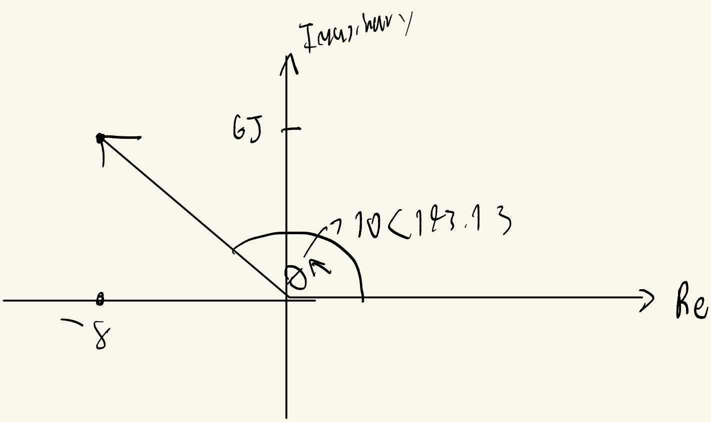
$$= 48 - 24j$$

$$V_{TH} = \frac{\Delta_2}{\Delta} = \frac{(48-24j)}{-2} \rightarrow \boxed{-24+12j}$$

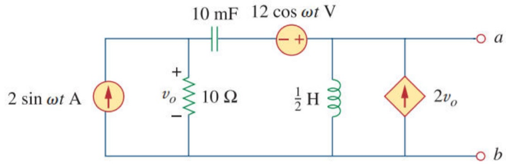
Thevenin Equivalent




Phasor diagram of $Z_{TH}$ $\rightarrow$ $-8+6j$



(b) (15%) Find the Norton equivalent at terminal a-b of the circuit below. And also draw the phasor diagram of the Norton equivalent impedance. Take $\omega = 10\ rad/s$.

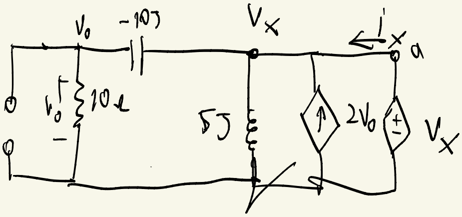



Find $I_N$ in phasor form

$$10\,mF = \frac{1}{(10\times10^{-3})(10)j} = -10j$$

$$\frac{1}{2}H = (10)\left(\frac{1}{2}\right)j = 5j$$

Find $Z_{TH}$ → close all independent source and apply 1 V → $1\angle 0°$




$$Z_{TH} = \frac{V_x}{I_x}$$

Source transform

KCL at $V_0$

$$\frac{V_0}{10} + \frac{V_0 - V_x}{-10J} = 0$$

$$\frac{V_0}{10} + \frac{1}{10}(JV_0 - JV_x) = 0$$

$$\left(\frac{1}{10} + \frac{J}{10}\right)V_0 = \frac{J}{10}V_x$$

$$\left(\frac{1+J}{10}\right)V_0 = \frac{J}{10}V_x$$

$$V_0 = \left(\frac{J}{1+J}\right)V_x$$

KCL at $V_x$

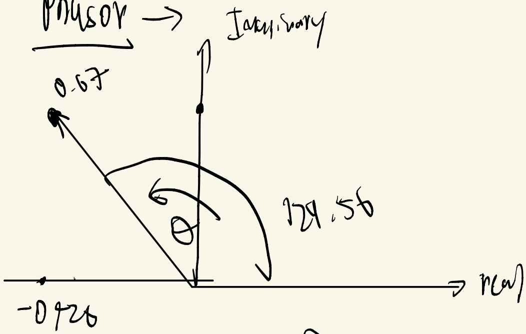
$$\frac{V_x - V_0}{-10J} + \frac{V_x}{5J} - 2V_0 - i_x = 0$$

$$\frac{V_x - \left(\frac{J}{1+J}\right)V_x}{-10J} + \frac{V_x}{5J} - 2\left(\frac{J}{1+J}\right)V_x = I_x$$

$$\frac{V_x\left(\frac{1}{2} - \frac{1}{2}j\right)}{-10J} + \left(\frac{1}{5J} - \frac{2J}{1+J}\right)V_x$$

$$V_x\left(\frac{-19}{20} - \frac{23}{20}j\right) = I_x$$

Phasor →




$$V_x = \frac{I_x}{\left(-\frac{19}{20} - \frac{23}{20}j\right)}$$

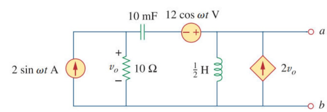
$$Z_{TH} = \frac{V_x}{I_x} = \frac{\dfrac{I_x}{\left(-\frac{19}{20} - \frac{23}{20}j\right)}}{I_x}$$

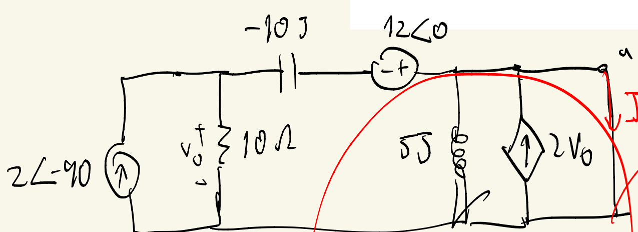
$$= \frac{1}{\left(-\frac{19}{20} - \frac{23}{20}j\right)}$$

$$= -\frac{38}{89} + \frac{46}{89}j \Rightarrow 0.67\angle 129.56^\circ$$

Find $I_N$ Short Circuit $I_{sc}$

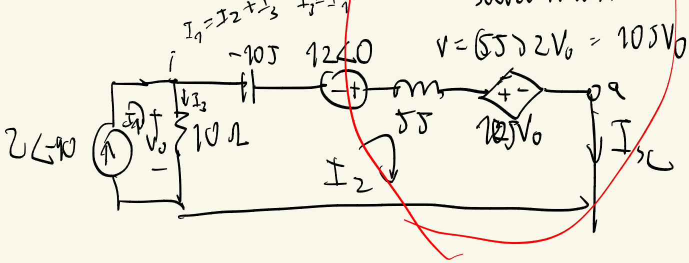
(b) (15%) Find the Norton equivalent at terminal a-b of the circuit below. And also draw the phasor diagram of the Norton equivalent impedance. Take $\omega = 10\ rad/s$.



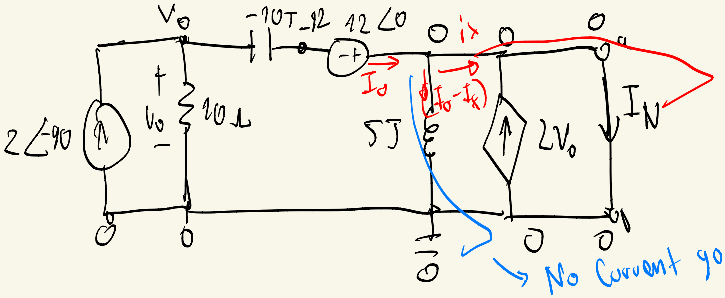

source transform

$V = (5j)\,2V_0 = 10jV_0$




\# WRONG

Can't use source transformation with the dependent source




$I_N = 2V_0 + I_0$

No Current goes down through $5J$ as both Top and bottom node $= 0$.

KCL at $V_0$ $\rightarrow$ $\left(-(-2J) + \dfrac{V_0}{10} + \dfrac{V_0 + 12}{-10J} = 0\right) - 10J$

$I_x + (I_0 - I_x) = I_0$

$-2V_0 + I_0 + 2V_0 = I_0$

$I_0 = I_0 = I_x$

$$-20J^2 - JV_0 + V_0 + 12 = 0$$

$$20 - JV_0 + V_0 + 12 = 0$$

$$(1 - J)V_0 = -32$$

$$V_0 = \frac{-32}{(1-J)} \approx -16 - 16J$$

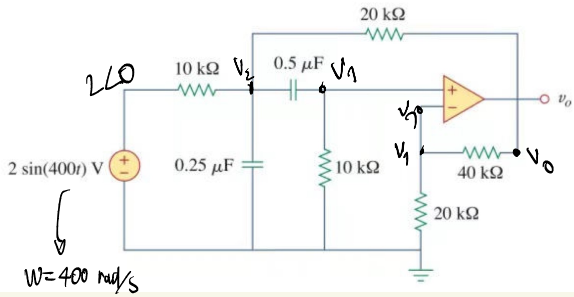
$I_N = 2V_0 + I_0$

$\rightarrow I_0 = \dfrac{V_0 + 12}{-10J}$

$$= 2(-16 - 16J) + \frac{(-16 - 16J) + 12}{-20J}$$

$$\boxed{I_N = 44.93 \angle -133.18^0}$$

**Exercise 5.4** (15%)
Determine $v_0(t)$ in the op amp circuit in the figure bellow.




$$0.5\mu F = \frac{1}{(400)(0.5\times10^{-6})(j)} = -5j\,K$$

$$0.25\mu F = \frac{1}{(400)(0.25\times10^{-6})j} = -10j\,K$$

KCL at $V_2$ $\rightarrow$ $\left(\frac{V_2-2}{10K} + \frac{V_2}{-10jK} + \frac{V_2-V_0}{20K} + \frac{V_2-V_1}{-5jK} = 0\right)20K$

$$\cancel{2V_2}-4 + \cancel{2jV_2} + \cancel{V_2} - V_0 + \cancel{4jV_2} - \cancel{4jV_1} = 0$$

$$(3+6j)V_2 - 4jV_1 - V_0 = 4 \rightarrow (1)$$

KCL at $V_1$ $\left(\frac{V_1-V_2}{-5jK} + \frac{V_1}{10K} = 0\right)10K$

$$2jV_1 - 2jV_2 + V_1 = 0$$

$$(1+2j)V_1 = 2jV_2 \rightarrow V_2 = (1-0.5j)V_1 \rightarrow (2)$$

KCL at $V_1$ $\rightarrow$ other side $\left(\frac{V_1-V_0}{40K} + \frac{V_1}{20K} = 0\right)40K$

$$V_1 - V_0 + 2V_1 = 0$$

$$3V_1 = V_0$$

$$V_1 = \frac{1}{3}V_0 \quad // \longrightarrow (3)$$

KCL at $V_0$ $\rightarrow$

$$\left( \frac{V_0 - V_1}{40K} + \frac{V_0 - V_2}{20K} = 0 \right) 40K$$

$$V_0 - V_1 + 2V_0 - 2V_2 = 0$$

$$3V_0 = V_1 + 2V_2$$

$$V_0 = \frac{1}{3}V_1 + \frac{2}{3}V_2$$

Sub (2) and (3) into (1)

$$(3+6j)(1-0.5j)\underbrace{V_1}_{\frac{1}{3}V_0} - 4j\left(\frac{1}{3}V_0\right) - V_0 = 4$$

$$\left(2 + \frac{3}{2}j\right)V_0 - \frac{4}{3}jV_0 - V_0 = 4$$

$$\left(1 + \frac{1}{6}j\right)V_0 = 4$$

$$V_0 = 3.9456 \angle -9.4623$$

$$V_0 = 3.9456 \sin(400t - 9.4623)$$

# Thevenin / Norton Equivalence

**10.59** Calculate the output impedance of the circuit shown in Fig. 10.102.

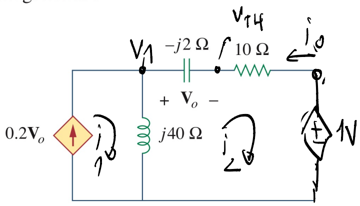



**Figure 10.102**
For Prob. 10.59.

out impedance $\Rightarrow Z_{TH}$

$\rightarrow i_2 = -i_0$

$$V_0 = -i_0(-j2) = 2j\,i_0$$

KVL at $i_1$ $\rightarrow$ $i_1 = 0.2V_0$

$$-j2(i_2) + 10i_2 + 1 + 40j(i_2 - i_1) = 0$$

$$2ji_0 - 10i_0 + 1 + 40j(-i_0) - 40ji_1 = 0$$

$$(-38j - 10)i_0 - 40j(0.2(2ji_0)) = -1$$

$$(-38j - 10)i_0 + 16i_0 = -1$$

$$(6 - 38j)i_0 = -1$$

$$i_0 = \frac{-3}{740} - \frac{19}{740}j$$

$$Z_{TH} = \frac{1}{\left(\frac{-3}{740} - \frac{19}{740}j\right)}$$

$$Z_{TH} = -6 + 38j$$

$$Z_{TH} = 38.47\angle 98.97$$

**10.60** Find the Thevenin equivalent of the circuit in Fig. 10.103 as seen from:

(a) terminals *a-b* (b) terminals *c-d*

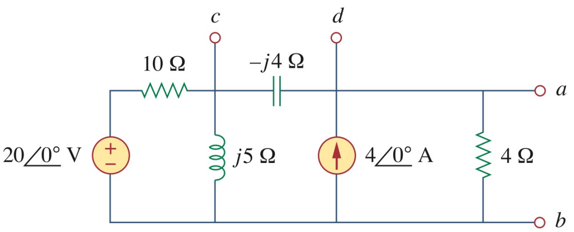
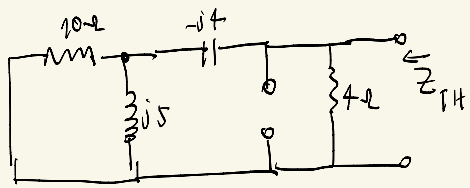



**Figure 10.103**
For Prob. 10.60.

at terminal ; a-b ; $V_{TH} = V_a - V_b$

Find $Z_{TH}$ → Close all independent source

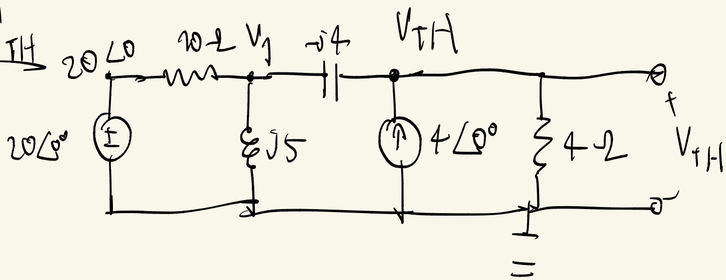



$10 // j5 \to \frac{50j}{10+j5} - j4 \quad // 4$

$2 // 4$

$= \frac{2 \times 4}{4+2} = \frac{8}{6} = \frac{4}{3}\ \Omega$

Find $V_{TH}$



KCL at $V_1$ $\rightarrow$ $\dfrac{V_1 - 20\angle 0}{10\,\Omega} + \dfrac{V_1}{j5} + \dfrac{V_1 - V_{TH}}{-j4} = 0$

$$\left(\frac{1}{10} + \frac{1}{20}j\right)V_1 + \frac{1}{4j}V_{TH} = 2 \rightarrow (1)$$

KCL at $V_{TH}$ $\rightarrow$ $\dfrac{V_{TH} - V_1}{-j4} - 4 + \dfrac{V_{TH}}{4} = 0$

$$\left(\frac{1}{4j}V_1\right) + \left(\frac{1}{4} + \frac{1}{4}j\right)V_{TH} = 4 \rightarrow (2)$$

find $V_{TH}$ $\rightarrow$ use $\Delta_2$

$$\begin{vmatrix} \left(\frac{1}{10} + \frac{1}{20}j\right) & \frac{1}{4j} \\ \frac{1}{4j} & \left(\frac{1}{4} + \frac{1}{4}j\right) \end{vmatrix} \begin{vmatrix} V_1 \\ V_{TH} \end{vmatrix} = \begin{vmatrix} 2 \\ 4 \end{vmatrix}$$

$$\Delta = \frac{3}{40} + \frac{3}{80}j$$

$$\Delta_2 = \begin{vmatrix} \left(\frac{1}{10} + \frac{1}{20}j\right) & 2 \\ \frac{1}{4j} & 4 \end{vmatrix} \Rightarrow \left(\frac{2}{5} + \frac{7}{10}j\right)$$

$$V_{TH} = \frac{\Delta_2}{\Delta} = 8 + \frac{16}{3}j \rightarrow 9.6148\angle 33.69^\circ$$ //

**10.62** Using Thevenin's theorem, find $v_o$ in the circuit of Fig. 10.105.

$\omega = 1$ ; $2H = 2j$

$\frac{1}{4}F = -4j$

$\frac{1}{8}F = -8j$

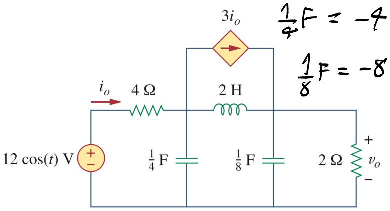
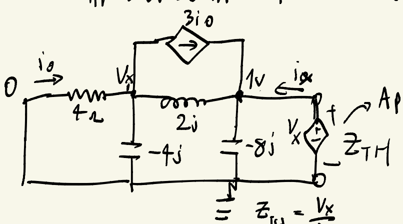



**Figure 10.105**
For Prob. 10.62.

Open circuit at $V_o$ as we want $V_o$

find $Z_{TH} \to$ close all independent source




Apply $V_x = 1V$

$Z_{TH} = \frac{V_x}{i_x}$

At Node 1 → $\frac{V_x}{4} + \frac{V_x}{-4j} + \frac{V_x - 1}{2} + 3i_o = 0$

$V_x$

$$\frac{V_x}{-j4} - \frac{2V_x}{4} = \frac{1 - V_x}{2j}$$

$$V_x = 0.4 + j0.8$$

At Node 1V → $\frac{1 - V_x}{2} - 3i_o + \frac{1}{-8j} - i_x = 0$

$$i_o = \frac{-V_x}{4}$$

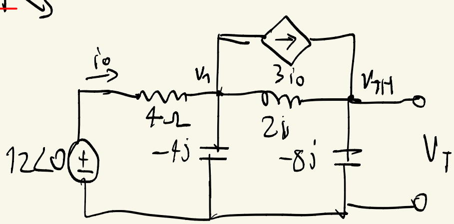
$$I_x = (0.75 + j0.5)V_x - j\frac{3}{8}$$

$$I_x = -0.1 + 0.425j$$

$$Z_{TH} = \frac{V}{I_x} = \frac{1}{(-0.1 + 0.425j)} = -0.5246 - j2.229$$

$$\boxed{Z_{TH} \Rightarrow 2.29\angle -103.24^0}$$

Find $V_{TH}$




KCL at $V_1$ $\quad \frac{V_1 - 12}{4} + \frac{V_1}{-4j} + 3i_o + \frac{V_1 - V_{TH}}{2j} = 0$

$$\left(\frac{1}{4} - \frac{1}{4}j\right)V_1 - \frac{1}{2j}V_{TH} = 3 - 3i_o \to (1)$$

$$i_o = \frac{12 - V_1}{4}$$

$$3 - 3i_o \Rightarrow 3 - 3\left(\frac{12 - V_1}{4}\right)$$

$$\Rightarrow 3 - \left(\frac{36 - 3V_1}{4}\right) = -6 + \frac{3}{4}V_1$$

$$\left(-\frac{1}{2} - \frac{1}{4}j\right)V_1 - \frac{1}{2j}V_{TH} = -6 \to (1)$$

KCL at $V_{TH}$

$$\frac{V_{TH} - V_1}{2j} - 3i_o + \frac{V_{TH}}{-8j} = 0$$

$$\frac{V_{TH} - V_1}{2j} - \frac{36}{4} + \frac{3V_1}{4} + \frac{V_{TH}}{-8j} = 0$$

$$\left(\frac{3}{4} + \frac{1}{2}j\right)V_1 - \frac{3}{8}jV_{TH} = 9 \to (2)$$

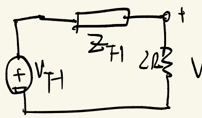
Find $V_{TH} \to \Delta 2$

$$\begin{vmatrix} -\frac{1}{2} - \frac{1}{4}j & -\frac{1}{2j} \\ \frac{3}{4} + \frac{1}{2}j & -\frac{3}{8}j \end{vmatrix} \begin{vmatrix} V_1 \\ V_{TH} \end{vmatrix} = \begin{vmatrix} -6 \\ 9 \end{vmatrix}$$

$$\Delta \Rightarrow \frac{5}{32} - \frac{3}{16}j$$

$$\Delta_2 \Rightarrow \begin{vmatrix} -\frac{1}{2} - \frac{1}{4}j & -6 \\ \frac{3}{4} + \frac{1}{2}j & 9 \end{vmatrix} \Rightarrow \frac{3}{4}j$$

$$\boxed{V_{TH} = \frac{\Delta_2}{\Delta} = -\frac{144}{61} + \frac{120}{61}j \approx 3.073\angle 140.194^\circ}$$




$\to V_o = V_{TH}\left(\frac{2}{2 + Z_{TH}}\right)$

$$V_o = \frac{2(3.073\angle -219.8^\circ)}{1.4754 - j2.229}$$

$$V_o \to 2.3\angle -163.3^0$$

$$\boxed{V_o = 2.3\cos(t - 163.3)\ V.}$$

**10.63** Obtain the Norton equivalent of the circuit depicted in Fig. 10.106 at terminals *a-b*.

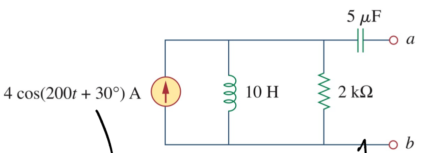


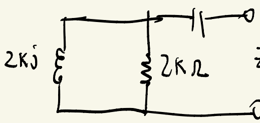

**Figure 10.106**
For Prob. 10.63.

$5\mu F = \dfrac{1}{(5\times10^{-6})(200)j} = -1kj$

$10H = 200(10)j = 2000j$

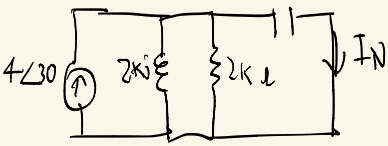
$w = 200$ rad/s

find $Z_n \rightarrow$ close all independent Source.

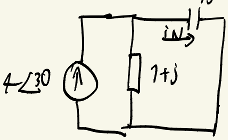



$Z_{TH} = Z_N$

$= \dfrac{4jK^2}{2Kj+2K} - 1Kj = 1K \rightarrow Z_{TH} = 1K\Omega = Z_N$




$j2 // 2 = \dfrac{4j}{2+2j} = 1+j$




$\rightarrow i_N = 4\angle30\left(\dfrac{1+j}{-1j+(1+j)}\right)$

$I_N = 5.657\angle75°$

**10.64** For the circuit shown in Fig. 10.107, find the Norton equivalent circuit at terminals *a-b*.

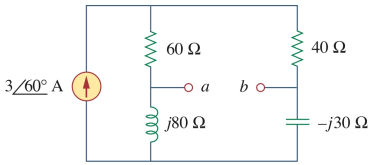
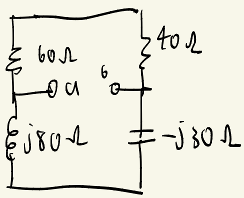


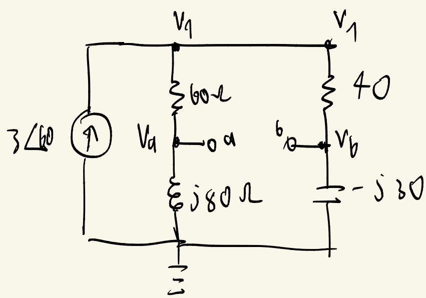

**Figure 10.107**
For Prob. 10.64.

Find $Z_{TH} = Z_N$




$Z_{TH} \to 100 // 50j = 20+j40 = 44.72 \angle 63.43$

Find $V_{TH} \to V_a - V_b$




KCL at $V_1$ → $-3\angle 60 + \dfrac{V_1 - V_a}{60} + \dfrac{V_1 - V_b}{40} = 0$

$$\left(\frac{1}{24}\right)V_1 - \frac{1}{60}V_a - \frac{1}{40}V_b = 3\angle 60 \to (1)$$

KCL at $V_a$ → $\dfrac{V_a - V_1}{60} + \dfrac{V_a}{j80} = 0$

$$\left(\frac{1}{60} - \frac{1}{80}j\right)V_a = \frac{V_1}{60}$$

$$V_1 = \left(1 - \frac{3}{4}j\right) V_a \rightarrow V_a = \left(\frac{16}{25} + \frac{12}{25}j\right) V_1$$

KCL at $V_b$ $\rightarrow$ $\frac{V_b - V_1}{40} + \frac{V_b}{-j30} = 0$

$$\left(\frac{1}{40} + \frac{1}{30}j\right) V_b = \frac{V_1}{40}$$

$$V_b = \left(\frac{9}{25} - \frac{12}{25}j\right) V_1$$

$$\left(\frac{1}{24}\right) V_1 - \frac{1}{60}\left(\frac{16}{25} + \frac{12}{25}j\right) V_1 - \frac{1}{40}\left(\frac{9}{25} - \frac{12}{25}j\right) V_1 = 3\angle 60$$

$$\left(\frac{11}{500} + \frac{1}{250}j\right) V_1 = 3\angle 60$$

$$V_1 = 86.7846 + 102.3953j$$

$$\Updownarrow$$

$$134.164 \angle 49.695$$

$$V_a = 6.431 + 107.738j$$

$$V_b = 80.354 - 4.8233j$$

$$V_{TH} = V_a - V_b \Rightarrow -73.923 + 111.96j$$

$$I_N = \frac{V_{TH}}{Z_{TH}} = 3\angle 60^\circ$$

**10.67** Find the Thevenin and Norton equivalent circuits at terminals *a-b* in the circuit of Fig. 10.110.

ML

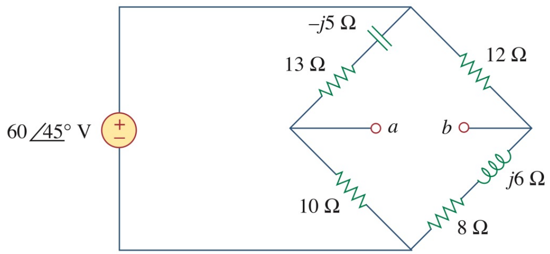
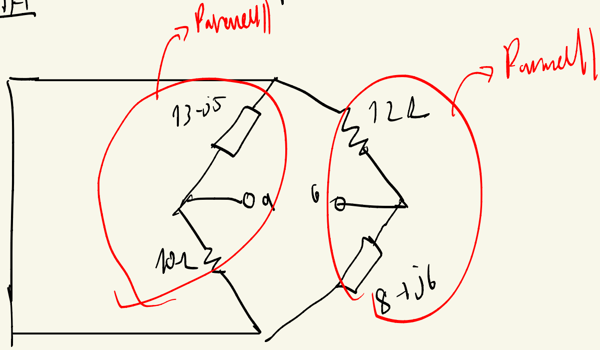



**Figure 10.110**
For Prob. 10.67.

$\underline{Z_{TH}}$ → Close all independent Source.



**10.68** Find the Thevenin equivalent at terminals *a-b* in the circuit of Fig. 10.111.

**ML**

Parallel to 1 Hence same $V_o$ as 1 H

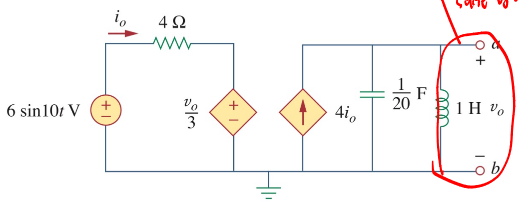
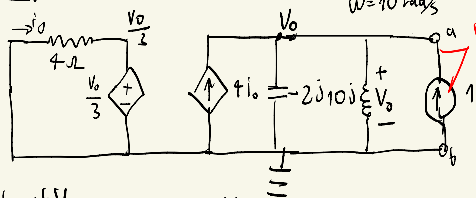



**Figure 10.111**
For Prob. 10.68.

Find $Z_{Th}$ → Close all independent Source

$w = 10$ rad/s




Use 1A to Apply J and $V = V_o$

KCL at $\frac{V_o}{3}$ → $i_o = \dfrac{-\frac{V_o}{3}}{4} = -\dfrac{V_o}{12}$

KCL at $V_o$ $\quad -4i_o - 1 + \dfrac{V_o}{-2j} + \dfrac{V_o}{10j} = 0$

$$\frac{V_o}{-2j} + \frac{V_o}{10j} = 1 + 4\left(\frac{-V_o}{12}\right)$$

$$\frac{V_o}{-2j} + \frac{V_o}{10j} = 1 + \frac{-V_o}{3}$$

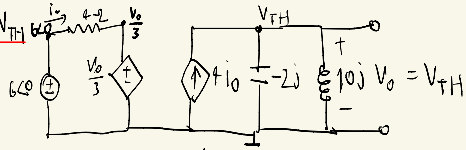
$$\frac{V_o}{-2j} + \frac{V_o}{10j} + \frac{V_o}{3} = 1$$

$$\left(\frac{1}{3} + \frac{2}{5}j\right)V_o = 1$$

$$V_o = \frac{75}{61} - \frac{90}{61}j \approx 1.92\angle -50.194^\circ$$

$$\boxed{Z_{TH} = \frac{V_o}{1A} = 1.2291 - 1.475j \approx 1.92\angle -50.194^\circ}$$

Find $V_{TH}$




KCL at $\frac{V_o}{3}$ → $i_o = \dfrac{6 - \frac{V_o}{3}}{4}$ → $V_{TH} = V_o$

KCL at $V_{TH}$ → $-4i_o + \dfrac{V_{TH}}{-2j} + \dfrac{V_{TH}}{10j} = 0$

$$-4\left(\frac{6 - \frac{V_o}{3}}{4}\right) + \frac{V_{TH}}{-2j} + \frac{V_{TH}}{10j} = 0$$

$$-6 + \frac{V_o}{3} + \frac{V_{TH}}{-2j} + \frac{V_{TH}}{10j} = 0$$

$$\left(\frac{1}{3} + \frac{2}{5}j\right)V_{TH} = 6$$

$$\boxed{V_{TH} = \left(\frac{450}{61} - \frac{540}{61}j\right) \approx 11.523\angle -50.194^\circ \text{ V}}$$

**10.76** Determine $\mathbf{V}_o$ and $\mathbf{I}_o$ in the op amp circuit of Fig. 10.119.

ML

$$I_0 = \frac{V_1 - V_0}{20K}$$

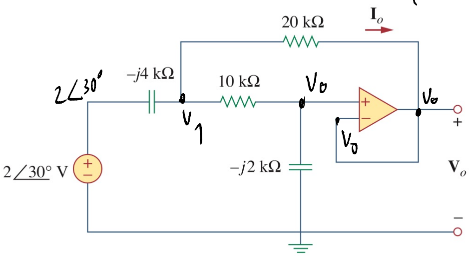



**Figure 10.119**
For Prob. 10.76.

KCL at $V_1$ $\rightarrow$ $$\frac{V_1 - 2\angle 30^\circ}{-j4} + \frac{V_1 - V_0}{20} + \frac{V_1 - V_0}{10} = 0$$

$$\left(\frac{3}{20} + \frac{1}{4}j\right)V_1 - \frac{3}{20}V_0 = \frac{\sqrt{3}}{4}j - \frac{1}{4} \rightarrow (1)$$

KCL at $V_0$ $\rightarrow$ $$\left(\frac{V_0 - V_1}{10} + \frac{V_0}{-j2} = 0\right)10$$

$$V_0 - V_1 + \frac{5V_0}{-j} = 0$$

$$(1+5j)V_0 = V_1$$

$$V_0 = \left(\frac{1}{1+5j}\right)V_1$$

$$\left(\frac{3}{20} + \frac{1}{4}j\right)V_1 - \frac{3}{20}\left(\frac{1}{1+5j}\right)V_1 = \frac{\sqrt{3}}{4}j - \frac{1}{4}$$

$$\left(\frac{15}{104} + \frac{29}{104}j\right)V_1 = \left(\frac{\sqrt{3}}{4}j - \frac{1}{4}\right)$$

$$V_1 = 0.859 + 1.341j$$

$V_0 = 0.2909 - 0.114j \Rightarrow 312.4 \angle -21.34^\circ$ mV

$$I_0 = \frac{V_1 - V_0}{20} = 0.0284 + 0.0727j$$

$\approx 0.078 \angle 68.67^\circ$ mA

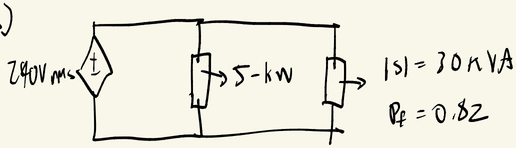
**Exercise 6.3** (30%) → real Power

A 240-V rms 60-Hz source supplies a parallel combination of a 5-kW heater and a 30-kVA induction motor whose power factor is 0.82. Determine:

$\downarrow |S|$

1. the system apparent power
2. the system reactive power
3. Sketch the power triangle for the current system and label $|S|$, $P$, and $Q$
4. the power factor of the current system
5. the kVA rating of a capacitor required to adjust the system power factor to 0.9 lagging
6. the value of the capacitor required

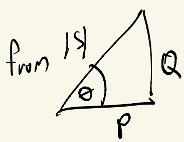
1.)




$$S_1 = 5\,kW + 0j \rightarrow |S_1| = \sqrt{5^2} = 5$$

real power

need to find $S_2 \rightarrow$ S 30 kVa and $P_f = 0.82$




$$\cos\theta = \frac{P}{|S|}$$

$$\cos\theta = 0.82$$

$$\theta = 34.91$$

$$P = \cos\theta\,|S|$$

$$P_f = \cos\theta = 0.82$$

$$P = 0.82 \times 30\,kVa = 24.6\,kVa$$

$$\tan\theta = \frac{Q}{P} \Rightarrow Q = P\tan\theta$$

$$= 24.6\tan(34.91)$$

$$Q = 17.1676\,kVAR$$

$$S_2 = P + jQ = 24.6 + 17.1676j$$

$$S_{total} = S_1 + S_2 \rightarrow 29.6 + 17.1676j$$

Apparent Power

$$= |S|$$

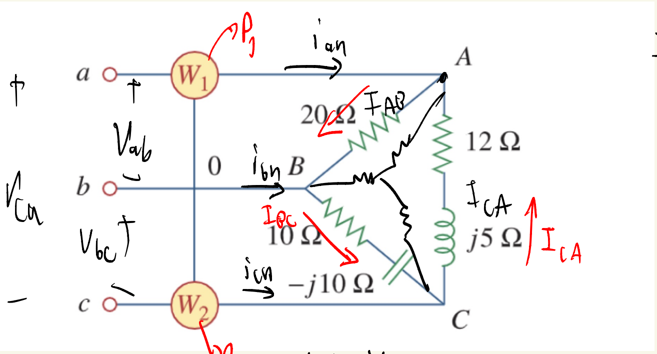
$$= \sqrt{29.6^2 + 17.1676^2}$$

$|S| = 34.22$ KVA

**Exercise 6.4** (20%)
Two wattmeters are properly connected to the unbalanced load supplied by a balanced source such that $V_{ab} = 208\angle 0°$ V with positive phase sequence.

Unbalance load → Cannot eq $P = V_p I_p \cos\theta$ → phase voltage/current

1. Determine the reading of each wattmeter.
2. Calculate the total apparent power absorbed by the load.




$P_2 \rightarrow V_c$ to $V_b$

1.) Need to find the reading of wattmeter at $W_1$ and $W_2$; Hence we need to find $P_1$ and $P_2$.

Power per phase for 3 Phase circuit is

$$P_p = V_p I_p \cos\theta \text{ or use } S = I_p^2 Z_p$$

<u>for Δ Connected load</u> $\rightarrow I_L = \sqrt{3} I_p$

$$V_L = V_p$$

$$V_{ab} = 208\angle 0$$

$$V_{bc} = 208\angle -120$$

$$V_{ca} = 208\angle 120$$

$$P_1 = |V_{ab}||I_{ab}|\cos(\theta_v - \theta_i)$$

$$I_{an} \Rightarrow I_{AB} - I_{CA}$$

$$I_{AB} = \frac{208\angle 0}{20} = 10.4\,\Omega\angle 0$$

$$I_{CA} \Rightarrow \frac{208\angle 120}{12 + j5} = 16\angle 97.38$$

$$I_{an} \Rightarrow 12.455 - 15.867j \approx 20.1716\angle -51.8697$$

always express in phasor form

$$P_1 = |208||20.4791| \cos(0 - (-51.8697))$$

$$= 2570 \text{ KW}$$

$$P_2 = |V_{cb}||I_{cn}| \cos(\theta_v - \theta_i)$$

flip from $V_{bc}$ to $V_{cb} \rightarrow 208\angle{-120+180}$

$$V_{cb} = 208\angle{+60}$$

$$\tilde{I}_{cn} = \tilde{I}_{CA} - \tilde{I}_{BC}$$

$$\tilde{I}_{CA} \Rightarrow \frac{208\angle 120}{12+j5} = 16\angle 97.38$$

$$\tilde{I}_{BC} \Rightarrow \frac{208\angle -120}{10-10j} = 14.708\angle -75^{\circ}$$

$$\tilde{I}_{cn} = 30.64\angle 101.0294$$

$$P_2 = |208||30.64) \cos(60 - 101.03)$$

$$= 4808 \text{ KW}$$

c.) $|S| = \sqrt{S^2}$

find complex Power = $P + jQ$

$P = P_T = 4.808 + 2.590 = 7.398$

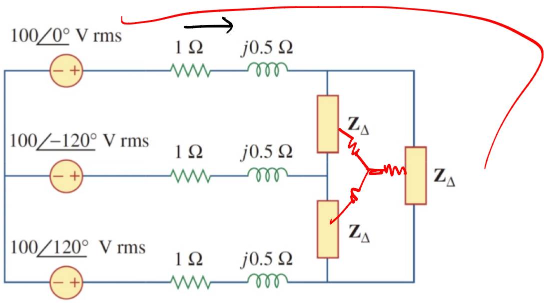
for Wattmeter $\longrightarrow$ $Q_T = \sqrt{3}\,(P_2 - P_1) = 3840.25$

$S_T = 7.398 + 3.840j$

$|S|_T = 8.335$ KVA

**Exercise 3.5** (20%) For the three-phase circuit below, find the average power absorbed by the delta-connected load with

$$Z_\Delta = 21 + j24\,\Omega.$$




For balance load we know

$Z_\Delta = 3Z_Y \rightarrow Z_Y = 3Z_\Delta$

$Z_Y = 7 + 8j\,\Omega.$

for Y-Connected load. $I_L = I_p$ ; $V_L = \sqrt{3}V_p$

Average power $\rightarrow$ $3I_p^2 Z_p$

$$I_L = \left(\frac{100\angle 0}{1 + j0.5 + 7 + 8j}\right) = 8.567\angle -46.736$$

$$= 3(8.567)^2(7 + 8j)$$

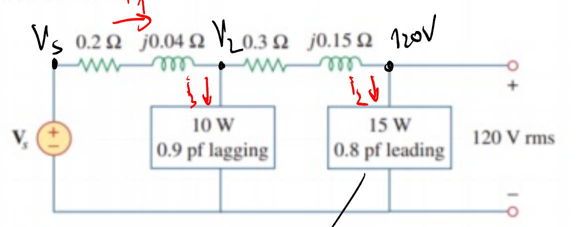
$$S \Rightarrow 1541.263 + 1761.446j$$

P Q

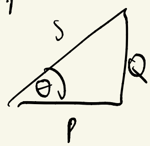
Hence the average power absorb by the load $Z_\Delta$

$= 1541.263$ W

**Exercise 6.1** (30%) For the circuit, find $V_s$.




for $S_1 = 15W$ and 0.8 pf leading




$\tan\theta = \frac{Q}{P} \rightarrow Q = P\tan\theta$

$\cos\theta = 0.8$

$\theta \Rightarrow -36.87$ ⇒ leading

$= 15(\tan(-36.87))$

$Q \Rightarrow -11.25$

$S_1 = 15 - 11.25j$

$S_2 = 10W \rightarrow Q = P\tan\theta$

$\cos\theta = 0.9$

$\theta = 25.842°$

$Q = 10\tan(25.842)$

$\Rightarrow 4.8432$ VAR

$S_2 = 10 + 4.8432j$

from $S_1 = \tilde{V}_{rms}\tilde{I}^*_{rms} \longrightarrow \tilde{I}^*_{rms} = \frac{S_1}{\tilde{V}_{rms}} = \frac{15 - 11.25j}{120}$

$$I_2^* \Rightarrow 0.125 - j0.09375$$

$$\tilde{I}_2 = 0.125 + 0.09375j$$

From the diagram $\rightarrow$ $i_2 = \dfrac{V_2 - 120}{0.3 + j0.15}$

$$V_2 = (0.125 + 0.09375j)(0.3 + j0.15) + 120$$

$$V_2 \Rightarrow 120.035 + 0.04687j$$

$$\downarrow$$

also $S_2 = 10 + 4.8432j$

$$S = V_{rms} I_3^*$$

$$I_B^* = \frac{S}{V_{rms}} = \frac{(10 + 4.8432j)}{(120.035 + 0.0228j)}$$

$$I_B^* = 0.09256 \angle 25.85$$

$$I_3 = 0.09256 \angle -25.82$$

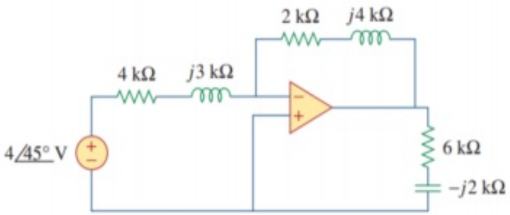
$$= 0.0837 - j0.0405$$

$$i = i_2 + i_3 = (0.2087 + 0.05325j)$$

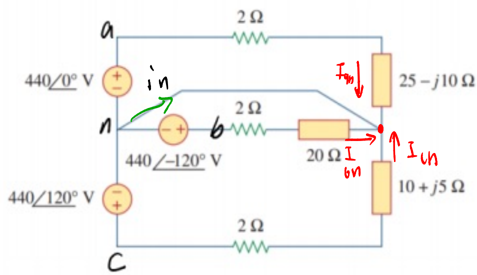
Also $i = \dfrac{V_s - V_2}{0.2 + j0.04} =$

$V_s \Rightarrow 120.06 \angle 0.03° \, V$

**Exercise 6.2** (15%) Obtain the average power absorbed by the 6-kΩ resistor in the circuit.



**Exercise 6.3** (20%) Determine the current in the neutral line.




$$i_n = -(i_a + i_b + i_c)$$

$$i_a = \frac{440\angle 0^\circ}{2+(25-j10)} \Rightarrow (14.33 + 5.3076j)$$

$$i_b = \frac{440\angle -120)}{2+20} = -10 - 17.32j$$

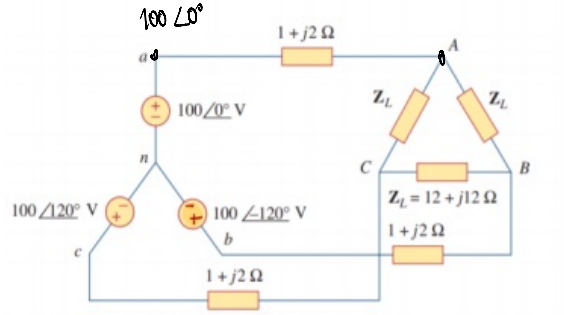
$$i_c = \frac{440\angle 120}{2+(10+j5)} \Rightarrow -4.35 + 33.565j$$

$$i_n = -(i_a + i_b + i_c)$$

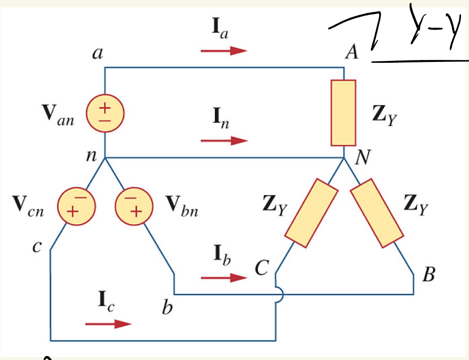
$$i_n = 0.02 - 21.553j$$

$$i_n = 21.553\angle -89.95^\circ$$

**Exercise 6.4** (15%) Obtain the line currents in the three-phase circuit.




Change to Y-Y Connection as it's easier → $Z_\Delta = 3Z_Y \rightarrow Z_Y = \frac{1}{3} Z_\Delta$




Y-Y Connect

$Z_Y = 4+4j$

$Z_L = 4+4j$

$Z_Y = Z_L + \cancel{Z_s} + Z_l \rightarrow 1+j2$

$Z_Y = 5+6j$

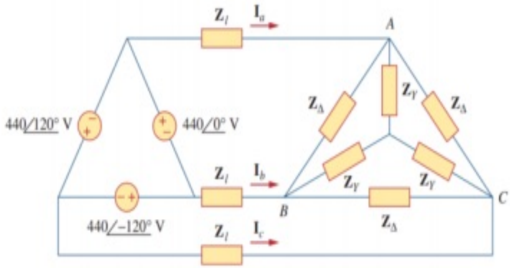
For Y-Y connected

$$I_a = \frac{V_{an}}{Z_Y} = \frac{100}{5+6j} = 12.8036 \angle -50.194 \text{ A}$$

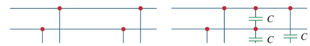
$$I_b = 12.8036 \angle -170.194 \text{ A}$$

$$I_c = 12.8036 \angle 69.8058$$

**Exercise 6.5** (20%) Find the line currents in the three-phase network. Take $Z_\Delta = 12 - j15\Omega$, $Z_Y = 4 + j6\Omega$, $Z_l = 2\Omega$.



**Example 12.8** Two balanced loads are connected to a 240-kV rms 60-Hz line, as shown in Fig. 12.22(a). Load 1 draws 30 kW at a power factor of 0.6 lagging, while load 2 draws 45 kVAR at a power factor of 0.8 lagging. Assuming the *abc* sequence, determine: (a) the complex, real, and reactive powers absorbed by the combined load, (b) the line current, and (c) the kVAR rating of the three capacitors Δ-connected in parallel with the load that will raise the power factor to 0.9 lagging and the capacitance of each capacitor.




Line voltage

KVAR = Reactive power

Load 1 → 30 kW ; pf = 0.6 → lagging $\theta_i < \theta_v$

$30\text{ kW} \to P$

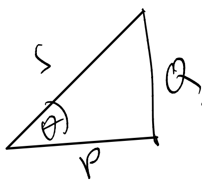
$P_f = \frac{P}{|S|} = 0.6 = \frac{30\text{ kW}}{|S|} \to |S| = \frac{30\text{ kW}}{0.6} = 50\text{ kV}$

$\cos\theta = 0.6 \to \theta = 53.13$

$\theta \to (\theta_v - \theta_i)$

Complex power load 1 → $S = |S|\angle\theta_v - \theta_i \Rightarrow 50\angle 53.13 \approx 30 + 40j$ kW

Load 2. $Q = 45$ KVAR $P_f = 0.8 \to \cos\theta = 0.8 \to \theta = 36.87$




$\tan\theta = \frac{Q}{P} \to P = \frac{Q}{\tan\theta} = \frac{45\text{ KVAR}}{\tan(36.87)}$

$P = 60$

$S = (60 + 45j)$ kW

Combine → $S_{total} = S_1 + S_2 = (30 + 40j) + (60 + 45j) = 90 + 85j$ KVA

$P = 90$ $Q$ $\theta = 43.36$

(b) The line Current → $P = \sqrt{3} V_L I_L \cos\theta$

$\frac{90}{\sqrt{3}(240)(\cos 43.36)} = I_L \to I_L = 0.2978$ A

(c) The kVAR rating = $Q_C \to P(\tan\theta_1 - \tan\theta_2)$

$= \theta_1 = 43.36$ ; $\theta_2 \to$ $\cos\theta = 0.9$

$\theta_2 = 25.842$

$= 90(\tan(43.36) - \tan(25.842))$

$Q_c = 41.4007$ KVAR

d) → Capacitance of each capacitor → $C = \dfrac{Q_c}{wV^2}$ → for each capacitor need to divide by 3

$= \dfrac{Q_c}{3} = \dfrac{41.4007}{3} = 13.8$

$C = \dfrac{13.8 \times 10^3}{(2\pi \times 60)(240 \times 10^3)^2}$

$= 6.355 \times 10^{-10}$ F

**Example 12.9** The unbalanced Y-load of Fig. 12.23 has balanced voltages of 100 V and the *acb* sequence. Calculate the line currents and the neutral current. Take $Z_A = 15\ \Omega$, $Z_B = 10 + j5\ \Omega$, $Z_C = 6 - j8\ \Omega$.

Phase voltage $V_p$

Balanced source
Unbalanced load

negative sequence.

$I_L$ ?
$I_N$ ?

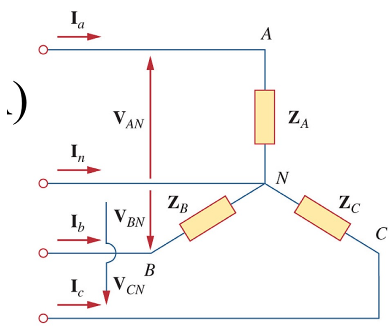



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Figure 12.23 Unbalanced three-phase Y-connected load.

Balance Voltage of 100V

$\tilde{V}_{an} \Rightarrow 100\angle 0$

$\tilde{V}_{bn} \Rightarrow 100\angle 120$

$V_{cn} \Rightarrow 100\angle -120$

$I_a = \dfrac{\tilde{V}_{an}}{Z_A} = \dfrac{100\angle 0}{15} = 6.667\angle 0$ ; $\tilde{I}_b = \dfrac{\tilde{V}_{BN}}{Z_B} = \dfrac{100\angle 120}{10+j5} = 8.9443\angle 93.43$

$\tilde{I}_c = \dfrac{V_{CN}}{Z_C} = \dfrac{100\angle -120}{6-j8} \approx 10\angle -66.87°$

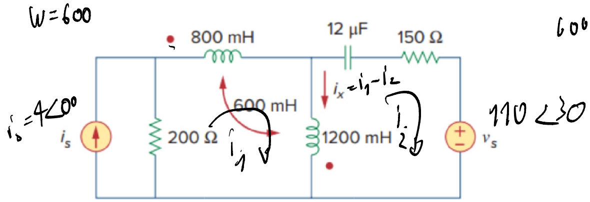
$\tilde{I}_n = -(\tilde{I}_a + \tilde{I}_b + \tilde{I}_c)$

$\tilde{I}_n = -10.0598 + j0.2079 \approx 10.0634\angle 178.77$

# Chapter 13 Problem.

**Exercise 7.1** (20%) Find the unknown current $i_x$ in the circuit below, where $i_s = 4\cos(600t)A$ and $v_s = 110\cos(600t + 30°)V$

$W = 600$




$600\,mH = j\omega M$
$= j(600)(600\times 10^{-3})$
$= 360j$

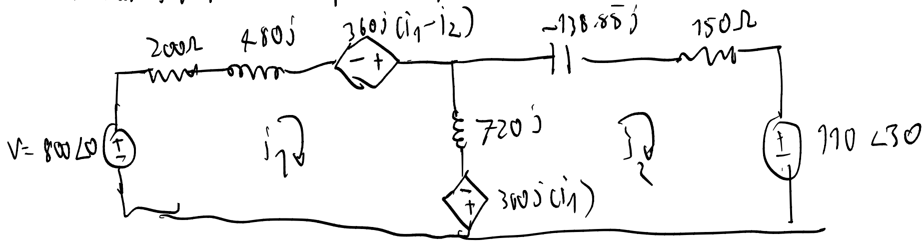
$V = M\frac{di}{dt}$

$800\,mH = \omega L j = (600)(800\times 10^{-3})(j) = 480j$

$1200\,mH \Rightarrow (1200\times 10^{-3})(600)(j) = 720j$

$12\,\mu F = \dfrac{1}{(600)(12\times 10^{-6})j} = -138.8\bar{8}j$

chose Mutual inductance to phasor form




$\underline{KVL\ at\ I_1} \rightarrow 200 i_1 + 480j\, i_1 - 360j(i_1 - i_2) + 720j(i_1 - i_2)$
$- 360j\, i_1 - 800\angle 0 = 0$

$(480j + 200) i_1 + (-360j) i_2 = 800 \rightarrow (1)$

$\underline{KVL\ at\ I_2}$

$-138.8\bar{8}j\, i_2 + 150 i_2 + 110 + 360j\, i_1 + 720j(i_2 - i_1)$
$= 0$

$(-360j) i_1 + (581.12j + 150) i_2 = -110 \rightarrow (2)$

$$\begin{vmatrix} (480j + 200) & -360j \\ -360j & (581.12j + 150) \end{vmatrix} \begin{vmatrix} i_1 \\ i_2 \end{vmatrix} = \begin{vmatrix} 800 \\ -110 \end{vmatrix}$$

$$\Delta = 222867 \angle 122.375^{\circ}$$

$$\Delta_1 = \begin{vmatrix} 800 & -360j \\ -110 & (581.12j + 150) \end{vmatrix} \Rightarrow 441101.2193 \angle 74.2432$$

$$\Delta_2 = \begin{vmatrix} (1480j+200) & 800 \\ -360j & -110 \end{vmatrix} \Rightarrow 236226.67 \angle 95.34373^{\circ}$$

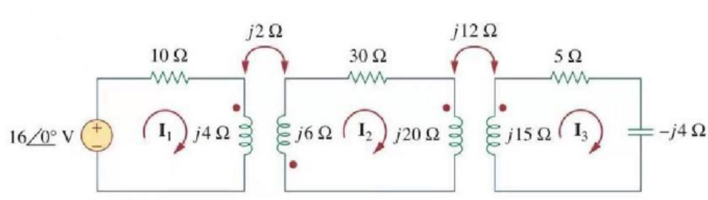
$$i_1 = \frac{\Delta_1}{\Delta} = 1.9828 \angle -48.1318\ A$$

$$i_2 = \frac{\Delta_2}{\Delta} = 1.05994 \angle -27.0313\ A$$

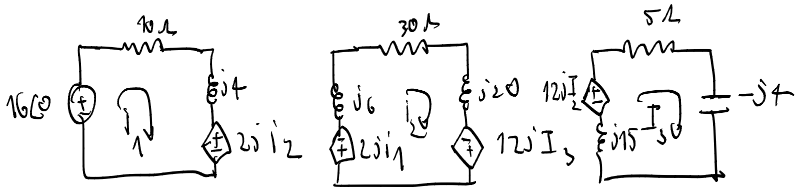
$$i_x = i_1 - i_2 \Rightarrow 1.0646 \angle -69.134^{\circ}$$

$$I_x = 1.1 \cos(600t - 66.0^{\circ})\ A$$

**Exercise 7.3** Determine $I_1$, $I_2$, $I_3$ in the circuit.




Phasor form




KVL $I_1$ $\quad 10i_1 + 4ji_1 + 2ji_2 - 16 = 0$

$$(10+4j)i_1 + 2ji_2 = 16 \rightarrow (1)$$

KVL $I_2$ $\quad 30i_2 + 20ji_2 - 12jI_3 + 2ji_1 + 6ji_2 = 0$

$$2ji_1 + (30+26j)i_2 - 12jI_3 = 0 \rightarrow (2)$$

KVL $I_3$ $\rightarrow$ $\quad 5I_3 - 4jI_3 + 15jI_3 - 12jI_2 = 0$

$$-12jI_2 + (11j+5)I_3 = 0 \rightarrow (3)$$

From (1)

$$(10+4j)i_1 = 16 - 2ji_2$$

$$i_1 = \frac{16}{10+4j} - \frac{2ji_2}{10+4j} \rightarrow \text{Plug to (2)}$$

$$2j\left(\frac{16}{10+4j} - \frac{2ji_2}{10+4j}\right) + (30+26j)i_2 - 12jI_3 = 0$$

$$\left(\frac{32}{29}+\frac{80}{29}j\right)+\left(\frac{880}{29}+\frac{750}{29}j\right)i_2-12j\,I_3=0$$

(4)

solve 4 and 3

$$\begin{vmatrix} -12j & (11j+5) \\ \left(\frac{880}{29}+\frac{750}{29}j\right) & -12j \end{vmatrix} \begin{vmatrix} I_2 \\ I_3 \end{vmatrix} = \begin{vmatrix} 0 \\ -\frac{32}{29}-\frac{80}{29}j \end{vmatrix}$$

$$\Delta = 463.24 \angle -99.39^0$$

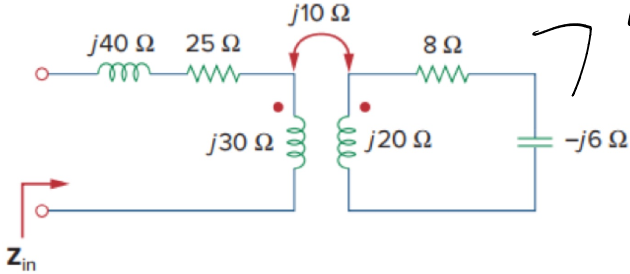
$$\Delta_1 = \begin{vmatrix} 0 & (11j+5) \\ -\frac{32}{29}-\frac{80}{29}j & -12j \end{vmatrix}$$

$$i_2 = 35.9 \angle 133.754^0$$

$$\Delta_2 => \begin{vmatrix} -12j & 0 \\ \left(\frac{880}{29}+\frac{750}{29}j\right) & -\frac{32}{29}-\frac{80}{29}j \end{vmatrix}$$

$$i_3 = 35.653 \angle 158.198^0$$

**Exercise 7.4** Find the input impedence $Z_{in}$ of the circuit below.




Linear transformer

$j\omega M = 10j$

$\omega M = 10$

$$Z_{in} = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L}$$

$$\Rightarrow (j40 + 25) + 30j + \frac{(\omega M)^2}{(8 - j6) + j20}$$

$$Z_1 = 70j + 25 + \frac{10^2}{8 + 14j}$$

$$Z_{in} = 70.452 \angle 66.53$$

**13.20** Determine currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ in the circuit of Fig. 13.89. Find the energy stored in the coupled coils at $t = 2$ ms. Take $\omega = 1{,}000$ rad/s.

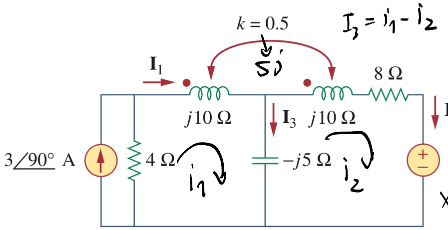



**Figure 13.89**
For Prob. 13.20.

$I_3 = i_1 - i_2$ we know K = Coefficient of Coupling

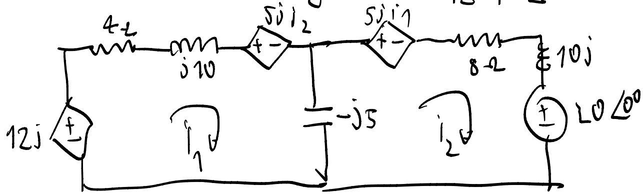
$0.5 = \frac{M}{\sqrt{L_1 L_2}}$ → $M = 0.5\sqrt{(10\times10^{-3})(10\times10^{-3})}$

$= 5\,mH$ → $wMj = 5j$

$X_{L_1} = jwL_1$ ; $X_{L_2} = jwL_2$

$L_1 = \frac{10j}{j(1000)} = 0.01$ or $10\,mH = L_2 = 10\,mH$

Energy Store → $W = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + M_{12} I_1 I_2$




KVL at $i_1$ → $4i_1 + 10j\,i_1 + 5j\,i_2 - 5j(i_1 - i_2) - 12j = 0$

$(4+5j)i_1 + 10j\,i_2 = 12j$ → (1)

KVL at $i_2$ → $5j\,i_1 + 8i_2 + 20 - 5j(i_2 - i_1) + 10j\,i_2 = 0$

$10j\,i_1 + (5j+8)i_2 = -20$ → (2)

$$\begin{vmatrix} (4+5j) & 10j \\ 10j & (5j+8) \end{vmatrix} \begin{vmatrix} i_1 \\ i_2 \end{vmatrix} = \begin{vmatrix} 12j \\ -20 \end{vmatrix}$$

$\Delta = 107 + 60j$

$$\Delta_1 = \begin{vmatrix} 12j & 10j \\ -20 & (5j+8) \end{vmatrix} = -60 + 296j$$

$$\Delta_2 = \begin{vmatrix} (4+5j) & 12j \\ 10j & -20 \end{vmatrix} = 40 - 100j$$

$$\dot{i}_1 = \frac{\Delta_1}{\Delta} = 2.46196 \angle 72.177^0$$

$$\dot{i}_2 = \frac{\Delta_2}{\Delta} \Rightarrow 0.878 \angle -97.48^0$$

$$\dot{i}_3 = \dot{i}_1 - \dot{i}_2 \Rightarrow 3.329 \angle 74.89^0$$

all RMS

Energy Store $\rightarrow \underbrace{W = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M_{12} I_1 I_2}$ $\rightarrow$ Need to use instantaneous Value.

$$i_1(t) = \underset{\text{Convert to peak}}{2.46196\sqrt{2}} \left(\cos(1000t + 72.177\right)$$

$$i_2(t) = 0.878\sqrt{2}\left(\cos(1000t - 97.48)\right)$$

at $t = 2ms = 0.002 = 1000t = 2\text{rad} = 114.6 \rightarrow 2\left(\frac{180}{\pi}\right)$ $\nearrow$ 1 rad

$$i_1(t) = 2.46196\sqrt{2}\left(\cos(114.6 + 72.177\right) = -3.45$$

$$i_2(t) = 1.19$$

$$W = \frac{1}{2}(0.01)(-3.45)^2 + \frac{1}{2}(0.01)(1.19)^2 + (5\times10^{-3})$$

$$(-3.45)(1.19)$$

$$W = 0.046j$$

**13.49** Find current $i_x$ in the ideal transformer circuit shown in Fig. 13.114.

ML

$\omega = 2$

turn ratio $= \dfrac{N_2}{N_1} = \dfrac{3}{2} = 3$

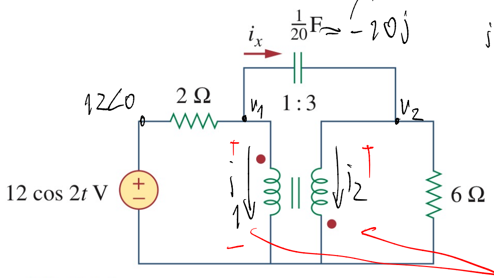



**Figure 13.114**
For Prob. 13.49.

$$i_x = \frac{V_1 - V_2}{-10j}$$

$$\frac{1}{20}f = \frac{1}{\left(\frac{1}{20}\right)(2)(j)} = -10j$$

$i_1$ enter and $i_2$ exit $= \dfrac{I_1}{I_2} = n \rightarrow I_1 = 3I_2$

also $\dfrac{V_2}{V_1} = -n \rightarrow \dfrac{V_2}{V_1} = -3$

$V_2 = -3V_1$

KCL at $V_1$ $\rightarrow$ $\dfrac{V_1 - 12\angle 0}{2} + \dfrac{V_1 - V_2}{-10j} + i_1 = 0$ ($i_x$)

$$\left(\frac{1}{2} + \frac{1}{10}j\right)V_1 + \left(\frac{-1}{10}j\right)V_2 + i_1 = 6 \rightarrow (1)$$

KCL at $V_2$

$$\frac{V_2 - V_1}{-10j} + \frac{V_2}{6} + i_2 = 0$$

$$\left(-\frac{1}{10}j\right)V_1 + \left(\frac{1}{6} + \frac{1}{10}j\right)V_2 + i_2 = 0 \rightarrow (2)$$

apply $V_2 = -3V_1$ and $i_1 = 3i_2$

$$\left(\frac{1}{2} + \frac{1}{10}j\right)V_1 + \left(-\frac{1}{10}j\right)(-3V_1) + 3i_2 = 6$$

$$\left(\frac{1}{2} + \frac{2}{5}j\right)V_1 + 3i_2 = 6 \rightarrow (3)$$

$$\left(\frac{-1}{10}j\right)V_1 + \left(\frac{1}{6} + \frac{1}{10}j\right)(-3V_1) + i_2 = 0$$

$$\left(\frac{-1}{2} - \frac{2}{5}j\right)V_1 + i_2 = 0 \rightarrow (4)$$

solve 3/4 $\rightarrow$

$$\begin{vmatrix} \left(\frac{1}{2} + \frac{2}{5}j\right) & 3 \\ \left(\frac{-1}{2} - \frac{2}{5}j\right) & 1 \end{vmatrix} \begin{vmatrix} V_1 \\ i_2 \end{vmatrix} = \begin{vmatrix} 6 \\ 0 \end{vmatrix}$$

$$\Delta \Rightarrow 2 + \frac{8}{5}j$$

$$\Delta_1 = \begin{vmatrix} 6 & 3 \\ 0 & 1 \end{vmatrix}$$

$$= 6$$

$$\Delta_2 = \begin{vmatrix} \left(\frac{1}{2} + \frac{2}{5}j\right) & 6 \\ \left(-\frac{1}{2} - \frac{2}{5}j\right) & 0 \end{vmatrix}$$

$$\Rightarrow \left(3 + \frac{12}{5}j\right)$$

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{6}{\left(2 + \frac{8}{5}j\right)} = 2.3426 \angle -38.66^0$$

$$i_2 = \frac{\Delta_2}{\Delta} = \frac{\left(3 + \frac{12}{5}j\right)}{\left(2 + \frac{8}{5}j\right)} = \frac{3}{2}\ A$$

$$V_1 = 1.83 - 1.46j$$

$$V_2 = -3V_1 = -5.487 + 4.39j$$

$$i_x = \frac{V_1 - V_2}{-10j} = 0.585 + 0.7317j$$

$$\downarrow$$

$$0.937 \angle 51.36^0$$

**13.12** Determine the equivalent $L_{eq}$ in the circuit of Fig. 13.81.

use $w = 1$

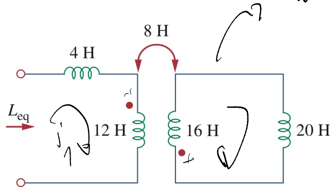


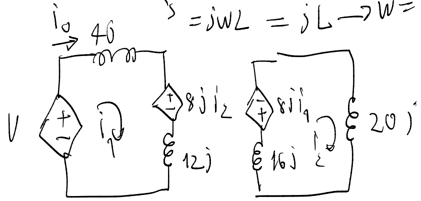

**Figure 13.81**
For Prob. 13.12.

$V \to$ apply $V = 1$

$$Z_{in} = \frac{1}{I_o} \to \frac{V}{I}$$

$$= jwL = jL \to w = 1$$

$$i_o = i_1$$




KVL $i_1$

$$4j i_1 + 8j i_2 + 12j i_1 - 1 = 0$$

$$16j i_1 + 8j i_2 = 1 \to (1)$$

KVL $i_2$ →

$$20j i_2 + 16j i_2 + 8j i_1 = 0$$

$$8j I_1 + 36j i_2 = 0 \to (2)$$

$$\begin{vmatrix} 16j & 8j \\ 8j & 36j \end{vmatrix} \begin{vmatrix} i_1 \\ i_2 \end{vmatrix} = \begin{matrix} 1 \\ 0 \end{matrix}$$

$$\Delta = -512$$

$$\Delta_1 = \begin{vmatrix} 1 & 8j \\ 0 & 36j \end{vmatrix} = 36j$$

$$\Delta_2 = \begin{vmatrix} 16j & 1 \\ 8j & 0 \end{vmatrix} = -8j$$

$$i_1 = \frac{\Delta_1}{\Delta} = -\frac{9}{128} j \; ; \quad i_2 = \frac{\Delta_2}{\Delta} = \frac{1}{64} j$$

$$\& \; i_1 = i_o$$

$$Z_{in} = \frac{1}{\left(\frac{-9}{128}j\right)} = \frac{128}{9}j$$

$$jL = Z_{in} \qquad \frac{1}{i_1} = jL_{eq}$$

$$L =$$

$$L_{eq} = \frac{1}{j\left(\frac{-9}{128}j\right)} = 14.2235$$

**13.9** Find $\mathbf{V}_x$ in the network shown in Fig. 13.78.

ML

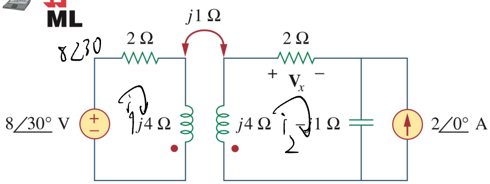
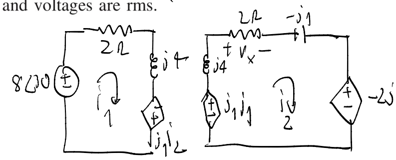



**Figure 13.78**
For Prob. 13.9.

s and voltages are rms.

Phasor form




KVL at $i_1 \rightarrow$ $2i_1 + 4ji_1 - ji_2 - 8\angle 30 = 0$

$(2+4j)i_1 - ji_2 = 8\angle 30° \rightarrow (1)$

KVL at $i_2 \rightarrow$

$2i_2 - ji_2 - 2j - ji_1 + 4ji_2 = 0$

$-ji_1 + (2+3j)i_2 = 2j \rightarrow (2)$

$$\begin{bmatrix} 2+4j & -j \\ -j & 2+3j \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 8\angle 30 \\ 2j \end{bmatrix}$$

$\Delta = -7 + 14j$

$\Delta_1 = \begin{vmatrix} 8\angle 30 & -j \\ 2j & 2+3j \end{vmatrix} \Rightarrow -0.1436 + 28.78j$

$\Delta_2 = \begin{vmatrix} 2+4j & 8\angle 30 \\ -j & 2j \end{vmatrix} \Rightarrow -12 + 10.93j$

$i_2 = \frac{\Delta_2}{\Delta} = 0.967 + 0.373j \approx 1.037\angle 21.106$

$V_x = i_2(2) = 1.935 + 0.747j$

**13.14** Obtain the Thevenin equivalent circuit for the circuit in Fig. 13.83 at terminals *a*-*b*.

$Z_{TH} = \frac{V_0}{1} \rightarrow$ Apply 1 A

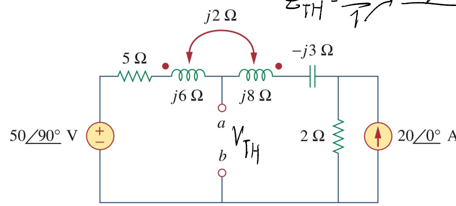
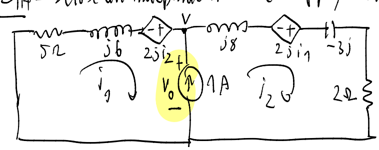



**Figure 13.83**

Find $Z_{TH}$ → Close all independent Source apply current 1A




$i_2 = i_1 + 1$

$1 = i_2 - i_1$

Super mesh $i_1, i_2 \rightarrow$ $5i_1 + 6ji_1 - 2ji_2 + 8ji_2 - 2ji_1 - 3ji_2 + 2i_2 = 0$

$(5+4j)i_1 + (3j+2)i_2 = 0 \rightarrow (1)$

also $I_2 - i_1 = 1 \rightarrow i_2 = i_1 - 1 \rightarrow (2)$

sub (2) to (1) give $i_1 = -(2+3j)/7+j7$

also from left loop KVL $\rightarrow (5+6j)i_1 - 2ji_2 + V_0 = 0$

$(5+6j)i_1 - 2j(i_1+1) + V_0 = 0$

$(5+4j)i_1 + 2j + V_0 = 0$

$V_0 = +2j - \left[(5+4j)\left(\frac{-(2+3j)}{7+7j}\right)\right]$

$V_0 = 1.5 + 3.7857j \approx 4.07\angle 68.385°$

Find $V_{TH}$

***13.22** Find current $\mathbf{I}_o$ in the circuit of Fig. 13.91.

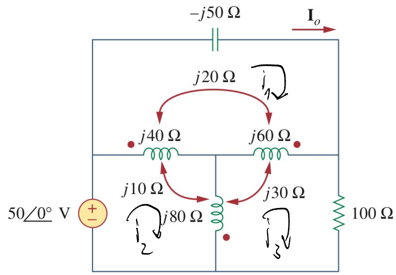
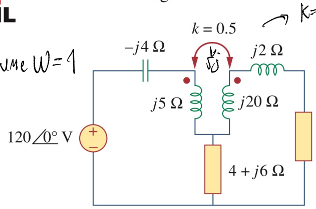



**Figure 13.91**
For Prob. 13.22.

**13.18** Find the Thevenin equivalent to the left of the load **Z** in the circuit of Fig. 13.87.

ML

assume $W = 1$




$K = 0.5 = \frac{M}{\sqrt{L_1 L_2}}$

$j w L_1 = j5$

$w L_1 = 5$

$L_1 = 5H$

$j w L_2 = 20j$

$L_2 = 20$

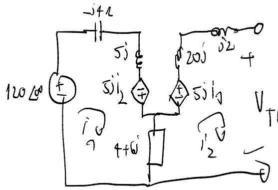
$M = 0.5\sqrt{100}$

$= 5H$

$j w M = 5j$

**Figure 13.87**
For Prob. 13.18.

Note that We need to find $Z_{TH} \rightarrow \frac{V_{TH}}{I_{sc}}$




$V_{TH} = V_{OC}$

note $i'_2$ now $= 0$

KVL Mesh $i_1 \rightarrow$ $-4ji_1 + j5i_1 - \cancel{5ji'_2} + (4+6j)(i_1 - \cancel{i_2}) = 120$

$(4+7j)i_1 = 120$

$i_1 = 14.804 \angle -60.255$

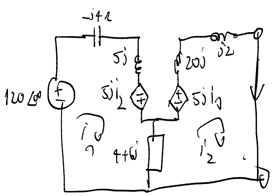
to find $V_{TH}$ $\rightarrow$ KVL at $i_2$ $\quad V_{TH} + (4+6j)(i_2 - i_1) - 5ji_1 = 0$

$V_{TH} = 5ji_1 + (4+6j)i_1$

$= (4+11j)i_1$

$V_{TH} = 174.22 \angle 9.76 V$

To find $i_{sc}$ $\rightarrow$ we short circuit




$i_{sc} = i_2$

$-4ji_1 + 5ji_1 - 5ji_2 + (4+6j)(i_1 - i_2) = 120$

$(4+7j)i_1 - (4+11j)i_2 = 120 \rightarrow (1)$

Mesh 2 $\quad j2i_2 + (4+6j)(i_2 - i_1) - 5ji_1 + 20ji_2 = 0$

$-(4+11j)I_1 + (4+j28)I_2 = 0 \rightarrow (2)$

solve (1) / (2) give

$I_2 = I_{sc} = 18.3997 \angle -75.2481 A$

$Z_{TH} = \frac{V_{TH}}{I_{sc}} \Rightarrow 11.32 \angle 85.01 \Omega$

# Chapter 14 Practice

**14.11** Sketch the Bode plots for

$$\mathbf{H}(\omega) = \frac{0.2(10 + j\omega)}{j\omega(2 + j\omega)}$$

$$H(w) = \frac{0.2 \times 10 \left(1 + \frac{jw}{10}\right)}{2 \times jw \left(1 + \frac{jw}{2}\right)}$$

Type 3 Zeros

Type 3 Pole

$$= \frac{\left(1 + \frac{jw}{10}\right)}{jw\left(1 + \frac{jw}{2}\right)}$$

$jw^{-1}$

Type 2 Pole

$(jw)^{-1} \to H(w) \Rightarrow -20\log_{10}(jw)$ → slope

$\phi = -90$

Type 3 Zeros → $H(w) \to 0 \quad w \le 10$

$20 \quad w \ge 10$

$\phi \to 0 \quad w \le 1$

$45 + 45\log_{10}\left(\frac{w}{10}\right) \quad 1 \le w \le 100$

45 db/decade

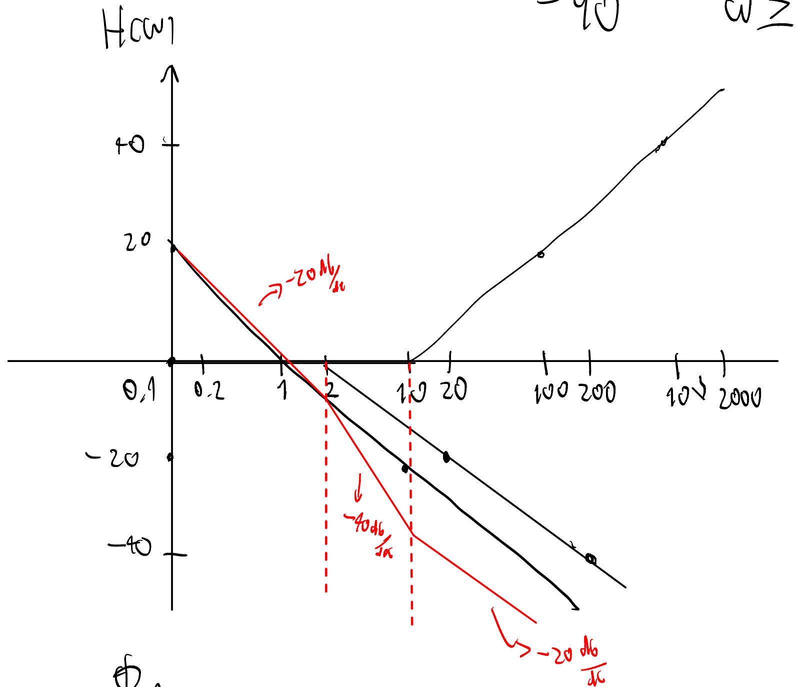
$90^\circ \quad w \ge 100$

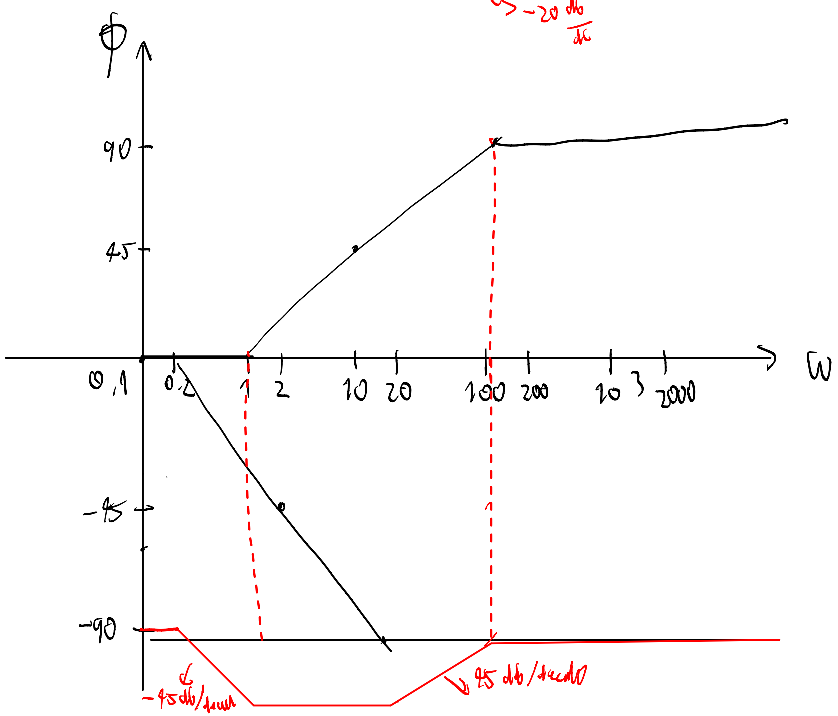
magnitude slope = 0

Type 3 Pole → $H(w) \to 0 \quad w \le 2$

$-20 \quad w \ge 2$

$$\phi \rightarrow \begin{cases} 0 & w \leq 0.2 \\ -45-45\log_{10}\left(\frac{w}{2}\right) & 0.2 \leq w \leq 20 \\ -90 & w \geq 20 \end{cases}$$



**14.12** A transfer function is given by

$$T(s) = \frac{100(s+10)}{s(s+10)}$$

Sketch the magnitude and phase Bode plots.

$s = jw$

$$T(s) = \frac{100(jw+10)}{jw(jw+10)}$$

$$= \frac{100 \times 10 \left(\frac{jw}{10}+1\right)}{jw \times \cancel{10} \left(\frac{jw}{10}+1\right)} = \frac{\overbrace{100}^{\text{Type 1}} \overbrace{\left(\frac{jw}{10}+1\right)}^{\text{Type 3 Zeros}}}{\underbrace{jw}_{\text{Type 2 } (jw)^{-1}} \underbrace{\left(\frac{jw}{10}+1\right)}_{\text{Type 3 Pole}}}$$

Type 1 $H(w) \to 20\log_{10}(\overbrace{100}^{K}) = 40$

$\phi \to$ as $K > 0 \to \phi = 0$

Type 2 $jw^{-1} \to H(w) = -20 \to$ slope

$\phi = -90$

Type 3 $H(w) \begin{cases} 0 & w \le 10 \\ 20 & w \ge 10 \end{cases}$ slope

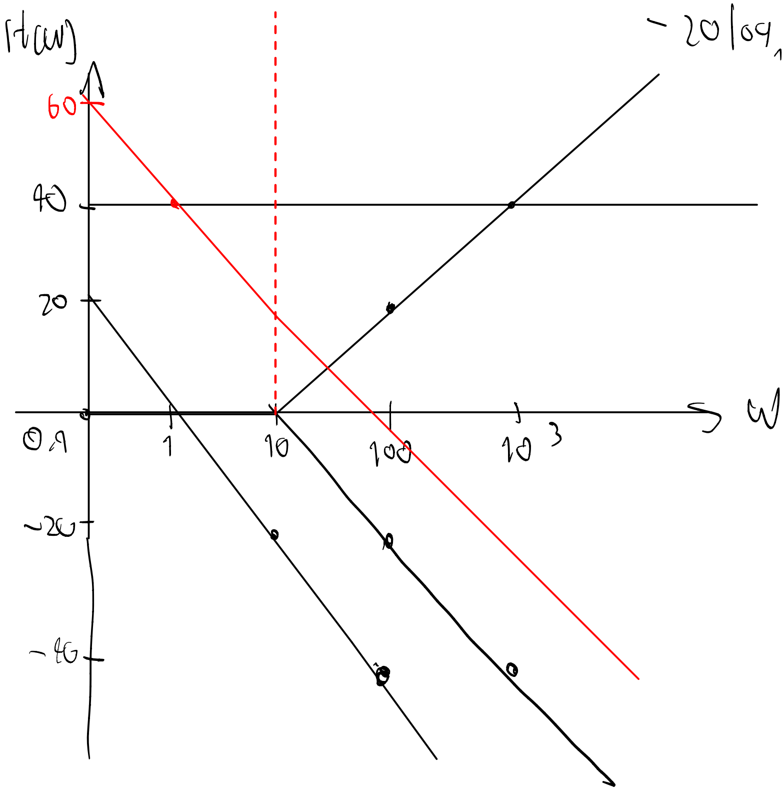
$\phi \begin{cases} 0 & w \le 1 \\ 45\text{deg/decade} & 1 \le w \le 100 \\ 90^\circ & w \ge 100 \end{cases}$

$90^\circ \to$ slope $= 0$

Type 3 Pole $\to H(w) \begin{cases} 0 & w \le 10 \\ -20 & w \ge 10 \end{cases}$

$\phi \begin{cases} 0 & w \le 1 \\ -45 & 1 \le w \le 100 \\ -90 & w \ge 100 \end{cases}$

$$H(\omega) = 20\log_{10}(100) - 20\log_{10}(j\omega) + 20\log_{10}\left(1+\frac{j\omega}{10}\right) - 20\log_{10}\left(1+\frac{j\omega}{10}\right)$$



**14.13** Construct the Bode plots for

$$G(s) = \frac{0.1(s+1)}{s^2(s+10)}, \qquad s = j\omega$$

Constant

$$G(s) = \frac{0.1(j\omega+1)}{j\omega^2(j\omega+10)} = \frac{0.1(j\omega+1)}{j\omega^2 \times 10(\frac{j\omega}{10}+1)} = \frac{\frac{1}{100}(j\omega+1)}{j\omega^2(\frac{j\omega}{10}+1)}$$

as constant $K>0 \rightarrow \phi=0$

$$H(\omega) \rightarrow 20\log_{10}\left(\frac{1}{100}\right) = -40$$

$$\phi = 0$$

Type 2 pole $(j\omega)^{-2}$

$$H(\omega) = -40 \ \frac{dB}{decade}$$

$$\phi = -180^\circ$$

Type 3 zero $\rightarrow \left(\frac{j\omega}{1}+1\right) \rightarrow z_1 = 1$

$H(\omega) \rightarrow$ 0 $\quad \omega \le 1$

20 $\quad \omega \ge 1$

$\phi \rightarrow$ 0 $\quad \omega \le 0.1$

$45 + 45\log\left(\frac{\omega}{10^{z}}\right)$ $\quad 0.1 \le \omega \le 10$

45 dB/dec

90 $\quad \omega \ge 10$

slope = 0 ; $90^\circ$

$z_1 = 10$

Type 3 Pole $\rightarrow \frac{1}{\left(\frac{j\omega}{10}+1\right)} \rightarrow H(\omega) \rightarrow$ 0 $\quad \omega \le 10$

$-20$ $\quad \omega \ge 10$

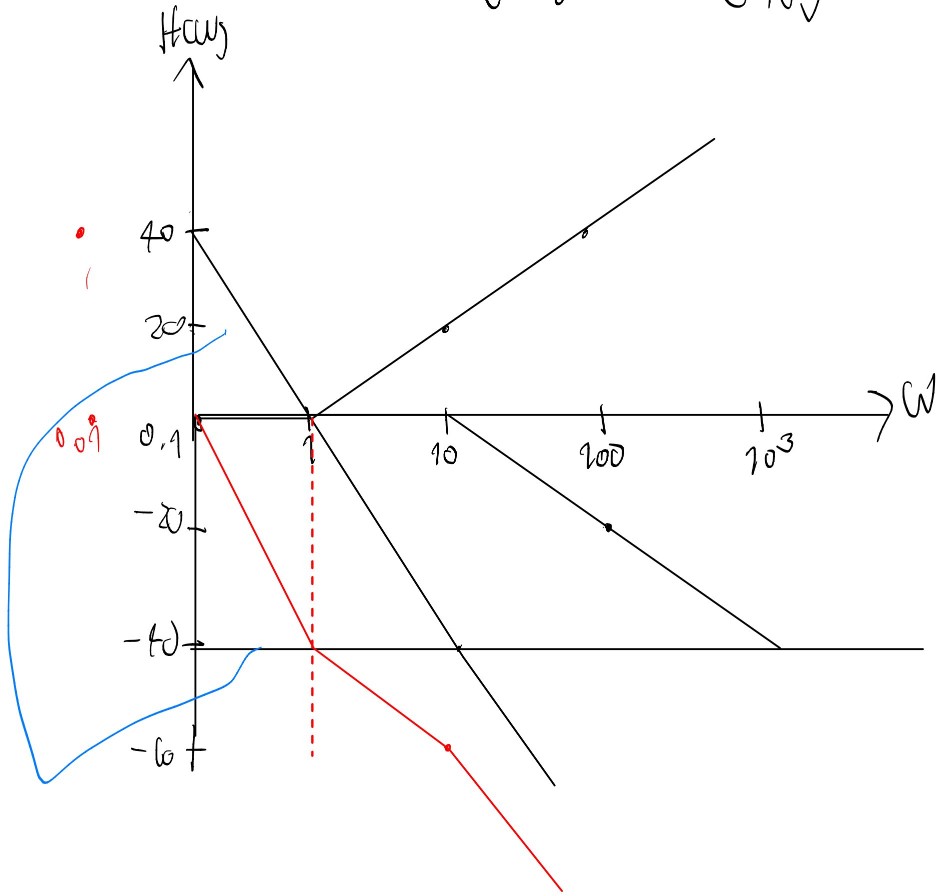
$\phi \rightarrow$ 0 $\quad \omega \le 1$

$-45$ $\quad 1 \le \omega \le 100$

$-90$ $\quad \omega \ge 100$

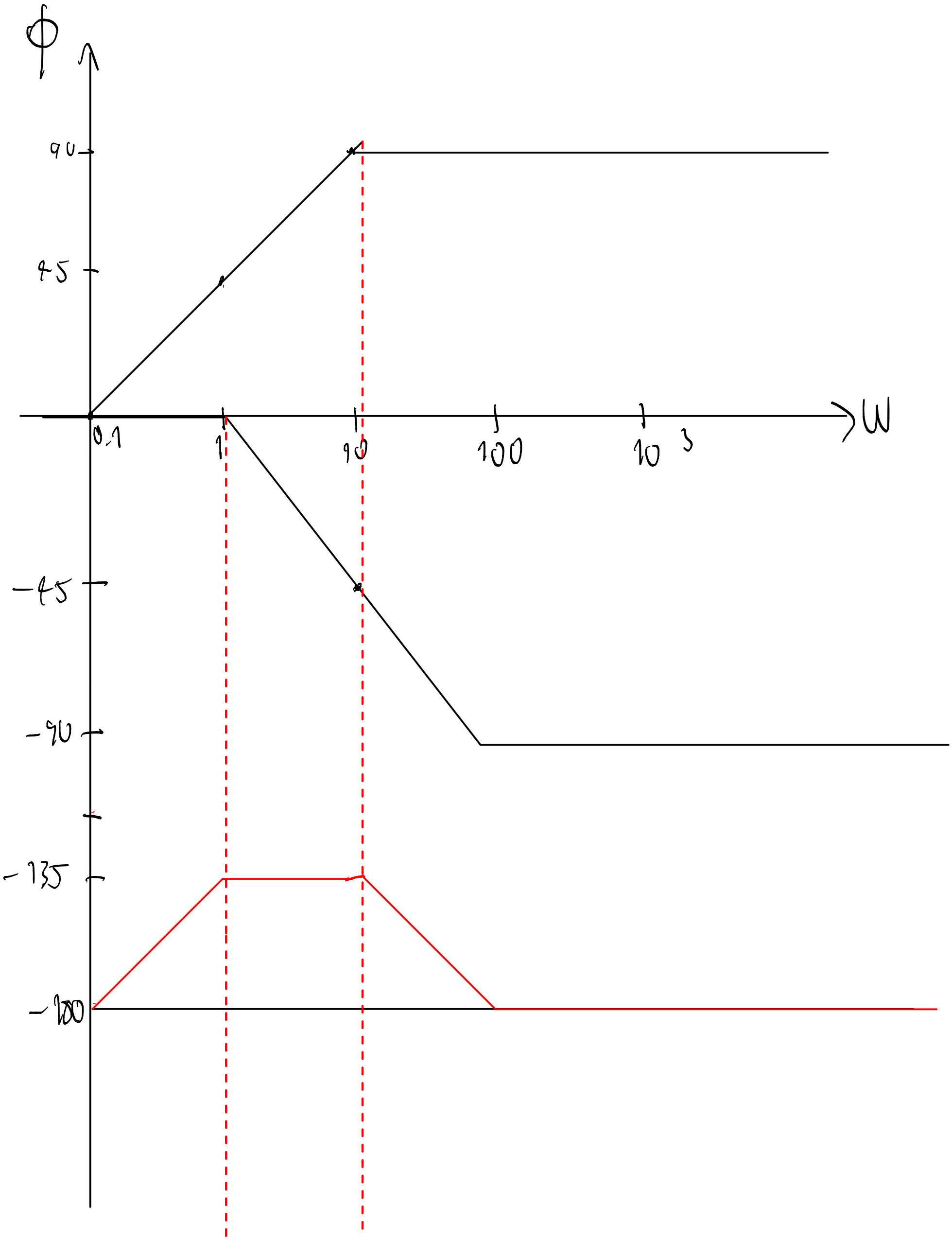
$$H(\omega) = 20\log_{10}\left(\frac{1}{100}\right) - 40\log_{10}|j\omega| + 20\log_{10}|j\omega+1| - 20\log_{10}\left(\frac{j\omega}{10}+1\right)$$

$$\phi \Rightarrow -180 + \tan^{-1}\left(\frac{\omega}{1}\right) - \tan^{-1}\left(\frac{\omega}{10}\right)$$




ถ้าแยก slope = -40 ; อันรวม => straight line = -40

ตอนรวมกัน slope เหมือนเดิมแค่ shift ลงมา -40 db



ϕ
90
45
-45
-90
-135
-180
0.1
1
10
100
10 3
W

**14.15** Construct the Bode magnitude and phase plots for

$$H(s) = \frac{2(s+1)}{(s+2)(s+10)}, \qquad s = j\omega$$

$$H(s) = \frac{2(jw+1)}{(jw+2)(jw+10)}$$

$$= \frac{2(jw+1)}{10 \times 2 \times \left(\frac{jw}{2}+1\right)\left(\frac{jw}{10}+1\right)} = \frac{\frac{1}{10}(jw+1)}{\left(\frac{jw}{2}+1\right)\left(\frac{jw}{10}+1\right)}$$

for constant $\rightarrow \frac{1}{10} \rightarrow 20\log_{10}\left(\frac{1}{10}\right) \rightarrow H(w) \rightarrow -20$

$\phi \rightarrow 0$ as $K>0$

for $jw+1$ $\rightarrow$ $H(w)\begin{cases} 0 & w \le 1 \\ 20 & w \ge 1 \end{cases}$

$\phi \begin{cases} 0 & w \le 0.1 \\ 45 & 0.1 \le w \le 10 \\ 90^\circ & w \ge 10 \end{cases}$

$\downarrow$ slope = 0

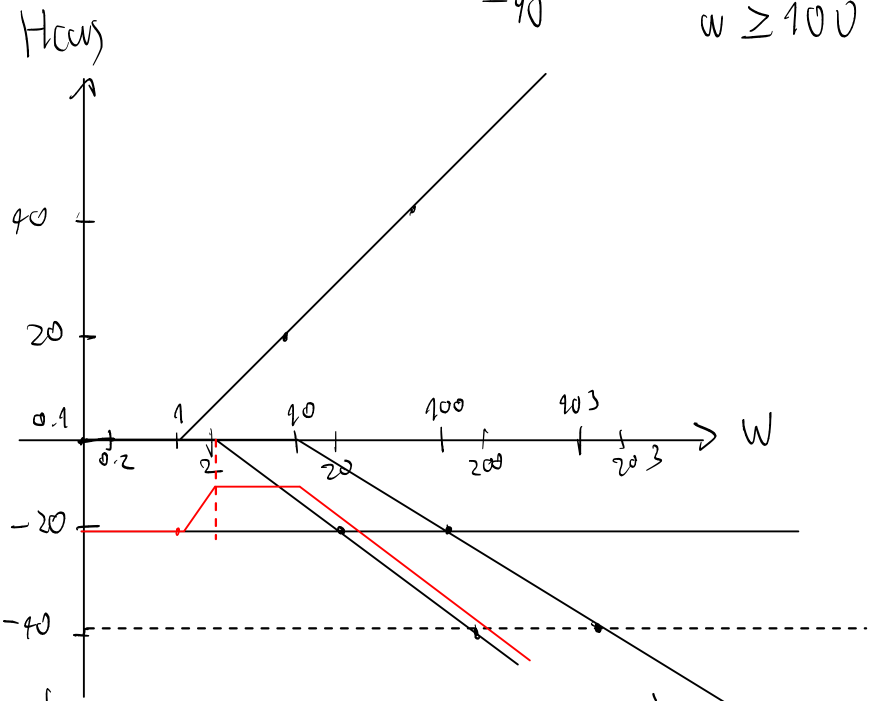
$\boxed{\frac{1}{\left(\frac{jw}{2}+1\right)}} \rightarrow H(w)\begin{cases} 0 & w \le 2 \\ -20 & w \ge 2 \end{cases}$

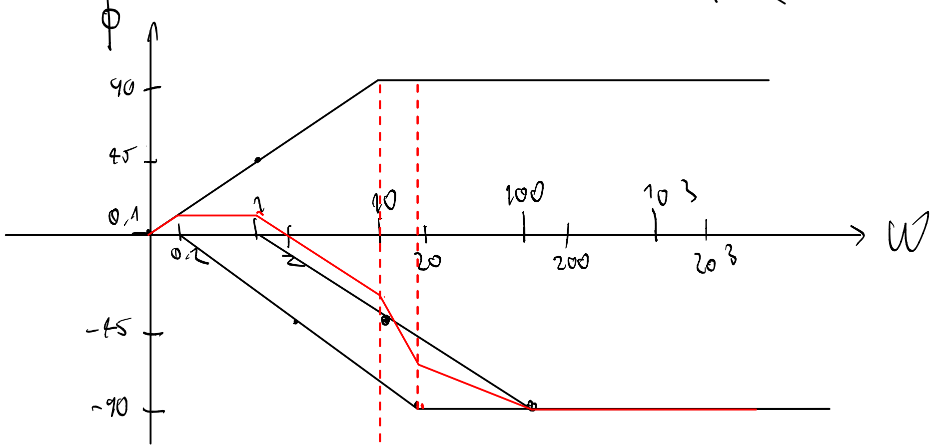
$\phi \begin{cases} 0 & w \le 0.2 \\ -45 & 0.2 \le w \le 20 \\ -90^\circ & w \ge 20 \end{cases}$

$\rightarrow$ slope = 0

for $\frac{1}{\left(\frac{j\omega}{10}+1\right)} \to H(\omega) \begin{cases} 0 & w \leq 10 \\ -20 & w \geq 10 \end{cases}$

$$\phi \begin{cases} 0 & w \leq 1 \\ -45 & 1 \leq w \leq 100 \\ -90 & w \geq 100 \end{cases}$$



**14.16** Sketch Bode magnitude and phase plots for

$$H(s) = \frac{1.6}{s(s^2 + s + 16)}, \quad s = j\omega$$

$$H(s) = \frac{1.6}{jw(jw^2 + jw + 16)} = \frac{1.6}{jw\left(\left(\frac{jw}{4}\right)^2 + \frac{jw}{16} + 1\right) \times 16}$$

$w_n = 1$

$$= \frac{0.1}{jw\left(\left(\frac{jw}{4}\right)^2 + \frac{jw}{16} + 1\right)}$$

Type 1 constant $\rightarrow 0.1$

$H(w) = 20 \log_{10}(0.1) \approx -20$

$\phi \Rightarrow 0$

Type 2 $jw^{-1}$

$H(w) \rightarrow -20 \rightarrow$ slope $dB/decade$

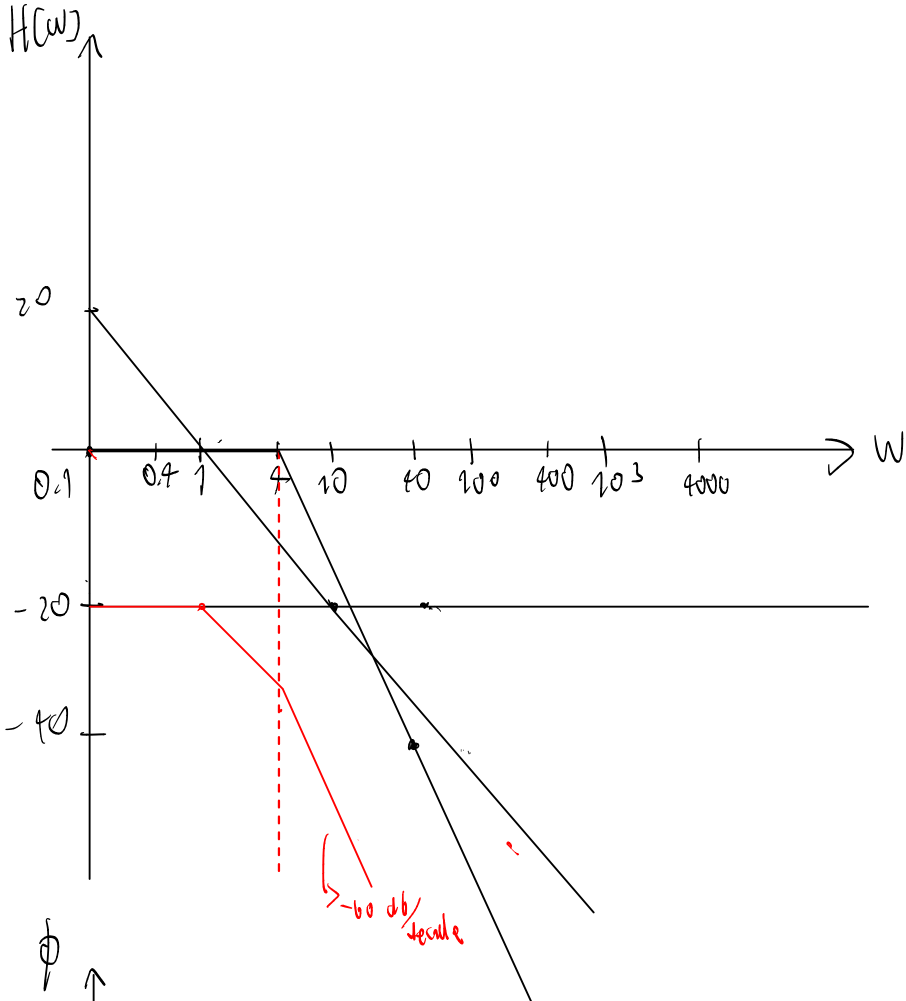
$\phi \rightarrow -90°$

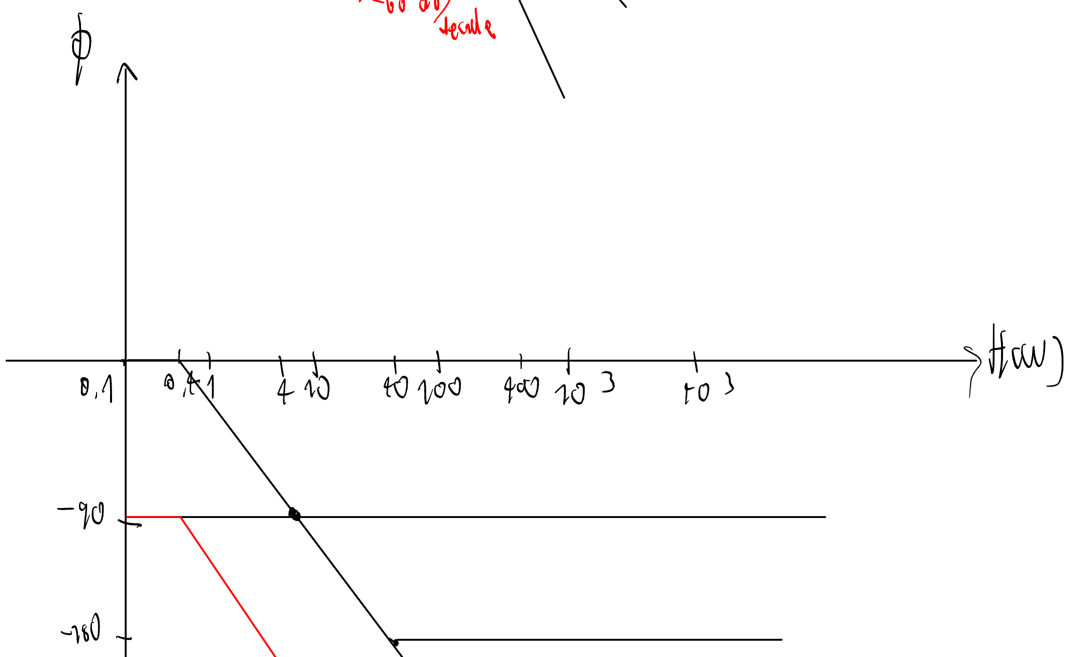
Type 4 $W_n = 4$

$$H(w) \begin{cases} 0 & w \leq 4 \\ -40 & w \geq 4 \end{cases}$$

$$\phi \begin{cases} 0 & w \leq 0.4 \\ -90 - 90 \log_{10}\left(\frac{w}{4}\right) & 0.4 \leq w \leq 40 \\ -180 & w \geq 40 \end{cases}$$

$\rightarrow$ slope $= 0$


H(w)
20
W
0.1
0.4
1
4
10
40
100
400
10 3
4000
-20
-40
-60 db/decade


ϕ
H(w)
0.1
0.4
1
4
10
40
100
400
10 3
10 3
-90
-180


**14.17** Sketch the Bode plots for

$$G(s) = \frac{s}{(s+2)^2(s+1)}, \qquad s = j\omega$$

$$G(s) = \frac{j\omega}{(j\omega+2)^2(j\omega+1)}$$

$$\downarrow$$

$$\frac{j\omega}{4\left(\frac{j\omega}{2}+1\right)^2(j\omega+1)} = \frac{\frac{1}{4}(j\omega)}{\left(\frac{j\omega}{2}+1\right)^2(j\omega+1)} \rightarrow \text{Type 2}$$

for type 1 $\rightarrow \frac{1}{4}$

$$H(\omega) = 20\log_{10}\left(\tfrac{1}{4}\right) = -12$$

$$\phi = 0 \quad \text{as } K > 0$$

for type 2 $\rightarrow$ $H(\omega) = 20 \rightarrow \frac{dB}{decade}$

$$\phi = 90$$

for type 3 pole $\left(\frac{j\omega}{2}+1\right)^2$

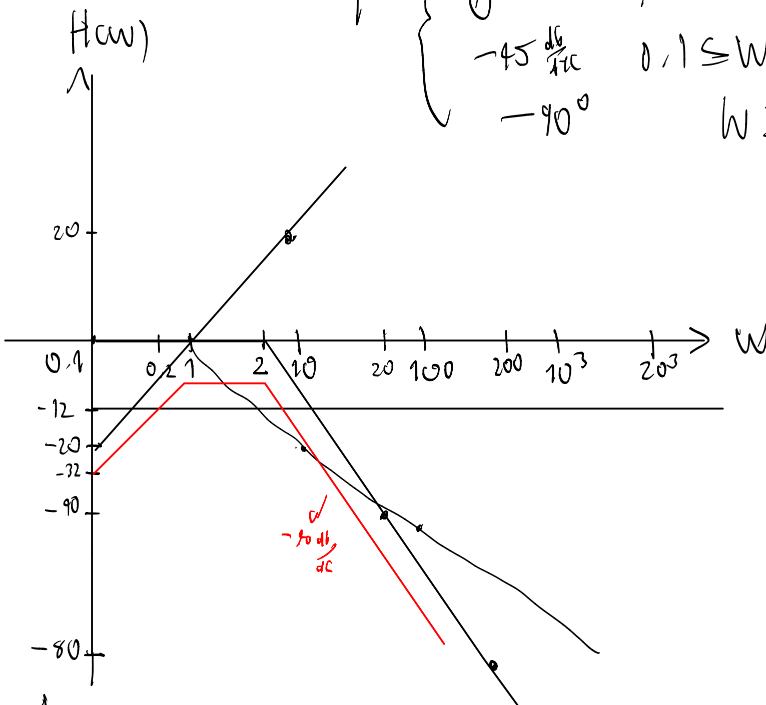
$$H(\omega) \begin{cases} 0 & \omega \le 2 \\ -40 & \omega \ge 2 \end{cases}$$

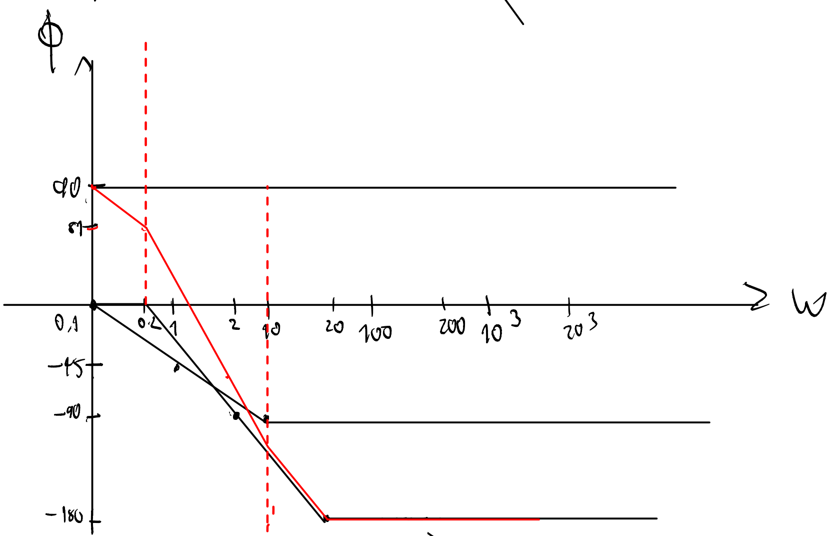
$$\phi \begin{cases} 0 & 0.2 \le \omega \\ -90\ \frac{dB}{dec} & 0.2 \le \omega \le 20 \\ -180^\circ & \omega \ge 20 \end{cases}$$

for type3 → $\frac{1}{jw+1}$ →

$$H(w)\begin{cases} 0 & w \le 1 \\ -20w & > 1 \end{cases}$$

$$\phi\begin{cases} 0 & 0,1 \le w \\ -45\,\frac{dB}{dec} & 0,1 \le w \le 10 \\ -90^\circ & w \ge 10 \end{cases}$$



**11.6** For the circuit in Fig. 11.38, $i_s = 6 \cos 10^3 t$ A. Find the average power absorbed by the 50-Ω resistor.

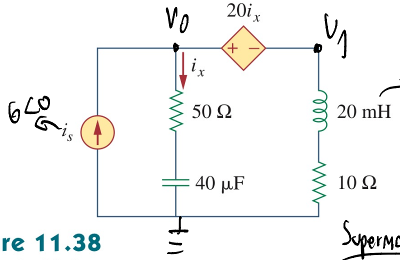



**Figure 11.38**
For Prob. 11.6.

$W = 1000$ rad/s

$20mH \Rightarrow (20\times10^{-3})(1000)j \Rightarrow 20j$

$40\mu F \Rightarrow \frac{1}{(40\times10^{-6})(1000)j} \Rightarrow -25j$

Supermesh $\rightarrow$ $V_0 - V_1 = 20i_x$

$V_1 = V_0 - 20i_x$

KCL at $V_0$

$$-6 + \frac{V_0}{50-25j} + \frac{V_1}{20j+10} = 0$$

$$\frac{V_0}{50-25j} + \frac{V_0 - 20i_x}{20j+10} = 6$$

also $i_x = \frac{V_0}{50-25j}$

solve and do later

**11.7** Given the circuit of Fig. 11.39, find the average power absorbed by the 10-Ω resistor.

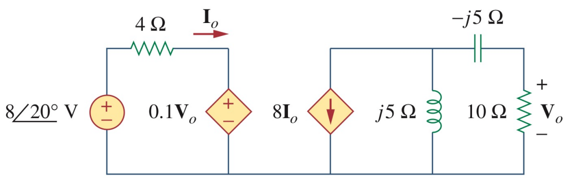



**Figure 11.39**
For Prob. 11.7.

**11.38** For the power system in Fig. 11.67, find: (a) the average power, (b) the reactive power, (c) the power factor. Note that 220 V is an rms value.

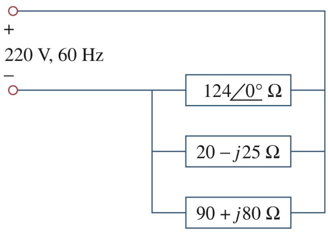



**Figure 11.67**
For Prob. 11.38.

We know from Complex Power

$S = P + jQ$

Average Power from Complex Power.

$S = P + jQ \rightarrow$

a) We Know Relation $\rightarrow S = \dfrac{V_{rms}^2}{Z^*} \rightarrow S_1 = \dfrac{(220)^2}{124} = 390.32$

$S_2 = \dfrac{(220)^2}{20 + j25} \Rightarrow 944.4 - 1180.5j$

$S_3 = \dfrac{V^2}{Z_3^*} \Rightarrow \dfrac{(220)^2}{90 - 80j} = 300 + 267.03j$

$S = S_1 + S_2 + S_3$ = Total Complex power $\Rightarrow 1872.6\angle{-29.796}$ VA

$S = 1634.7 - 913.47j$

Average Power $P$

$= 1634.7$ W

b) Reactive $\Rightarrow$ 913.47 VA

c) $P_f = \dfrac{P}{|S|} \rightarrow \dfrac{1634.7}{1872.61} \Rightarrow 0.8732$

**11.41** Obtain the power factor for each of the circuits in Fig. 11.68. Specify each power factor as leading or lagging.

load impedence angle

We know Pf angle = $Z \angle \theta_v - \theta_i$

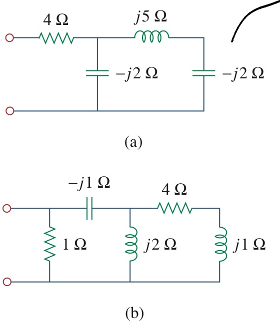



**Figure 11.68**
For Prob. 11.41.

a) $Z_{eq} \Rightarrow 4 - 6j \approx 2\sqrt{13} \angle -56.309$

$= \cos(-56.309)$ leading $\rightarrow \theta_i > \theta_v$

$Pf = 0.5547$

**11.42** A 110-V rms, 60-Hz source is applied to a load impedance **Z**. The apparent power entering the load is 120 VA at a power factor of 0.707 lagging.

(a) Calculate the complex power.

(b) Find the rms current supplied to the load.

(c) Determine **Z**.

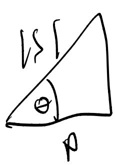
(d) Assuming that $\mathbf{Z} = R + j\omega L$, find the values of $R$ and $L$.

(a.) $S = \tilde{V}_{rms} \tilde{I}^{*}_{rms}$ $\qquad \vec{I}_{rms} = \dfrac{\tilde{V}}{R}$

$P_f = \dfrac{P}{|S|} \rightarrow Power = (0.707)(120)$

$\Rightarrow 84.84$ Watt !

to find Q →




→ $Q = P \tan\theta$

$Q = 84.84 \tan 45$

$\cos\theta = 0.707 \rightarrow \theta \Rightarrow 45°$

$Q = 84.84$

Ans → $S = P + jQ \Rightarrow 84.84 + 84.84j$

✓

(6.)  $S = \tilde{V}_{rms} \tilde{I}_{rms}^{*}$

$$\tilde{I}_{rms}^{*} = \frac{S}{\tilde{V}_{rms}} = \frac{(84.84 + 84.84j)}{110} \Rightarrow 0.7713 + 0.7727j$$

or $1.0907 \angle 45$

$$\underline{\tilde{I}_{rms} \Rightarrow 1.0907 \angle -45^{\circ}}$$

(c.)  $S = (I_{rms})^2 Z$

$$Z = \frac{84.84 + 84.84j}{(1.0907)^2} \Rightarrow 71.3965 + 71.3965j$$

$R = 71.3965$

$WL = 71.3965$

$$L = \frac{71.3965}{(2\pi \times 60)}$$

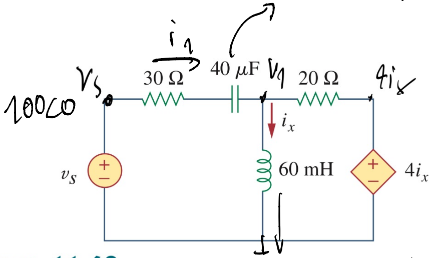
$$\underline{= 0.1894 \, H}$$

**11.44** Find the complex power delivered by $v_s$ to the network in Fig. 11.69. Let $v_s = 100 \cos 2000t$ V.

don't know if it is RMS or not use

$S = \frac{1}{2} V_s I_o^*$

$40\mu F = \frac{1}{\omega C j} = \frac{1}{2000(40 \times 10^{-6})j} = -12.5j$




$\rightarrow S = V_s I_1^*$

**Figure 11.69**
For Prob. 11.44.

$60mH = L\omega j = (60 \times 10^{-3})(2000)(j) = 120j$

KCL at $V_1$ $\rightarrow$ $\frac{V_1 - 100}{30 - 12.5j} + \frac{V_1}{120j} + \frac{V_1 - 4i_x}{20} = 0$

also $i_x = \frac{V_1}{120j}$

$\frac{V_1 - 100}{30 - 12.5j} + \frac{V_1}{120j} + \frac{V_1 - 4\left(\frac{V_1}{120j}\right)}{20} = 0$

$\frac{V_1 - \frac{V_1}{30j}}{20}$

$\frac{1 V_1}{30 - 12.5j} + \frac{1}{120j} V_1 + \frac{1}{20} V_1 - \frac{1}{600j} V_1$

$= \frac{100}{30 - 12.5j}$

$\left(\frac{53}{676} + \frac{131}{25350} j\right) V_1 = \left(\frac{480}{169} + \frac{200}{101} j\right)$

$V_1 = 37.06 + 12.652j$

$\approx 39.16 \angle 18.85°$

$i_1 = \frac{100 - (39.16 \angle 18.85°)}{30 - 12.5j}$

$1.937 + 0.3855j$

or $1.9754 \angle 11.259°$

$$S = \frac{1}{2} V_s \tilde{I}^*$$
$$= \frac{1}{2}(100)(1.9754 \angle -11.2580)$$

**11.46** For the following voltage and current phasors, calculate the complex power, apparent power, real power, and reactive power. Specify whether the pf is leading or lagging.

(a) $\mathbf{V} = 220\angle 30°$ V rms, $\mathbf{I} = 0.5\angle 60°$ A rms

(b) $\mathbf{V} = 250\angle -10°$ V rms, $\mathbf{I} = 6.2\angle -25°$ A rms

(c) $\mathbf{V} = 120\angle 0°$ V rms, $\mathbf{I} = 2.4\angle -15°$ A rms

(d) $\mathbf{V} = 160\angle 45°$ V rms, $\mathbf{I} = 8.5\angle 90°$ A rms

(a.) Complex Power → $S = \tilde{V}_{rms} \tilde{I}_{rms}^*$

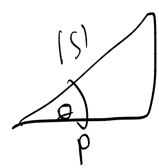
$$= 220\angle 30 \, (0.5\angle -60)$$
$$= 110\angle -30 \qquad \frac{1}{4} - \frac{\sqrt{3}}{4} j$$

$$S = 95.2628 - 55j$$

as $Q < 0$ → It's leading

real power = 95.2628  -j Reactive Power => 55 kVA

$|S| = 110$




$$P_f = \frac{P}{|S|} = \frac{95.2628}{110} = 0.866$$

$$\cos\theta = 0.866$$

**11.52** In the circuit of Fig. 11.71, device *A* receives 2 kW at 0.8 pf lagging, device *B* receives 3 kVA at 0.4 pf leading, while device *C* is inductive and consumes 1 kW and receives 500 VAR.

(a) Determine the power factor of the entire system.

(b) Find **I** given that $\mathbf{V}_s = 120\angle 45°$ V rms.

2kW; 0.8 pf lag $\rightarrow$ $P = 2K$

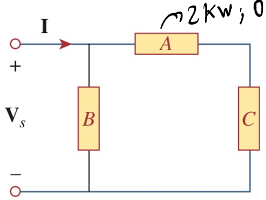



**Figure 11.71**
For Prob. 11.52.

$S = P + jQ$ $\qquad \cos\theta = 0.8$

$\qquad\qquad\qquad\qquad \theta = 36.87$

$Q = P\tan\theta$

$= 2\tan(36.87) = 1.5$

$S_1 = (2 + 1.5j)$ KW

At B $\quad |S| = 3$ kVA $-$ $P_f = 0.4$

$P_f = \frac{P}{|S|} \rightarrow P = 0.4 \times 3 = 1.2$ KW

$\cos\theta = 0.4$

$\theta = -66.422 \rightarrow$ leading

$Q = P\tan\theta \rightarrow 1.2\tan(-66.422)$

$\Rightarrow -2.7495$

$S_2 = (1.2 - 2.7495j)$ KW

At C. $\quad$ 1KW ; $+500 \rightarrow Q$

$S_3 = (1 + 0.5j)$ KW

$S = 4.2 - 0.7495j$

$Pf = \frac{P}{|S|} = \frac{4.2}{4.26635} = 0.9844$

(b) I

$S = \bar{V}_{rms}\bar{I}^*_{rms}$

$\bar{I}^*_{rms} = \frac{S}{\bar{V}_{rms}}$

$\bar{I}_{rms} = 35.55\angle 55.1$

**11.54** For the network in Fig. 11.73, find the complex power absorbed by each element.

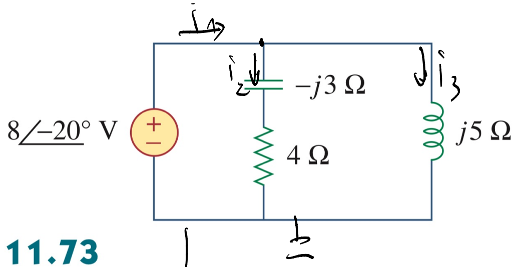



**Figure 11.73**
For Prob. 11.54.

$$i_1 = i_2 + i_3$$

$$i_2 = \frac{8\angle -20}{4 - j3}$$

$$i_2 = 1.6\angle 16.87^0$$

$$i_3 = \frac{8\angle -20}{j5}$$

$$\Rightarrow 1.6\angle -110°$$

$$i_1 = i_2 + i_3 \Rightarrow 1.431\angle -46.565$$

Complex Power by the Source.

$S = V I^* \Rightarrow$

for Complex Power it's

in $V_{rms}$

**11.57** For the circuit in Fig. 11.76, find the average, reactive, and complex power delivered by the dependent current source.

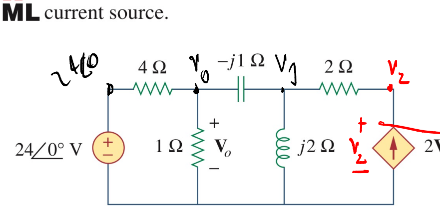



**Figure 11.76**
For Prob. 11.57.

$S =$

Need to find Voltage $V_{rms}$ at the current source.

to find Complex power.

$$\frac{V_0 - 24}{4} + \frac{V_0 - V_1}{-j1} + \frac{V_0}{1} = 0$$

$$\frac{V_0}{1} + \frac{V_0 - V_1}{-j1} = \frac{24 - V_0}{4}$$

$$24 = (5 + j4)V_0 - j4V_1 \rightarrow (1)$$

At node 1.

$$\frac{V_1 - V_0}{-j1} - 2V_0 + \frac{V_1}{j2} = 0$$

$$\frac{V_0 - V_1}{-j1} + 2V_0 = \frac{V_1}{j2}$$

$$V_1 = (2 - j4)V_0 \rightarrow (2)$$

$$V_0 = \frac{-24}{11+j4} \quad ; \quad V_1 = \frac{(-24)(2-j4)}{11+j4}$$

also We know $\rightarrow$ $2V_0 = \dfrac{V_2 - V_1}{2}$

$$V_2 - V_1 = 4V_0$$

$$V_2 = 4V_0 + V_1$$

$$V_2 = \frac{(-24)(6-j4)}{11+j4}$$

$$S = V_2 I^* = V_2 (2V_0^*)$$

Ans

**11.58** Obtain the complex power delivered to the 10-kΩ resistor in Fig. 11.77 below.

ML

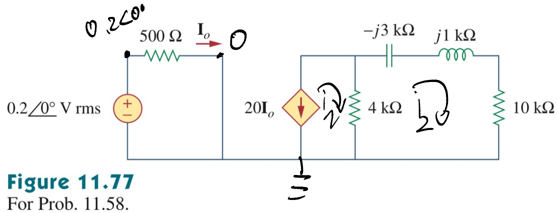



**Figure 11.77**
For Prob. 11.58.

$$I_o = \frac{0.2}{500} \Rightarrow \frac{1}{2500}\ A$$

$$i_1 = -20 I_o \Rightarrow -\frac{1}{125}$$

KVL $i_2$

$$i_2(-2j) + 10 i_2 + 4(i_2 - i_1) = 0$$

$$-2j i_2 + 10 i_2 + 4 i_2 + \frac{4}{125} = 0$$

$$(14 - 2j) i_2 = \frac{-4}{125}$$

$$i_2 = (2.2627 \times 10^{-3}) \angle -171.87°$$

$$S = I_{rms}^2 Z \Rightarrow \left| (2.2627 \times 10^{-3}) \right|^2 (10 \times 10^3)$$

$$\boxed{S = 0.0512\ VA}$$

W

Complex Power delever to resistor always = watts.

**11.60** For the circuit in Fig. 11.79, find $\mathbf{V}_o$ and the input power factor.

The complex, real, reactive power of the source equal to the sum of the complex, real, reactive power of each individual load.

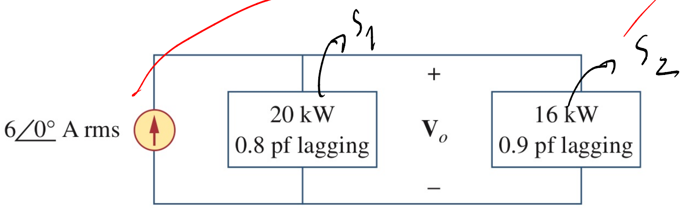



**Figure 11.79**
For Prob. 11.60.

Conservation of AC Power

$$S_{Total} = S_1 + S_2 \longrightarrow \text{for } S_1 \rightarrow P = 20\text{ kW} \; ; \; P_f = 0.8$$

$$Q = P\tan\theta \rightarrow \cos\theta = 0.8$$

$$\theta = 36.87$$

$$= 20\tan 36.87$$

$$Q = 15$$

$$S_1 = (20 + 15j)\text{ kVA}$$

for $S_2 \rightarrow P = 16\text{ kW} \; ; \; P_f = 0.9$

$$Q = P\tan\theta \rightarrow \cos\theta = 0.9$$

$$\theta = 25.842^\circ$$

$$Q = 16\tan 25.842$$

$$= 7.749$$

$$S_2 = (16 + 7.749j)\text{ kVA}$$

$$S_T = S_1 + S_2 \Rightarrow (20 + 15j) + (16 + 7.749j)$$

$= 36 + 22.75j$

$\downarrow$

$= 42.585 \angle 32.29^{0}$

We know $S = \tilde{V}_{rms} \tilde{I}^{*}_{rms}$

$$\tilde{V}_0 = \frac{S}{\tilde{I}^{*}_{rms}} = \frac{42.585 \angle 32.29}{6}$$

$$\approx 6 + 3.79j$$

$\downarrow$

$$\tilde{V}_0 = 7.0975 \angle 32.29$$

$P_f = \cos\theta \Rightarrow 0.845$ //

$\downarrow$

$S = 42.585 \angle 32.29$

$\downarrow \qquad \downarrow$

$|S| \quad (\theta_v - \theta_i)$

**11.61** Given the circuit in Fig. 11.80, find $\mathbf{I}_o$ and the overall complex power supplied.

Capacitive → Q = negative

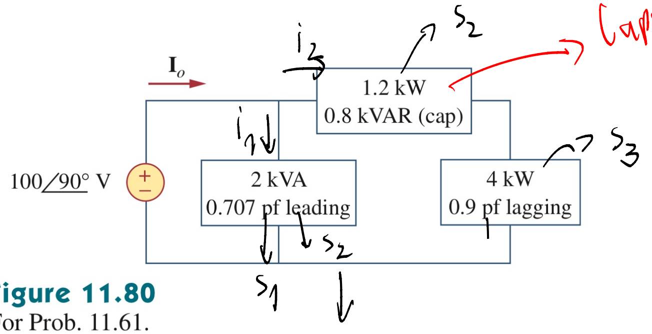



**Figure 11.80**
For Prob. 11.61.

We know $I_o = I_1 + I_2$

$S_2 = (1.2 - 0.8j)$ kVA ✱

$S_3 = 4\text{kW}\ ;\ 0.9$ pf lagging

$\cos\theta = 0.9$

$\theta = 25.841$

$Q = 4\tan 25.841$

$Q = 1.937$

$S_3 = (4 + 1.937j)$ KVA ✱

$S_4 = S_2 + S_3$

$\Rightarrow 5.2 + 1.137j$

$S_4 = V_{rms} I^*$

$I_2^* = \frac{S_4}{V_{rms}}$

$I_2^* = 11.37 - 52j$

$I_2 = 11.37 + 52j$

$S_1 = |S| = 2$ kVA → $P_f = 0.707$

$\cos\theta = 0.707$

$\theta = -45$

$P_f = \frac{P}{|S|}$

$P = P_f|S| = 0.707 \times 2$

$\Rightarrow 1.414$

$Q = P\tan\theta \rightarrow 1.414\tan(-45)$

$S_1 = 1.414 - 1.414j$ KVA

find $i_1 \Rightarrow S_1 = \tilde{V}_{rms}\tilde{I}^*_{rms}$

$$\tilde{I}^*_{rms_1} = \frac{S_1}{\tilde{V}_{rms}} \Rightarrow \frac{(1.414 - 1.414j)\times 10^3}{100\angle 90}$$

$$\tilde{I}_1 = -14.142 + 14.142j$$

$$\boxed{I_0 = I_1 + I_2 \Rightarrow 66.2\angle 92.4^\circ}$$

$$S_0 = V_0 \tilde{I}_0^*$$

$$= (100\angle 90)(66.2\angle -92.4)$$

$$S_0 = 6.62\angle -2.4 \,//$$

**11.62** For the circuit in Fig. 11.81, find $\mathbf{V}_s$.

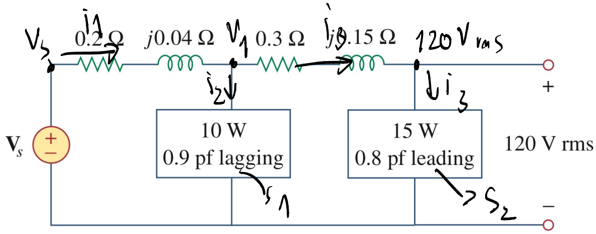



**Figure 11.81**
For Prob. 11.62.

$i_1 = i_2 + i_3 \rightarrow \quad S_0 = \tilde{V}_s \tilde{I}_1^*$

$$\tilde{V}_s = \frac{S_0}{\tilde{I}_1^*}$$

find $I_3$ $\rightarrow$ 15 W ; 0.8 Pf leading

$P = 15 \qquad \cos\theta = 0.8$

$$\theta = -36.87^\circ$$

$Q = 15 \tan(-36.87)$

$\Rightarrow -11.25$

$S_2 = 15 - 11.25j$

$\downarrow$

$S_2 = \tilde{V}_{rms} \tilde{I}_3^*$

$$\tilde{I}_3^* = \frac{S_2}{\tilde{V}_{rms}} = \frac{15 - 11.25j}{120}$$

$\tilde{I}_3 \Rightarrow 0.156 \angle 36.87$ //

find $I_2$ ~~also~~ need $V_1$ $\rightarrow$ $i_3 = \dfrac{V_1 - 120}{0.3 + 0.15j}$

$$i_3(0.3+0.15j)+120 = V_1$$

$$\boxed{V_1 = 120.02 + 0.0469j}$$

$$S_1 = \tilde{V}_{rms} \tilde{I}_2^*$$

$$\tilde{I}_2^* = \frac{S_1}{\tilde{V}_{rms}} = \frac{10 + 4.843j}{(120.02 + 0.0469j)}$$

10W ; 0.9 pf lagging

$\cos\theta = 0.9$

$\theta = 25.842$

$Q = 10 \tan 25.842 = 4.843$

$S_1 = 10 + 4.843j$

$$\tilde{I}_2 = 0.0926 \angle -25.8185$$

$$\tilde{I}_1 = \tilde{I}_2 + \tilde{I}_3 = 0.2087 + 0.053j$$

Now We know $\rightarrow$ $i_1 = \dfrac{V_s - V_1}{0.2 + 0.04j}$ $\quad i_1(0.2+0.04j)+V_1 = V_s$

**11.63** Find $\mathbf{I}_o$ in the circuit of Fig. 11.82.

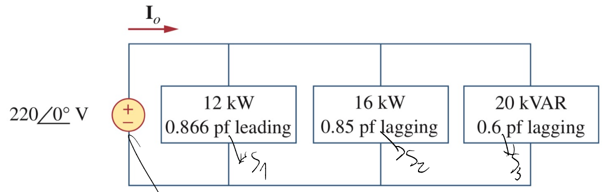



**Figure 11.82**
For Prob. 11.63.

The complex power from the source is equal to the sum of each individual complex power from each load.

From $S_1$ → 12 kW ; 0.866 pf leading

$$P = 12\ \text{kW} \rightarrow Q = P\tan\theta \rightarrow \cos\theta = 0.866$$

$$\theta = -30$$

$$Q = 12\tan(-30)$$

$$\Rightarrow -6.928$$

$$S_1 = (12 - 6.928j)\ \text{kVA}$$

$$S_2 \rightarrow \quad P = 16\ \text{kW} \quad \cos\theta = 0.85$$

$$\theta \simeq 31.788^\circ$$

$$Q = 16\tan 31.788 \Rightarrow 9.916$$

$$S_2 = (16 + 9.916j)\ \text{kVA}$$

$$S_3 \rightarrow \quad Q = 20\ \text{kVAR} \rightarrow P_f = 0.6$$

$$\cos\theta = 0.6$$

$$\theta = 53.13$$

$$Q = P\tan\theta \rightarrow P = \frac{Q}{\tan\theta} = \frac{20}{\tan 53.13} = 15$$

$$S_3 = (15 + 20j)\ \text{kVAR}$$

$$S_T = 43 + 22.988j \rightarrow S = V_{rms} I_o^*$$

$$\Downarrow$$

$$\tilde{I}_0^* = \frac{S}{\tilde{V}_{rms}}$$

$$\Rightarrow \frac{(43 + 22.9881j)}{220}$$

$$\tilde{I}_0 = (0.2216 \angle -28.13^{\circ}) \text{ A}$$

**10.48** Find $i_o$ in the circuit of Fig. 10.93 using superposition.

ML

$$20\mu F = \frac{1}{(20\times10^{-6})2000j} = -25j$$

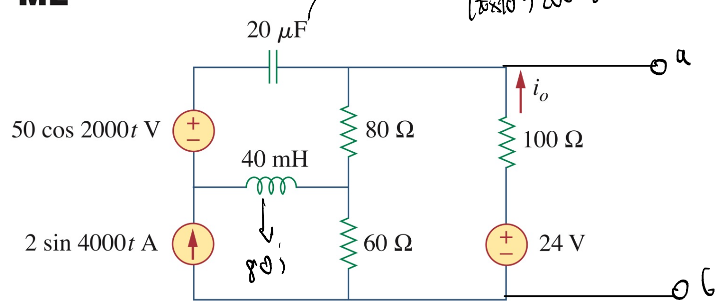
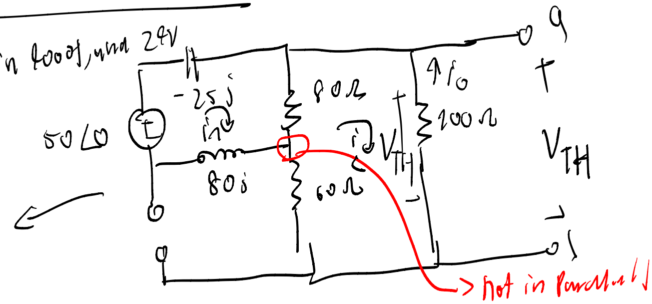



**Figure 10.93**

Find Thevenin / Norton Equivalent.

first use W = 2000 close 2 sin 4000t, and 24V




$Z_{eq}$ → close 50∠0

$$\rightarrow 100 \,/\!/\, [(55j \,/\!/\, 80) + 60]$$

$$= 41.35 \angle 12.78^{\circ}$$

$V_{TH}$ → KVL → $i_1$ → $-25j i_1 - 50 + 80(i_1 - i_2) + 80j i_1 = 0$

$$55j\, i_1 - 50 + 80(i_1 - i_2) = 0$$

~~FVL~~ $i_2 \rightarrow$ $$100\, i_2 + 60\, i_2 + 80(i_2 - i_1) = 0$$

$$\Downarrow$$

$$80(i_2 - i_1) = -160\, i_2$$

$$80(i_1 - i_2) = 50 - 55j\, i_1$$

$$80(i_2 - i_1) = -50 + 55j\, i_1$$

$$-160\, i_2 = -50 + 55j\, i_1$$

$$-160\, i_2 + 50 = 55j\, i_1$$

$$i_1 = \frac{-160\, i_2 + 50}{55j}$$

$$80\left(i_2 - \left(\frac{-160\, i_2 + 50}{55j}\right)\right) = -160\, i_2$$

$$80\left(i_2 + \left(\frac{160\, i_2}{55j} - \frac{50}{55j}\right)\right) = -160\, i_2$$

$$i_2 + \frac{160\, i_2}{55j} - \frac{50}{55j} = -2\, i_2$$

$$\left(3 - \frac{32}{11} j\right) i_2 = \frac{50}{55j}$$

$$i_2$$

$$V_{TH} = i_2 \times 100 = 15.144 - j15.618 \ \text{V}$$

**10.48** Find $i_o$ in the circuit of Fig. 10.93 using superposition.

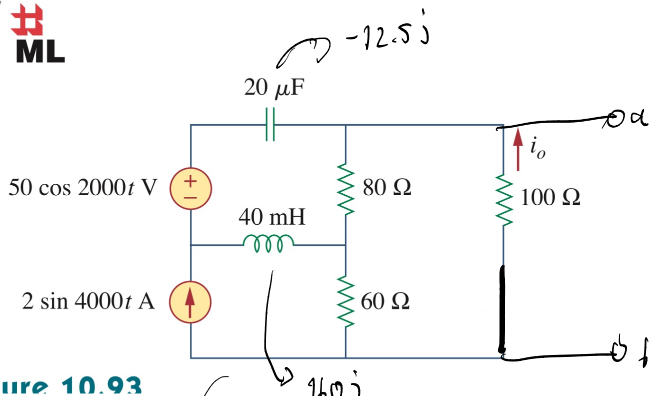
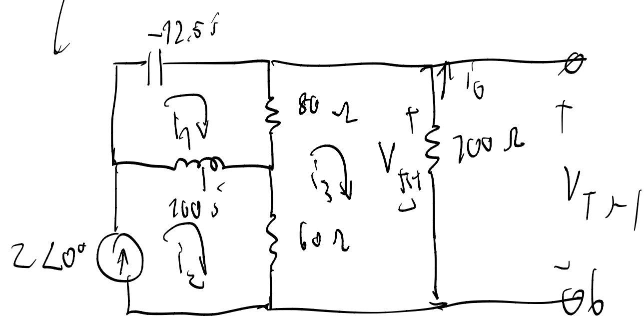



**Figure 10.93**

for $W = 4000$

close 50 cos 2000t and 24 V




$Z_{th}$ close $2\angle 0 \longrightarrow$ $100 \,//\, \left[ (147.5j // 80) + 60 \right]$

$\Downarrow$

$55.124 + 6.662j \ \Omega$

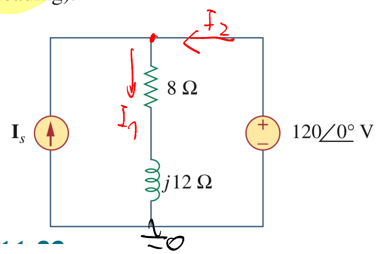
Find $V_{TH}$

KVL $i_1 \rightarrow$ $-12.5j\, i_1 + 80(i_1 - i_3) + 160j(i_1 - i_2) = 0$

KVL $i_2 \Rightarrow$ $i_2 \Rightarrow 2\angle 0°$

KVL $i_3 \rightarrow$ $100 i_3 + 60(i_3 - i_2) + 80(i_3 - i_1) = 0$

**11.64** Determine $\mathbf{I}_s$ in the circuit of Fig. 11.83, if the voltage source supplies 2.5 kW and 0.4 kVAR (leading).




$$I_1 = I_s + I_2 \rightarrow I_s = I_1 - I_2$$

We know $120\angle 0°$V supplies 2.5 kW and 0.4 KVAR

$$S = (2.5 - 0.4j)\ \text{KVA}$$

leading

$$S = \tilde{V}_{rms} \tilde{I}_{rms}^{*}$$

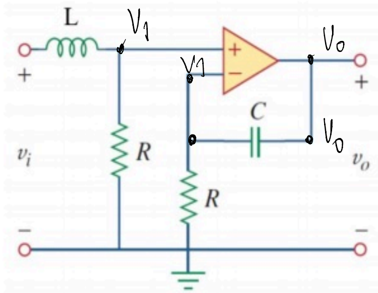
$$\tilde{I}_2^{*} = \frac{S}{V_{rms}} = \frac{(2500 - 400j)}{120}$$

$$\tilde{I}_2 = \frac{125}{6} + \frac{10}{3}j$$

$$I_1 = \frac{120 - 0}{8 + 12j} \Rightarrow \frac{60}{13} - \frac{90}{13}j$$

$$I_s = I_1 - I_2 \Rightarrow 19.19\angle -147.61$$

Q5. Please draw a **Bode plot** of the transfer function $H(\omega) = V_o/V_i$ [Total 18 points]




$$H(\omega) = \frac{V_o}{V_i}$$

KCL at $V_1$ $\rightarrow$ $\dfrac{V_1 - V_i}{j\omega L} + \dfrac{V_1}{R} = 0$

$$\frac{V_1}{R} = \frac{V_i - V_1}{j\omega L}$$

$$\frac{(j\omega L)V_1}{R} + V_1 = V_i$$

$$V_1\left(\frac{j\omega L}{R} + 1\right) = V_i \rightarrow (1)$$

KCL at $V_1$ on other side: $\dfrac{V_1 - V_o}{\frac{1}{j\omega C}} + \dfrac{V_1}{R} = 0$

$$j\omega C(V_1 - V_o) + \frac{V_1}{R} = 0$$

$$\frac{V_1}{R} = j\omega C(V_o - V_1)$$

$$\frac{V_1}{R} = j\omega C V_o - j\omega C V_1$$

$$\frac{V_1}{R} + j\omega C V_1 = j\omega C V_o$$

$$V_o = V_1\left(\frac{1}{j\omega C R} + 1\right) \rightarrow (2)$$

$\downarrow$

$$H(\omega) = \frac{V_o}{V_i} \Rightarrow \frac{\cancel{Y_1}\left(\frac{1}{j\omega CR}+1\right)}{\cancel{Y_1}\left(\frac{j\omega L}{R}+1\right)}$$

$$H(\omega) = \frac{\left(\frac{1}{j\omega CR}+1\right)}{\left(\frac{j\omega L}{R}+1\right)} \times \frac{j\omega CR}{j\omega CR}$$

$$= \frac{1+j\omega CR}{j\omega CR\left(\frac{j\omega L}{R}+1\right)} = \frac{1+j\omega CR}{(j\omega)^2 CL + j\omega CR}$$

$$\downarrow$$

$$\frac{\left(1+\frac{j\omega}{\frac{1}{RC}}\right)}{j\omega C\underbrace{\frac{(j\omega L+R)}{R}}} \rightarrow \left(\frac{j\omega}{\frac{R}{L}}+1\right)$$

$$H(\omega) = \frac{\overbrace{\frac{1}{RC}}^{\text{Type 1}}\overbrace{\left(1+\frac{j\omega}{\frac{1}{RC}}\right)}^{\text{Type 3}}}{\underbrace{j\omega}_{\text{Type 3}\to(j\omega)^{-1}}\underbrace{\left(\frac{j\omega}{\frac{R}{L}}+1\right)}_{\text{Type 3}\to\text{Pole}}}$$

for Type 1 $\rightarrow \frac{1}{RC} \rightarrow 20\log_{10}\left|\frac{1}{RC}\right|$

for $\frac{1}{RC} > 0 \rightarrow \phi = 0$

for type 2 $(j\omega)^{-1} \rightarrow$ slope $\Rightarrow -20\log_{10}(j\omega)$

$\phi = -90$

for type 3 zeros $\rightarrow z_1 = \frac{1}{RC}$ $\quad H(\omega) \begin{cases} 0 \rightarrow \omega \leq \frac{1}{RC} \\ 20 \rightarrow \omega \geq \frac{1}{RC} \end{cases}$

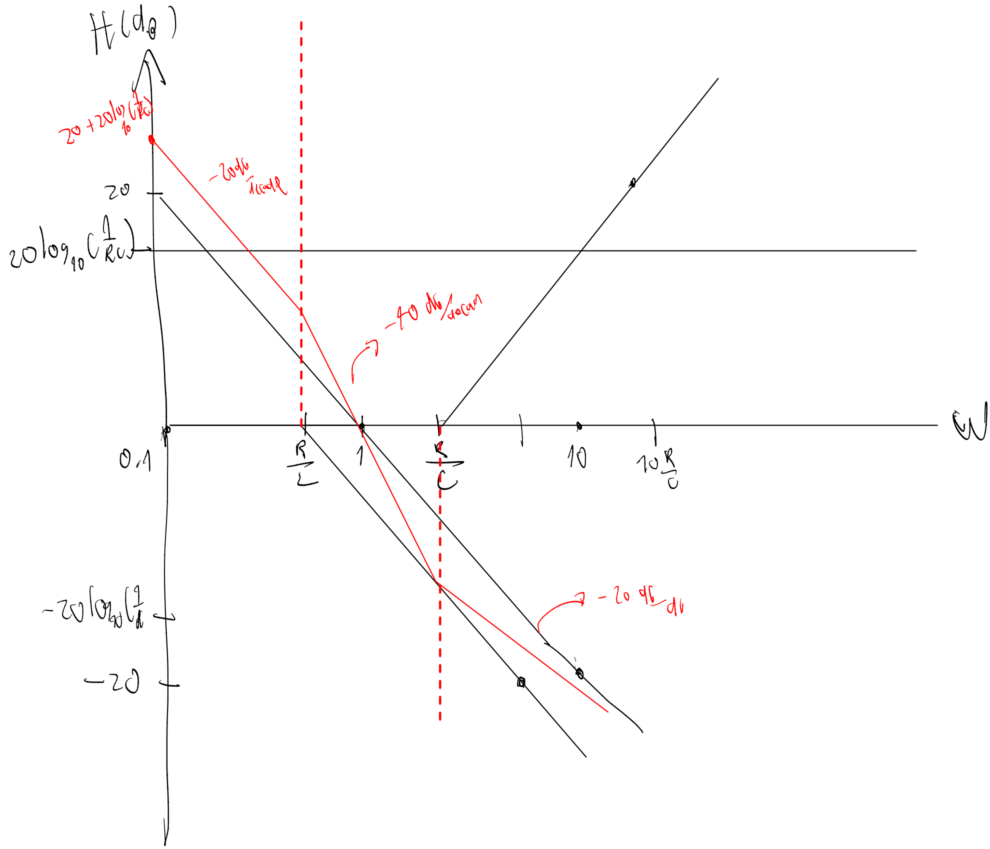
$$\phi \rightarrow \begin{cases} 0 & \omega \leq 0.1\frac{1}{RC} \\ 45+45\log_{10}\left(\frac{\omega}{\frac{1}{RC}}\right) & 0.1\frac{1}{RC} \leq \omega \leq \frac{10}{RC} \end{cases}$$

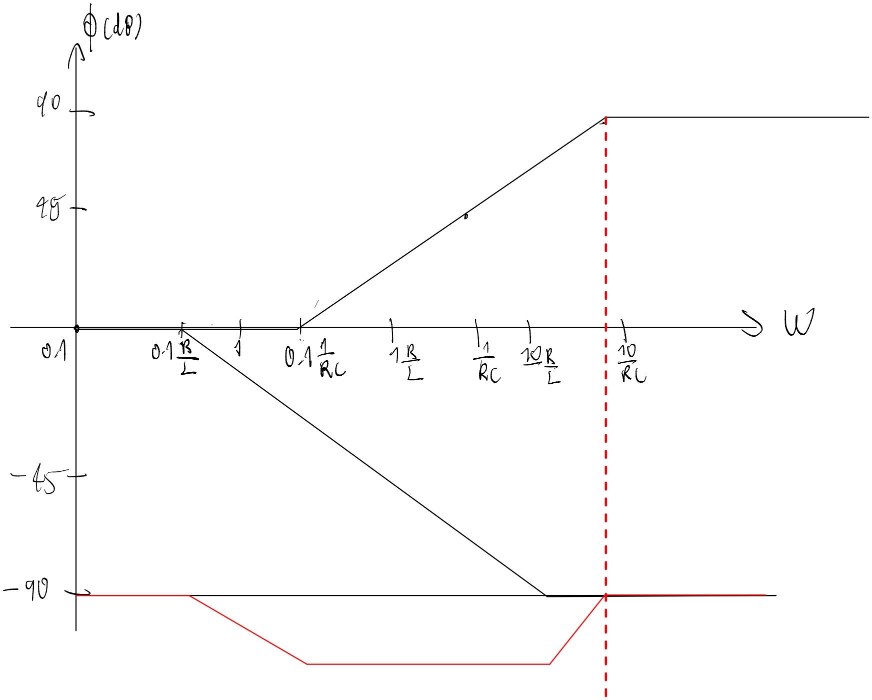
90 $\quad w \geq \frac{10}{RC}$

for type 4 zeros $\quad z_1 = \frac{R}{L}$

$$H(w) \to \begin{cases} 0 & w \leq \frac{R}{L} \\ -20 & w \geq \frac{R}{L} \end{cases}$$

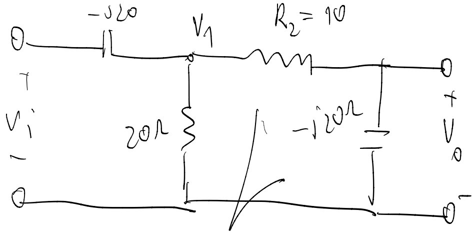
$$\phi \to \begin{cases} 0 & w \leq 0.1\frac{R}{L} \\ -45 & 0.1\frac{R}{L} \leq w \leq 10\frac{R}{L} \\ -90 & w \geq \frac{R}{L} \end{cases}$$






$$V_L = 100 \angle 45 \left( \frac{j1}{1+3j} \right)$$

$$= \quad V_L = 50.596 \angle 63.43°$$




$$Z = 20 // 10 - 20j = \frac{140}{13} - \frac{80}{13}j$$

$$V_1 = V_i\left(\frac{\left(\frac{140}{13} - \frac{80}{13}j\right)}{\left(\frac{140}{13} - \frac{80}{13}j\right) - 20j}\right)$$

$$V_i = \frac{\left(\frac{140}{13} - \frac{340}{13}j\right)}{\left(\frac{140}{13} - \frac{80}{13}j\right)} V_1 \quad //$$

$$V_0 = V_1\left(\frac{-j20}{10 - 20j}\right) \Rightarrow \left(\frac{4}{5} - \frac{2}{5}j\right)V_1$$

$$V_0 = \left(\frac{2\sqrt{5}}{5} \angle -26.565\right)V_1$$

$V_i =$

$|S| = 6$ ; $P_f = 0.750 \rightarrow 41.4096$

$$P_f = \frac{P}{|S|} \rightarrow 0.750(6) = 4.5 \text{ kW}$$

$S_1 = 4.5 +$

$$C = \frac{4500(\tan(41.4096) - \tan(11.4783))}{(2\pi \times 60)(240)^2}$$

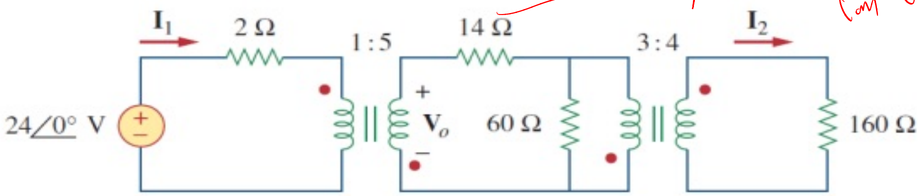
$P_{f_2} = 0.980 \rightarrow 11.4783$

leading ลดลง หาใช้ Capacitance

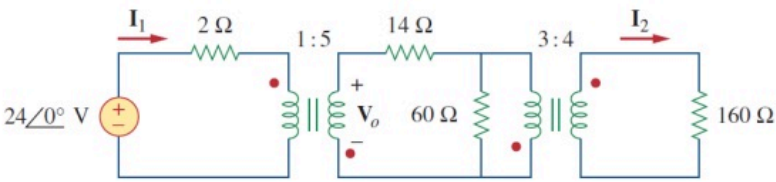
Feed into transformer ต่อไป

Can use reflection

3.[8%] For the circuit below, find $\mathbf{I_1}$, $\mathbf{I_2}$ and $\mathbf{V_o}$.



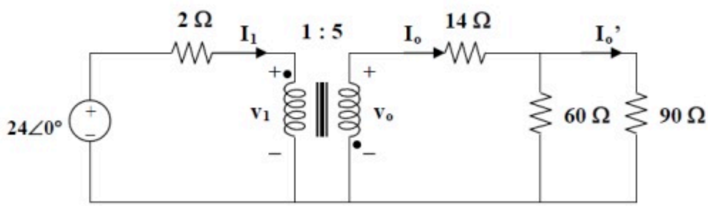

3.[8%] For the circuit below, find $\mathbf{I_1}$, $\mathbf{I_2}$ and $\mathbf{V_o}$.




**Solution:**

We reflect the 160-ohm load to the middle circuit.

$$Z_R = Z_L/n^2 = 160/(4/3)^2 = 90 \text{ ohms, where } n = 4/3$$




$$14 + 60||90 = 14 + 36 = 50 \text{ ohms}$$

We reflect this to the primary side.

$$Z_R' = Z_L'/(n')^2 = 50/5^2 = 2 \text{ ohms when } n' = 5$$

$$I_1 = 24/(2 + 2) = \mathbf{6A}$$

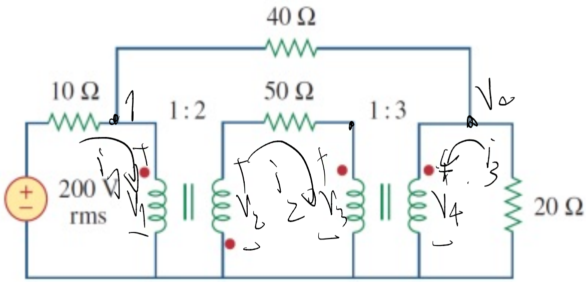
$$24 = 2I_1 + v_1 \text{ or } v_1 = 24 - 2I_1 = 12 \text{ V}$$

$$v_o = -nv_1 = \mathbf{-60 \text{ V}}, \ I_o = -I_1/n_1 = -6/5 = -1.2$$

$$I_o' = [60/(60 + 90)]I_o = -0.48A$$

$$I_2 = -I_o'/n = 0.48/(4/3) = \mathbf{360 \text{ mA}}$$

4.[10%] Calculate the average power dissipated by the 20Ω resistor in the circuit below.




$$P = \frac{V_{rms}^2}{R}$$

$$\boxed{P = \frac{V_4^2}{20}}$$

KCL at Node 1-

$$\frac{V_1 - 200}{10} + I_1 + \frac{V_1 - V_4}{40} = 0$$

$$\frac{200 - V_1}{10} = \frac{V_1 - V_4}{40} + I_1 \rightarrow (1)$$

$$200 = 1.25 V_1 - 0.25 V_4 + 10 I_1 \rightarrow (1)$$

KCL at Node 2.

$$\frac{V_4 - V_1}{40} + I_3 + \frac{V_4}{20} = 0$$

$$\frac{V_1 - V_4}{40} = \frac{V_4}{20} + I_3$$

$$V_1 = 3V_4 + 40 I_3 \rightarrow (2)$$

also at the terminal $\rightarrow \frac{V_2}{V_1} = -2 \rightarrow V_2 = -2V_1$

$$\rightarrow \frac{I_1}{I_2} = -2 \rightarrow I_1 = -2I_2$$

KVL at the main loop. $\rightarrow$ $50 i_2 + V_3 - V_2 = 0$

$\checkmark$

$V_3 = V_2 - 50 I_2 \rightarrow (3)$

At the Second Terminal. $\rightarrow$ $\frac{V_4}{V_3} = 3 \rightarrow V_3 = \frac{1}{3} V_4$

$\frac{I_2}{I_3} = -3$

$I_2 = -3 I_3$

$\frac{1}{3} V_4 = V_2 - 50 I_2$

$V_4 = 3V_2 - 150(-3I_3)$

$\downarrow$

$V_2 = -2V_1$

We have seven equations and seven unknowns. Combining (1) and (2) leads to

$$200 = 3.5V_4 + 10I_1 + 50I_3$$

But from (4) and (7), $I_1 = -2I_2 = -2(-3I_3) = 6I_3$. Hence

$$200 = 3.5V_4 + 110I_3 \qquad (8)$$

$V_4 = -6V_1 + 450 I_3$

From (5), (6), (3), and (7),

$$V_4 = 3(V_2 - 50I_2) = 3V_2 - 150I_2 = -6V_1 + 450I_3$$

Substituting for $V_1$ in (2) gives

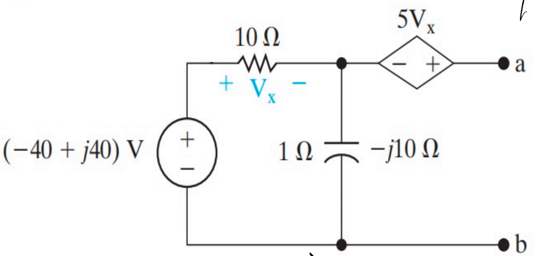
$$V_4 = -6(3V_4 + 40I_3) + 450I_3 \longrightarrow I_3 = \frac{19}{210}V_4 \qquad (9)$$

Substituting (9) into (8) yields

$$\boxed{200 = 13.452V_4 \longrightarrow V_4 = 14.87 \qquad P = \frac{V^2_4}{20} = \underline{11.05\text{ W}}}$$

**9.48** Find the Norton equivalent with respect to terminals a,b in the circuit of Fig. P9.48.

**Figure P9.48**



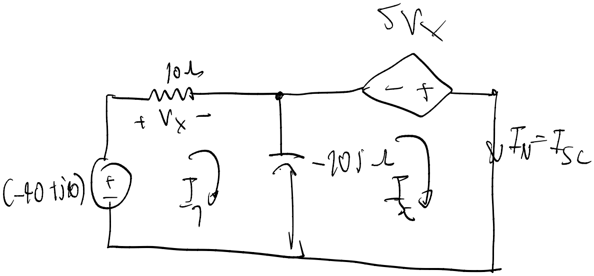

find $I_{sc} = I_N$

$Z_N = Z_{TH}$

$Z_{TH} = \frac{V_{TH}}{I_{SC}}$

$I = \frac{-5V_x - (-V_{oc})}{-j10}$

$-j10I = -5V_x + V_{oc}$




$V_x - 10j(i_1 - i_{sc}) = -40 + j40 \rightarrow (1)$

$V_x - 10j\,i_1 + 10j\,i_{sc} = -40 + 40j$

KVL at Mesh 2

$-5V_x = 10j(I_{sc} - I_1)$

$V_x = -2j(I_{sc} - I_1) \rightarrow (2)$

also $V_x = 10i_1$

Solve give $I_N = I_{sc} = 6 + j4$ A

Sketch bode Plot. $\rightarrow$ $G(s) \rightarrow \dfrac{s}{(s+2)^2(s+1)}$

$\rightarrow$ Type 2 $\rightarrow$ Zeros

$$\frac{jw}{(jw+2)^2(jw+1)}$$

$\downarrow 2$

$$\left(1+\frac{jw}{2}\right)\left(1+\frac{jw}{1}\right)$$

$$= \frac{\frac{1}{\cancel{4}}(jw)}{\left(1+\frac{jw}{2}\right)^2(jw+1)}$$