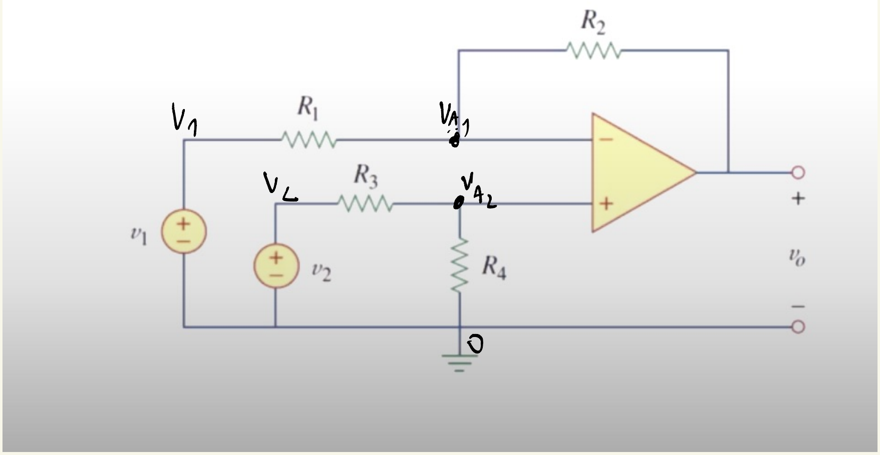
Q1. For the difference amplifier circuit, please derive the equation below. (8 points)

$$v_o = \left(\frac{R_2}{R_1} + 1\right)\frac{R_4}{R_3 + R_4} v_2 - \frac{R_2}{R_1} v_1$$



At node V_{A_1} $\rightarrow$ $\frac{V_{A_1} - V_1}{R_1} + \frac{V_{A_1} - V_o}{R_2} = 0$

$V_{A_1} = V_{A_2}$

At node V_{A_2} $\rightarrow$ $\frac{V_{A_2} - V_2}{R_3} + \frac{V_{A_2}}{R_4} = 0$

$$\frac{V_A}{R_3} - \frac{V_2}{R_3} + \frac{V_A}{R_4} = 0$$

$$V_A \left(\frac{R_4 V_A + R_3 V_A}{R_3 R_4}\right) = \frac{+V_2}{R_3}$$

$$V_A = \frac{\cancel{R_3} R_4}{R_4 + R_3}\left(\frac{+V_2}{\cancel{R_3}}\right)$$

$$V_A = \frac{+V_2 R_4}{R_4 + R_3}$$

$$\frac{\left(\frac{V_2 R_4}{R_4+R_3}\right)-V_1}{R_1}+\frac{\left(\frac{V_2 R_4}{R_4+R_3}\right)-V_0}{R_2}=0$$

$$= \frac{V_2 R_4}{R_1(R_4+R_3)}-\frac{V_1}{R_1}+\frac{V_2 R_4}{R_2(R_4+R_3)}-\frac{V_0}{R_2}=0$$

$$\frac{V_0}{R_2}=\left(\frac{R_2 R_4 V_2+R_1 R_4 V_2}{R_1 R_2 (R_4+R_3)}\right)-\frac{V_1}{R_1}$$

$$V_0=\left(\frac{R_2}{R_1}+1\right)\frac{R_4 V_2}{R_3+R_4}-\frac{V_1}{R_1}R_2$$

Q3. The circuit model of a dc voltage source is shown below. The following voltage measurements are made at the terminals of the source: (1) With the terminals of the source open, the voltage is measured at 50 mV, and (2) with a 15 MΩ resistor connected to the terminals, the voltage is measured at 48.75 mV. All measurements are made with a digital voltmeter that has a meter resistance of 10 MΩ. (10 points)

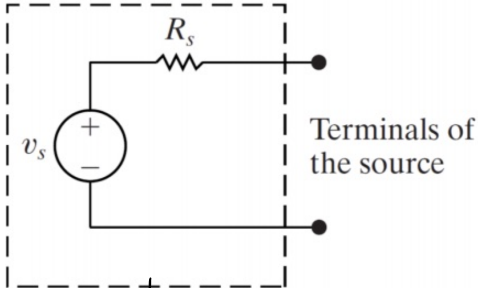
R_L

V_L

(a) What is the internal voltage of the source v_s in millivolts?

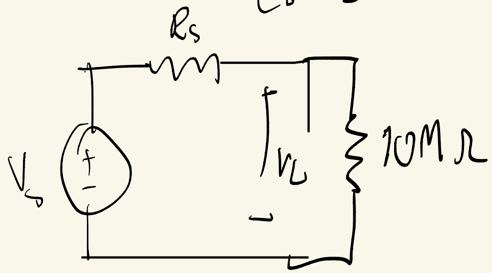
(b) What is the internal resistance of the source R_s in kilo-ohms?

$V_s = 50mV$



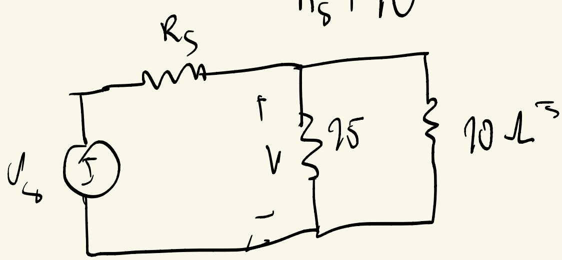
(a) when a practical voltage source is connected to the load then → $V_L = \left(\frac{R_L}{R_s + R_L}\right) V_s$

1st case



$$50mV = \frac{10V_s}{R_s + 10} \rightarrow (2)$$

2nd case



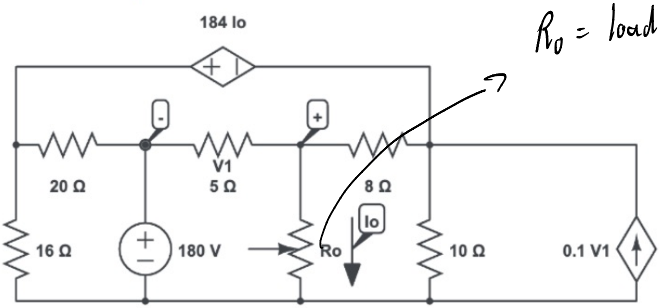
$$48.75 = \frac{6V_s}{R_s + 6}$$

Q4. The rheostat R_o in the circuit below has been adjusted so that the maximum power is delivered to R_o. (18 points)

(a) Find the value of R_o.

(b) Find the maximum power delivered to R_o.

R_o = load resistance ; as it consume power output from the circuit

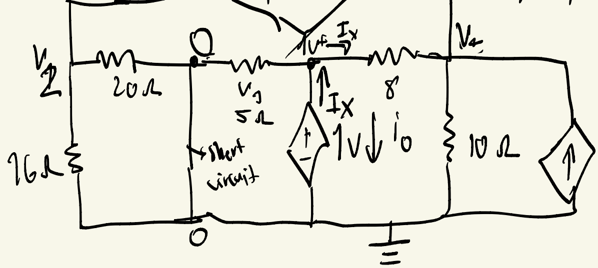


Max power deliver to the load when $R_o = R_{TH}$

(a) Find R_{TH} $\longrightarrow$ Close all indepndent source, and open circuit load resistance which is R_o

$0 - 1 = V_1$

$V_1 = -1$



$R_{TH} = \dfrac{V}{I_x}$

Node Analysis at Supermesh V_2 / V_4 $\longrightarrow$

$$\frac{V_2}{16} + \frac{V_2 - 0}{20} + \frac{V_4 - 1}{8} + \frac{V_4}{10} - 0.1V_1 = 0 \rightarrow (1)$$

also $V_2 - V_4 = 184\, I_o \rightarrow (2)$; also $V_1 = 1 - 0$

Node Analysis at 1 $\rightarrow$ $\dfrac{1-0}{5} - I_x + \dfrac{1 - V_4}{8} = 0 \rightarrow (3)$

$$I_x = \frac{1}{5} + \frac{1 - V_4}{8} \rightarrow (4)$$

also $I_x = -I_o \Rightarrow I_o' = -I_x$
$\rightarrow (5)$

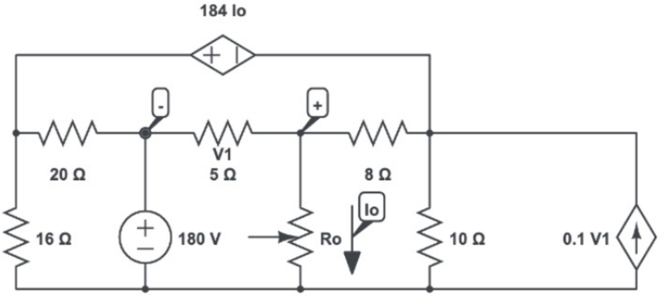
Solve for $I_x \rightarrow$ get R_{TH} #

$R_{TH} \Rightarrow 27.45\ \Omega$ #

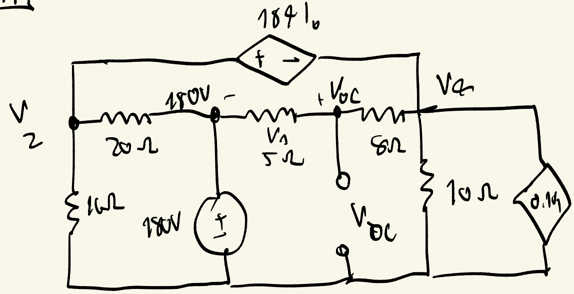
(6.) Q4. The rheostat R_o in the circuit below has been adjusted so that the maximum power is delivered to R_o. (18 points)

(a) Find the value of R_o.

(b) Find the maximum power delivered to R_o.



find V_{TH} or V_{oc}



Make sure to write down equation

$$\frac{V_2}{16} + \frac{V_2 - 180}{20} + \frac{V_4 - V_{oc}}{8} + \frac{V_4}{10} - 0.1V_1 =$$

$$V_1 = V_{oc} - 180$$

Q5. Inductors are initially charged and are connected to the black box at t = 0. If $i_1(0) = 6$ A, $i_2(0) = -3$ A, and $v(t) = 50e^{-100t}$ mV when $t \geq 0$, please answer (a)-(d). (16 points)

(a) The energy initially stored in each inductor.

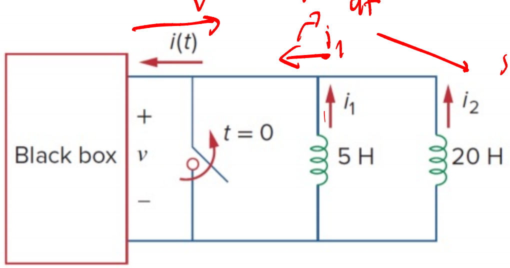
(b) The total energy delivered to the black box from t = 0 to t = ∞.

(c) $i_1(t)$ and $i_2(t)$ when $t \geq 0$.

(d) $i(t)$, $t \geq 0$.

$V = L\frac{di}{dt}$

Volts from the capacitor is in the opposite ways of the current i_1 hence Volts opposite to intended i_1 direction



so $-\frac{V}{L} = \frac{di}{dt}$

$-\frac{V}{L}\,dt = di$

$i_1(t) = -\frac{1}{L}\int_0^t v\,dt$

(a.) for each inductor → $W_{\text{inductor of 5 H}} \rightarrow \frac{1}{2}Li^2 = \frac{1}{2}5(6)^2$

$= \frac{1}{2} \times 5 \times 36$

$= 90\,J$

$W_{\text{inductor of 20H}} \rightarrow = \frac{1}{2}Li^2 = \frac{1}{\cancel{2}}(\cancel{20}^{10})(-3)^2$

$= 90\,J$

$\frac{5 \times 20}{5+20} = \frac{100}{25} = 4$

(b.) $W = W_1 + W_2$ → All energy will be transfer to the black box

$\Rightarrow W = W_1 + W_2 = 90 + 90$

$= 180\,J$

(c.) $V = L\frac{di_1}{dt}$ = $Vdt = L\,di$ $v(t) = \text{negative}$

Inductor → supplying power → so its needed to be negative for $v(t)$ as $P = v(t)$

for inductor = 5 or $i_1(t)$

$\frac{V}{L}dt = di$

$i(t) = \frac{1}{L}\int_0^t v\,dt$

$i_1(0) = 6$

$$= \frac{1}{5}\int_0^t 50e^{-100t}\,dt + i(0)$$

$$= \cancel{\frac{1}{5}(50)}^{10}\left(-\frac{1}{100}\right)\left[e^{-100t} - e^{-100(0)}\right]$$

$$= +\frac{1}{10}\left[e^{-100t} - 1\right]$$

$v(t) = mV$

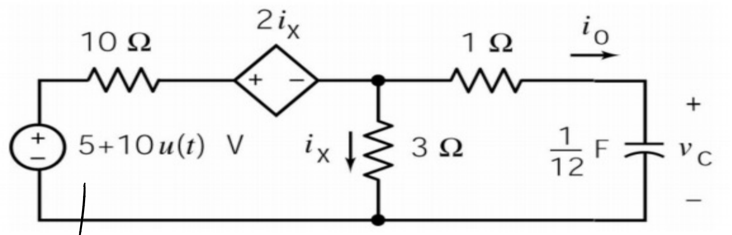
$V = -L\frac{di_1}{dt}$

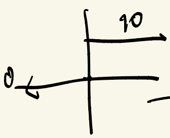
$$i_1(t) = 1\times10^{-4}(e^{-100t} - 1) + 6$$

(d.) = $i(t) = i_1(t) + i_2(t)$

$i_2(t)$ solve the same way as $i_1(t)$

Q6. Please determine a capacitor current i_0 when $t > 0$. (18 points)





→ When $t>0 = 5+10 = 15$

when $t<0 = 5$

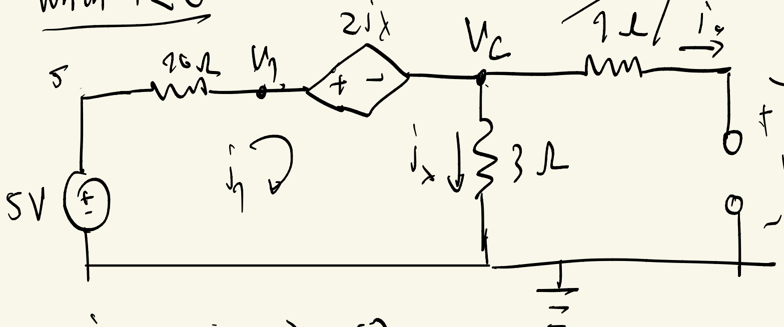
RC circuit with step input

find i_c

when $t<0$

$= 2(\frac{1}{3}) = \frac{2}{3}$

no current flow through



Open circuit.

$i_0(0^+) = i_0(0^-) = 0$

also $i_x = i_1$

KVL at i_1 $\quad 10i_1 + 2i_x + 3i_1 - 5 = 0$

$10i_x + 2i_x + 3i_x - 5 = 0$

$15i_x = 5$

$$i_x = \frac{1}{3}\,A \rightarrow (1)$$

KCL at V_2 supernode V_2 and V_1

$$\frac{V_1 - 5}{10} + \frac{V_C}{3} = 0 \rightarrow (2)$$

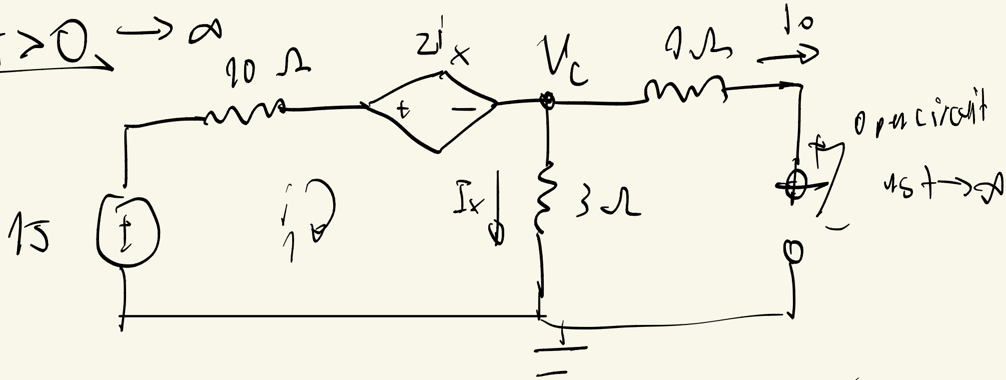
also $V_1 - V_C = 2i_x = V_1 - V_C = \frac{2}{3} \rightarrow (3)$

also $i_x = \frac{V_C - 0}{3} = \frac{1}{3}(3) = V_C$

$$V_C = 1$$

$$V_C(0^+) = V_C(0^-) = \boxed{V_0 = 1\ V}$$

When $t > 0 \rightarrow \infty$



same KVL i_1 also $i_1 = I_x$

$$10 i_1 + 2 i_x + 3 i_1 - 15 = 0$$

$$10 i_x + 2 i_x + 3 i_x - 15 = 0$$

$$15 i_x = 15$$

$$i_x = 1 . A$$

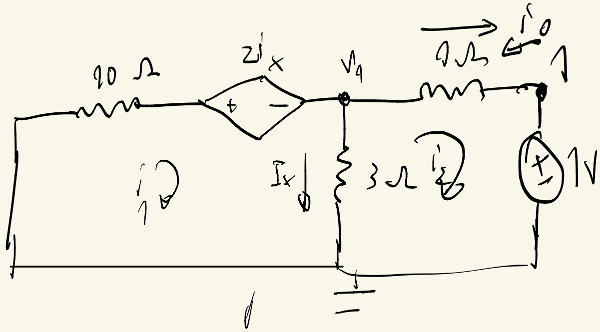
also $I_x = \frac{V_c - 0}{3}$

$$1 \times 3 = V_C$$

$$3 = V_C(\infty)$$

Response for R_C Circuit with step input $\rightarrow v(\infty) + [v(0^+) - V(\infty)] e^{-\frac{t}{\tau}}$

$\tau \Rightarrow RC \longrightarrow$



Mesh analysis →

$$10i_1 + 2i_x + 3(i_1 - i_2) = 0$$

$$I_x = i_1$$

$$10I_x + 2I_x + 3I_x - 3i_2 = 0$$

$$13I_x - 3i_2 = 0 \rightarrow (1)$$

from mesh 2 → $$1i_2 + 1V + 3(i_2 - I_x) = 0$$

also $i_2 = -i_0$

$$4i_2 - 3I_x = -1V \rightarrow (2)$$

$$i_x = i_0 = \frac{13}{43}$$

$$\tau = RC \Rightarrow \frac{13}{43}\left(\frac{1}{12}\right) = \frac{13}{516}$$

$$= 0.025$$

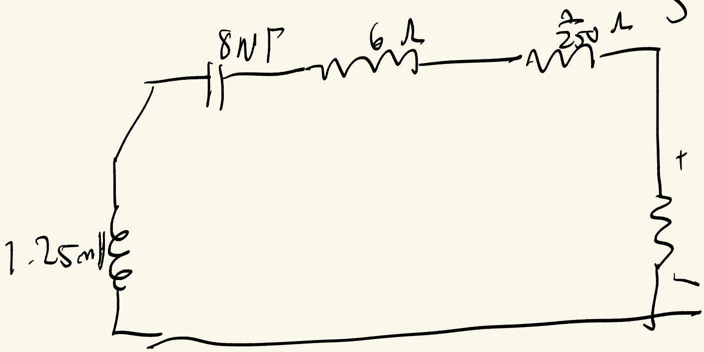
$$v(t) = 3 + [1 - 3]e^{-\frac{t}{\tau}}$$

$$= 3 - 2e^{-4t}$$

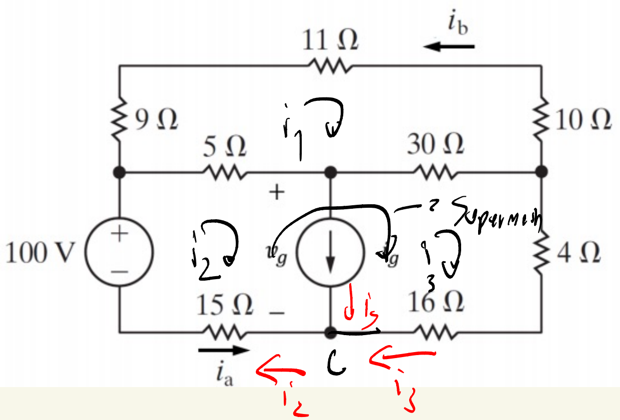
$$i(t) = c\frac{dv}{dt} = \frac{1}{12}\left[8e^{-4t}\right]$$

$$= \frac{2}{3}e^{-4t}$$

when $t<0 \Rightarrow V(0^+) = V(0^-) = \frac{2}{5}$



1. The currents i_a and i_b in the circuit are 4 A and -2 A respectively. Find i_g and v_g. [8 points]



Supermesh between i_2 / i_3.

From KCL at C

$$-i_g - i_3 + i_2 =$$

$$i_g = i_2 - i_3 \quad \rightarrow (1)$$

Supermesh $\rightarrow$

$$i_2 = i_g \; ; \; i_a = 4 \; ; \; i_b = -2$$

$$5(i_2 - i_1) + 30(i_3 - i_1) + 4i_3 + 16i_3 + 15i_2 - 100 = 0$$

$$\cancel{5}i_2 - \cancel{5}i_1 + 30i_3 - \cancel{30}i_1 + 4i_3 + 16i_3 + \cancel{15}i_2 = 100$$

$$-35i_1 + 20i_2 + 50i_3 = 100 \rightarrow (2)$$

Mesh 1

$$11i_1 + 10i_1 + 30(i_1 - i_3) + 5(i_1 - i_2) + 9i_1 = 0 \rightarrow (3)$$

$$i_1 = -i_b = -(-2) = 2$$

sub i_1 to (3)

$$22 + 20 + 60 - 30 i_3 + 10 - 5 i_2 + 28 = 0$$

$$-30 i_3 - 5 i_2 = -130 \rightarrow (4)$$

$$-35 i_1 + 20 i_2 + 50 i_3 = 100 \rightarrow (2)$$

$$\downarrow$$

$$i_1 = 2 \rightarrow 20 i_2 + 50 i_3 = 170 \rightarrow (5)$$

Combine / solve (4) (5)

$$\Rightarrow \quad i_2 = -4 \ ; \ i_3 = 5$$

$$i_g = i_2 = -4$$

$$i_g = i_2 - i_3 = -4 - 5 = -9 \text{ A}$$

\#

From mesh i_2

$$5(i_2 - i_1) + V_g + 15i_2 - 100 = 0$$

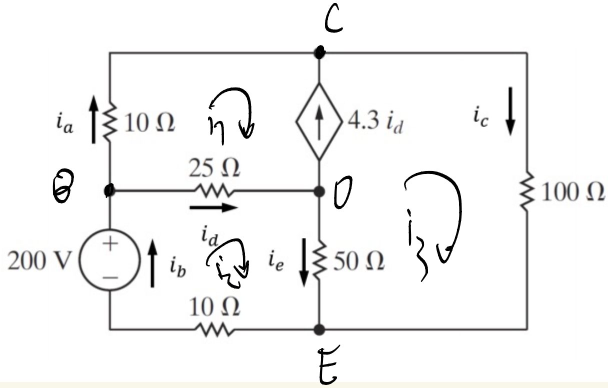
$$5i_2 - 5i_1 + V_g + 15i_2 - 100 = 0$$

$$-20 - 10 + V_g - 60 - 100 = 0$$

$$V_g = 190 \text{ V}$$

2. (a) Use the mesh-current method to find the branch currents in $i_a \sim i_e$ in the circuit below. [8 points]

(b) Check your solution by showing that the total power generated in the circuit equals the total power consumed. [8 points]



Mesh i_1/i_3 → Super mesh

also $i_1 = i_a$ $\qquad i_c = i_3$

KCl at C $\quad -i_1 - 4.3 i_d + i_c = 0$

$$-i_a - 4.3 i_d + i_c = 0$$

$$i_c = i_a + 4.3 i_d \rightarrow (1)$$

for supermesh i_1/i_3 $\qquad$ or $i_3 = i_1 + 4.3 i_d$

$$100 i_3 + 50(i_3 - i_2) + 25(i_1 - i_2) + 10 i_1 = 0$$

$$100 i_3 + 50 i_3 - 50 i_2 + 25 i_1 - 25 i_2 + 10 i_1 = 0$$

$$35 i_1 - 75 i_2 + 150 i_3 = 0 \rightarrow (2)$$

Mesh i_2 $\quad 25(i_2-i_1)+50(i_2-i_3)+10i_2-200=0$

$$25i_2-25i_1+50i_2-50i_3+10i_2-200=0$$

$$-25i_1+85i_2-50i_3=200 \rightarrow (3)$$

also KCL at B $\quad i_b=i_a+i_d \iff i_b=i_1+i_d \rightarrow (4)$

KCL at D $\quad -i_d+4.3i_d+i_e=0 \rightarrow$

$$3.3i_d=-i_e$$

$$i_d=\frac{-10}{33}i_e \rightarrow (5)$$

//

KCL at A $\quad -i_e-i_3+i_2=0$

$$i_2=i_e+i_3$$

$$\rightarrow i_e=-3.3i_d$$

$$i_2=-3.3i_d+i_3$$

$$i_d=\frac{i_3-i_2}{3.3} \rightarrow (6)$$

Sub (6) to (1) $i_3 = i_1 + 4.3\left(\frac{i_3 - i_2}{3.3}\right)$

$$= i_3 = i_1 + \frac{43}{33} i_3 - \frac{43}{33} i_2$$

$$0 = i_1 - \frac{43}{33} i_2 + \frac{10}{33} i_3 \rightarrow (7)$$

Solve (2) (3) (7) give

$$i_1 = 5.7A ; i_2 = 4.6 \ A ; i_3 = 0.97A$$

Hence $i_a = 5.7A$; $i_c \Rightarrow 0.97A$

also $i_c = i_a + 4.3 i_d$

$$\frac{i_c - i_a}{4.3} = i_d \rightarrow i_d = -1.1 A. \rightarrow i_d$$

also $i_e = -3.3 i_d \Rightarrow 3.63 A$

$$i_b \Rightarrow i_a + i_d = 5.7 - 1.1$$

$i_6 \Rightarrow 4.6\,A$

6.) Kirchoff power law.

total power generated = total power consumed

Power dissipated

by the independent source

Voltage Source / Current Source

by All resistors

check Power dissipated

Power dissipated $\rightarrow \sum P_{dis} \Rightarrow \sum I^2 R_L$

=

Come all current with their corresponding Resistance R_L. from independent source

Power generated $\rightarrow$ Supply $= \sum = P_{voltage} + P_{current}$

$P_{voltage} = V i_6 = -(200)\,4.6$

$= -920\,W$

(negative as its supply

$$P_{current} = V_{current}(4.3 I_d)$$

$$V_{current\ from\ loop\ i_1} = 84.5(4.3(-1.1))$$

$$= -399.685\ W$$

$$P_{total} = -1319.685 \Leftarrow P_{Dis}$$

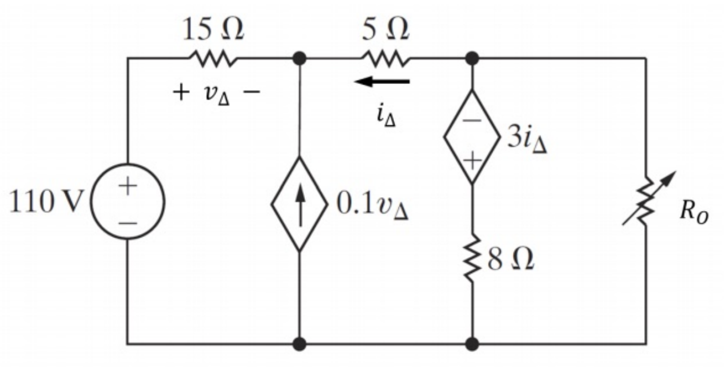
negative as its must

supply power

3. The variable resistor (R_O) in the circuit is adjusted until it absorbs maximum power from the circuit.

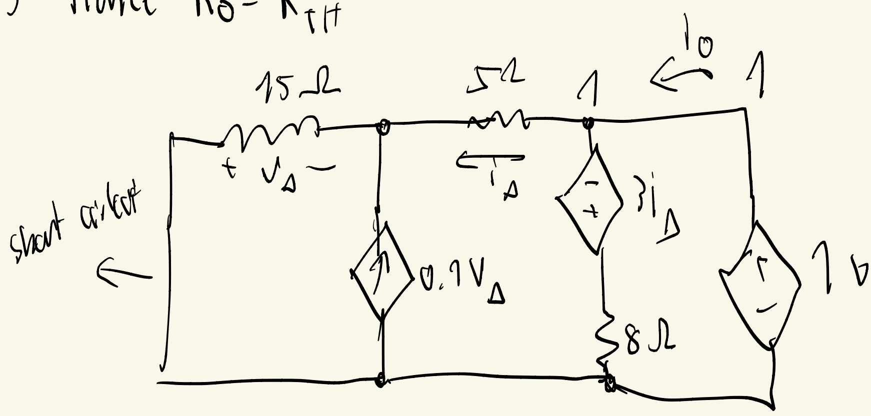
(a) Find the value of R_O. [12 points]

(b) Find the maximum power delivered to R_O. [4 points]

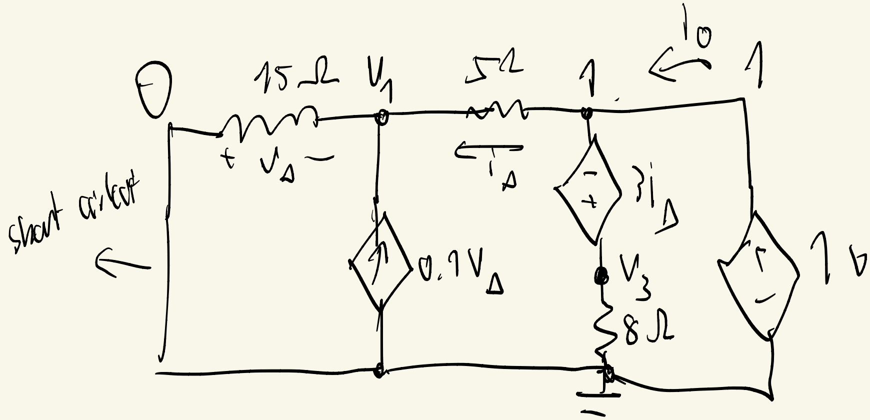


Max power transfer to the load when $R_{TH} = R_o$

(a.) Hence $R_o = R_{TH}$



$R_{TH} = \frac{V}{i_o} \rightarrow$ Apply $1V = \frac{1}{i_o}$



$$R_{TH} = \frac{V}{i_0} \rightarrow \text{Apply } 1V = \frac{1}{i_0}$$

find i_0

At node V_1

$$15\left(\frac{V_1-0}{15} - 0.1V_\Delta + \frac{V_1-1}{5} = 0\right)$$

$$V_1 - 1.5V_\Delta + 3V_1 - 3 = 0$$

$$4V_1 - 3 - 1.5V_\Delta = 0 \rightarrow (1)$$

also $\quad 0 - V_1 = V_\Delta$

$$4V_1 - 3 + 1.5V_1 = 0$$

$$5.5V_1 - 3 = 0 \rightarrow (1)$$

$$V_1 = \frac{3}{5.5}\ V$$ ##

also $i_\Delta = \frac{1 - V_1}{5} = \frac{1 - \frac{3}{5.5}}{5} = \frac{1}{11}$

<u>Supernode V_2 / V_3</u>

$$\frac{1 - V_1}{5} + \frac{V_3 - 0}{8} - i_o = 0 \quad \rightarrow (2)$$

also $V_3 - 1 = 3 i_\Delta$

$$V_3 = 3\left(\frac{1}{11}\right) + 1 = \frac{14}{11}\ V$$

<u>solve (2)</u>

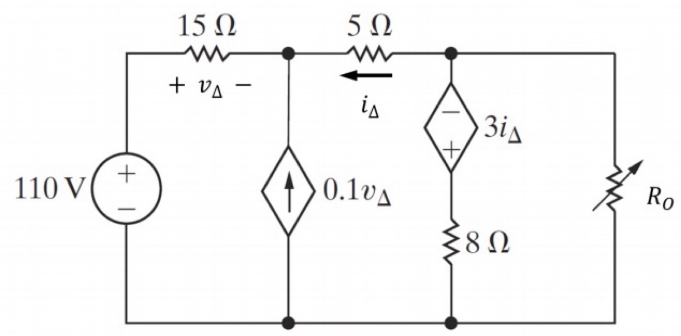
$$\frac{1}{11} + \frac{\frac{14}{11}}{8} = i_o$$

$$i_o = \frac{1}{4}$$

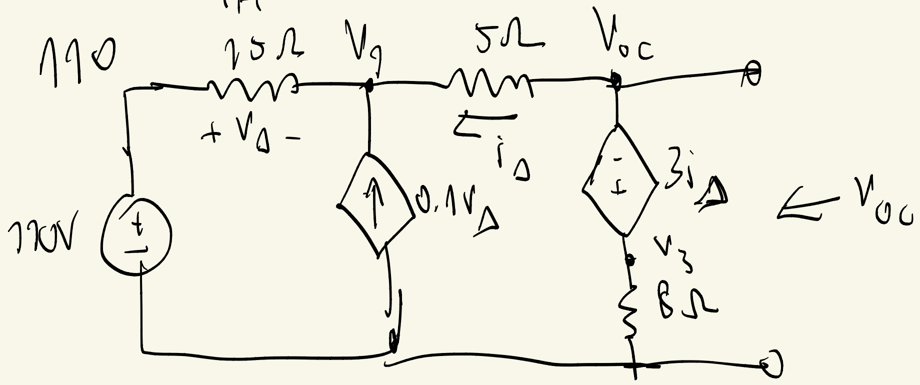
Hence $\boxed{R_{TH} = \frac{1}{i_o} = \frac{1}{\frac{1}{4}} = 4\,\Omega} \Rightarrow = R_o$

6.)

(b) Find the maximum power delivered to R_O. [4 points]



$$P_{max} = \frac{V_{TH}^2}{4R_{TH}} \Rightarrow \text{Need } V_{TH} = V_{oc}$$



Supernode V_{oc} and V_3

$$40\left(\frac{V_{oc}-V_1}{5} + \frac{V_3-0}{8} = 0\right)$$

$$8V_{oc} - 8V_1 + 5V_3 = 0 \rightarrow (1)$$

also $\rightarrow$ $i_\Delta = \dfrac{V_{oc} - V_1}{5\Omega}$ $\quad i_\Delta = \dfrac{V_{oc} - 110 + V_\Delta}{5\Omega}$

also $\rightarrow$ $V_3 - V_{oc} = 3 i_\Delta$

KCL at Node V_1 $\left(\dfrac{V_1 - 110}{15} + \dfrac{V_1 - V_{oc}}{5} - 0.1 V_\Delta = 0\right) 15$

$V_1 - 110 + 3V_1 - 3V_{oc} - 1.5 V_\Delta = 0$

$4V_1 - 3V_{oc} - 1.5V_\Delta = 110 \rightarrow (2)$

also $110 - V_1 = V_\Delta$

$V_1 = 110 - V_\Delta$

sub to (1)/2

$8V_{oc} - 8(110 - V_\Delta) + 5V_3 = 0 \rightarrow (3)$

$4(110 - V_\Delta) - 3V_{oc} - 1.5V_\Delta = 110 \rightarrow (4)$

$$8V_{oc} - 880 + 8V_\Delta + 5V_2 = 0 \qquad V_3 = V_{oc} + 3i_\Delta$$

$$8V_{oc} + 8V_\Delta + 5V_3 = 880 \qquad = V_{oc} + 3\left(\frac{V_{oc} - 110 + V_\Delta}{5}\right)$$

(5)

$$-3V_{oc} - 5.5V_\Delta = -330 \quad =$$

(6)

$$= V_{oc} + \frac{3}{5}V_{oc} - \frac{330}{5} + \frac{3V_\Delta}{5}$$

from $(5) \rightarrow 8V_{oc} + 8V_\Delta + 5\left(V_{oc} + \frac{3}{5}V_{oc} - \frac{330}{5} + \frac{3V_\Delta}{5}\right) = 880$

$$\cancel{8}V_{oc} + 8V_\Delta + \cancel{5V_{oc}} + \cancel{3V_{oc}} - 330 + 3V_\Delta = 880$$

$$16V_{oc} + 11V_\Delta = 1210 \rightarrow (7)$$

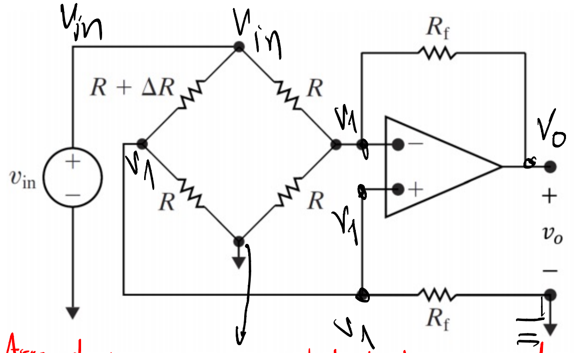
$$V_{oc} = 55\,V \qquad V_\Delta = 30$$

$$P_{max} = \frac{(V_{TH})^2}{4R_{TH}} = \frac{(55)^2}{4(4)} = 189.0625\,W$$

4. Show that if $\Delta R \ll R$, the output voltage of the op amp is approximately

$$v_o \approx \frac{R_f(R+R_f)}{R^2(R+2R_f)}(-\Delta R)v_{in}$$

for the circuit below. [20 points]



Arrow down mean ; Connected to the ground.

<u>At node V_1 for inverting amplifier.</u>

$$\frac{V_1 - V_{in}}{R} + \frac{V_1}{R} + \frac{V_1 - V_o}{R_f} = 0$$

$$\frac{V_1}{R} + \frac{V_1}{R} + \frac{V_1}{R_f} = \frac{V_{in}}{R} + \frac{V_o}{R_f}$$

$$= V_1\left(\frac{2R_f + R}{RR_f}\right) = \frac{V_{in}}{R} + \frac{V_o}{R_f} \rightarrow (1)$$

<u>KCL at node V_1 for non inverting amplifier.</u>

$$\frac{V_1 - 0}{R_f} + \frac{V_1}{R} + \frac{V_1 - V_{in}}{R + \Delta R} = 0 \rightarrow (2)$$

also

<u>From (2)</u> $\rightarrow$

$$\left(\frac{V_1}{R_f} + \frac{V_1}{R} + \frac{V_1}{R+\Delta R} - \frac{V_{in}}{R+\Delta R} = 0\right)(R_f)(R)(R+\Delta R)$$

$$= V_1(R)(R+\Delta R) + V_1(R_f)(R+\Delta R) + V_1(R_f)(R) - V_{in}(R_f)(R) = 0$$

$$V_1(R^2+R\Delta R) + V_1(R_fR + R_f\Delta R) + V_1 R_f R = V_{in}(R_f)(R)$$

$$= V_1(R^2+R\Delta R+2R_fR+R_f\Delta R) = V_{in}(R_f)R$$

$$V_1 = \frac{V_{in}(R_f)R}{R^2+R\Delta R+2R_fR+R_f\Delta R}$$

$\downarrow$

Plug to (1)

$$\frac{V_{in}\cancel{(R_f)}\cancel{(R)}}{R^2+R\Delta R+2R_fR+R_f\Delta R}\left(\frac{2R_f+R}{\cancel{RR_f}}\right) \Rightarrow \frac{V_{in}}{R}+\frac{V_o}{R_f}$$

$$\frac{V_{in}(2R_f+R)}{R^2+R\Delta R+2R_fR+R_f\Delta R} \Rightarrow \frac{V_{in}}{R}+\frac{V_o}{R_f}$$

$$\frac{V_o}{R_f} = \frac{V_{in}(2R_f+R)}{R^2+R\Delta R+2R_fR+R_f\Delta R} - \frac{V_{in}}{R}$$

$\downarrow$

$$\frac{V_o}{R_f} \Rightarrow \frac{V_{in}\,(\cancel{2R_f + R})R - V_{in}\,(\cancel{R^2} + R\Delta R + \cancel{2R_f R} + R_f \Delta R)}{R(R^2 + R\Delta R + 2R_f R + R_f \Delta R)}$$

$$\frac{\cancel{2R_f R V_{in} + R^2 V_{in} - V_{in} R^2} - V_{in} R\Delta R - \cancel{V_{in} 2R_f R} - V_{in} R_f \Delta R}{R(R^2 + R\Delta R + 2R_f R + R_f \Delta R)}$$

$$V_o \Rightarrow \frac{-(\Delta R)(R + R_f)\,R_f\,V_{in}}{R(R^2 + 2RR_f + \cancel{R\Delta R + R_f \Delta R})}$$

$R + R_f >> \Delta R$ so fast such that two terminal are eliminated,

$$\text{Hence } V_o \Rightarrow \frac{(-\Delta R)\,R_f\,(R + R_f)\,V_{in}}{R^2(R + 2R_f)}$$

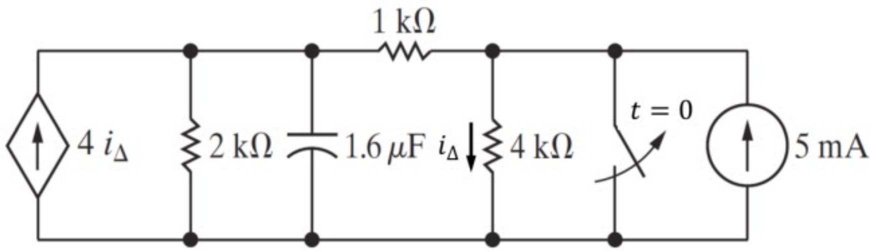
##

5. The switch in the circuit below has been closed for a long time. The maximum voltage rating of the 1.6 μF capacitor is 14.4 kV. How long after the switch is opened does the voltage across the capacitor reach the maximum voltage rating? Please follow steps below.

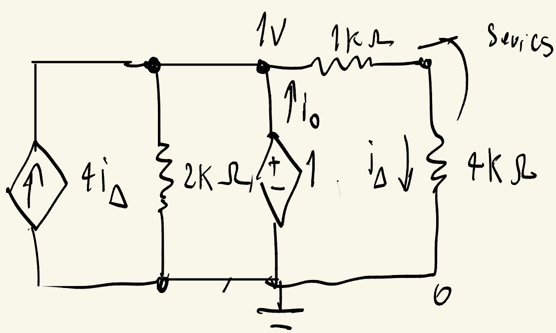
(a) Find Thevenin equivalent circuit seen from the capacitor. [10 points]

(b) Derive a capacitor voltage in the Thevenin circuit, i.e. *RC* circuit. [6 points]

(c) Find time for the capacitor to reach the maximum voltage rating. [4 points]



(a.) Find R_{TH} Close all independent source



KCL at 1V → $\left(-4i_\Delta + \frac{1-0}{2k} - i_o + \frac{1-0}{5k} = 0\right) 20K$

$$-40i_\Delta + 5 - 10K i_o + 2 = 0$$

$$40K i_\Delta + 10K i_o = 7 \rightarrow (1)$$

also $i_\Delta = \frac{1-0}{5} = \frac{1}{5k}$

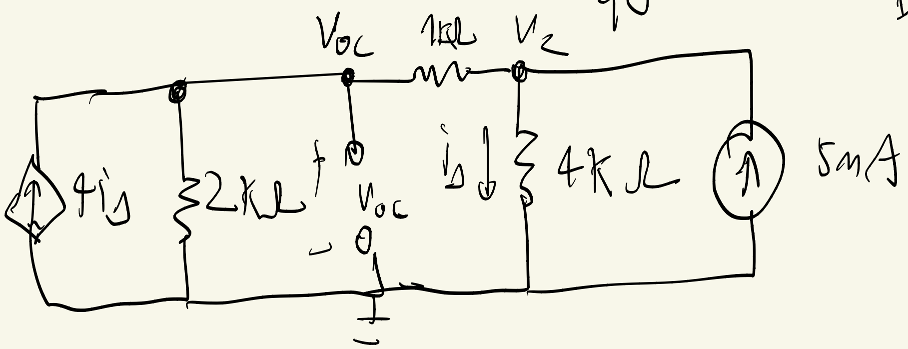
Plug i_Δ to (1) $\rightarrow$ $\overset{8}{\cancel{40}}\left(\frac{1}{\cancel{5k}}\right) + 10i_o' = 7$

$\frac{1}{1}\left(10k\,i_o' = 7 - 8\right)$

$i_o' = \frac{-1}{10k} = -1 \times 10^{-4}$

Hence $R_{TH} = \frac{1}{i_o'} = \frac{1}{\frac{-1}{10}} \Rightarrow -10\,k\Omega$

Find V_{TH}



$\left(-4i_\Delta + \frac{V_{oc}}{2k\Omega} + \frac{V_{oc}-V_2}{1k\Omega} = 0\right)^{10k}$

$-40k\,i_\Delta + 5V_{oc} + 10V_{oc} - 10V_2 = 0$

$15V_{oc} - 10V_2 = 40k\,i_\Delta \rightarrow (1)$

KCL at $V_2 \rightarrow$ $\left(\frac{V_2 - V_{oc}}{1K\Omega} + \frac{V_2}{4K} - 5mA = 0\right) 4K$

$$4V_2 - 4V_{oc} + V_2 = 20$$

$$-4V_{oc} + 5V_2 = 20 \rightarrow (2)$$

also $i_\Delta \Rightarrow \frac{V_2 - 0}{4K} \rightarrow$ sub to (?)

$$15V_{oc} - 10V_2 = \cancel{40K}^{20}\left(\frac{V_2}{\cancel{4K}}\right)$$

$$15V_{oc} = 20V_2$$

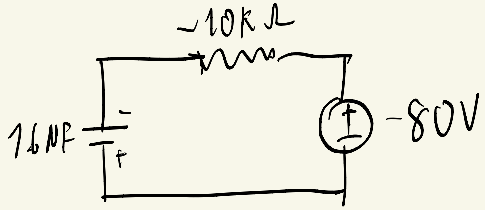
$$V_2 = \frac{3}{4}V_{oc}$$

$$-4V_{oc} + 5\left(\frac{3}{4}V_{oc}\right) = 20$$

$$-4V_{oc} + \frac{15}{4}V_{oc} = 20 \rightarrow \frac{1}{6}V_{oc} = 20$$

$V_{oc} = V_{TH} = -80V$

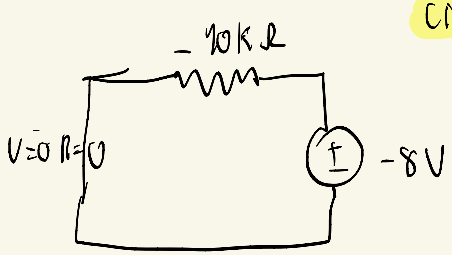
Thevenin equivalent Circuit.



6.) Capacitor Voltage →

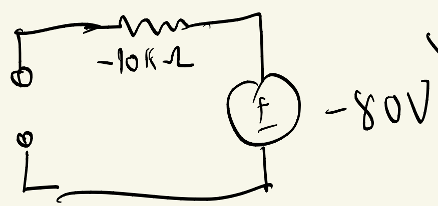
when $t<0$ or when $t=0$ $V_c(0^+) = V_c(0^-)$

Capacitor behave like a short Circuit.



→ Hence $V_c(0^+) = V_c(0^-)$

when $t \to \infty$: Capacitor act as a open circuit.



$V(\infty)$

$T = RC = -10000 \times (1.6 \times 10^{-6}) = -16\,ms$

$V_C(t) = V_C(\infty) + [V_C(0^+) - V_C(\infty)]\, e^{-t/T}$

$= -80 + 80\, e^{62.5t}$

c.) Max Voltage rating $= 14.4\ kV$

$= 14400$

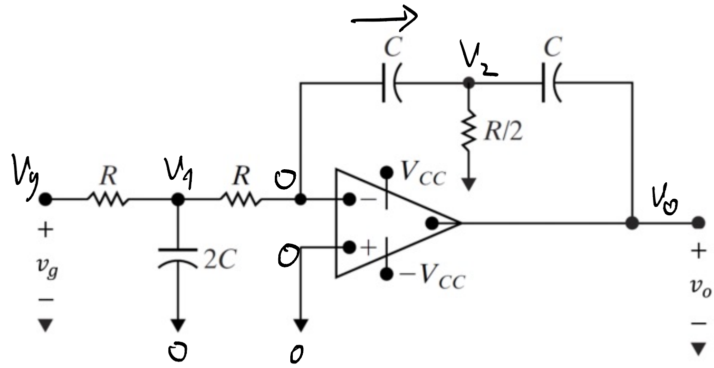
Hence $14400 + 80 = 80 e^{62.5t}$

$181 = e^{62.5t}$

$62.5t = \ln(181)$

$t = 83.02\ ms$

6. Derive the differential equation that relates the output voltage to the input voltage for the circuit shown below. [20 points]



KCL at V_1 → $\frac{V_1 - V_g}{R} + 2C\frac{dV_1}{dt} + \frac{V_1}{R} = 0$

V in $i = C\frac{dv}{dt}$ is the voltage across the capacitor from positive to negative terminal

that the same as node analysis

KCL at 0 $\frac{0 - V_1}{R} + C\frac{d(0-V_2)}{dt} = 0$

Positive terminal

KCL at V_2 $-C\frac{d(0-V_2)}{dt} + \frac{V_2}{\frac{R}{2}} + C\frac{d(V_2-V_o)}{dt} = 0$

from → (1) $V_g = 2RC\frac{dV_1}{dt} + 2V_1 \rightarrow (1)$

From (2) → $V_1 = -RC\frac{dV_o}{dt} \rightarrow (2)$

from (3) $\longrightarrow$ $$C\frac{dV_2}{dt} + 2\frac{V_2}{R} + C\frac{dV_2}{dt} - C\frac{dV_0}{dt}$$

$$\frac{dV_0}{dt} = \frac{dV_2}{dt} + \frac{2}{RC}V_2 + \frac{dV_2}{dt}$$

$$\frac{dV_0}{dt} = 2\frac{dV_2}{dt} + \frac{2V_2}{RC} \to (3)$$

$$\frac{d^2V_0}{dt^2} = 2\frac{d^2V_2}{dt^2} + 2\frac{dV_2}{dt}\frac{1}{RC}$$

from (2)

$$-\frac{V_1}{RC} = \frac{dV_2}{dt}$$

also $$\frac{d^2V_2}{dt^2} = -\frac{1}{RC}\frac{dV_1}{dt}$$

$$\frac{d^2V_0}{dt^2} = -\frac{2}{RC}\frac{dV_1}{dt} - \frac{2V_1}{R^2C^2}$$

$$= \frac{-1}{(RC)^2}\left(2RC\frac{dV_1}{dt} + 2V_1\right)$$

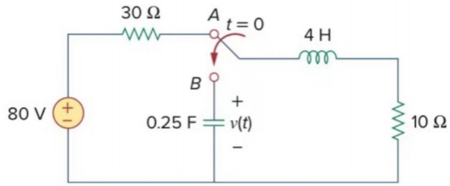
$$2RC \; xx = \frac{2}{RC}$$

$$v_g = 2RC\frac{dv_1}{dt} + 2V_1$$

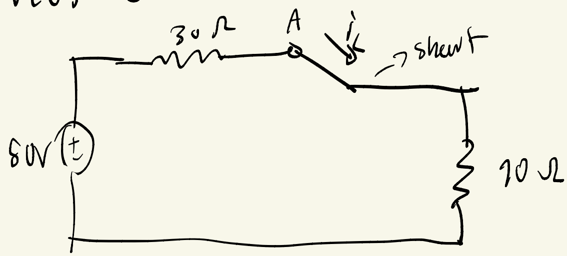
$$\boxed{\frac{d^2v_o}{dt^2} = \frac{-v_g}{R^2C^2}}$$

Exercise 4.3 (20%)

The switch in the following figure moves form position A to position B at $t = 0$ (please note that the switch must connect to point B before it breaks the connection at A, a make-before-break switch). Let $v(0) = 0V$, find $v(t)$ for $t > 0$.

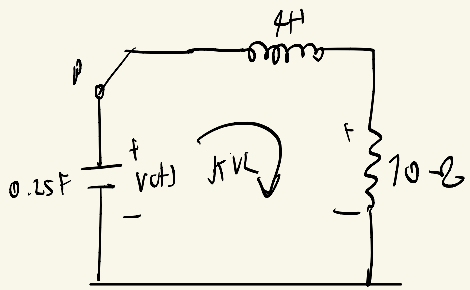


When $t<0$. $V(0) = 0\,V$



$i(0^+) = i(0^-) = \frac{V}{R} = \frac{80\cancel{0}}{4\cancel{0}} = 2\,A$

When $t>0$.



RLC Series circuit

do KVL

$v = L\frac{di}{dt}$

$\frac{v}{L}dt = di \rightarrow i = \int_{-\infty}^{\infty} \frac{v}{L}dt$

$i_c = C\frac{dv_c}{dt}$

$L\frac{di_L}{dt} + 10 i_L + V_c(t) = 0$

$\frac{i_c dt}{C} = dv_c$

$V_c = \int_{-\infty}^{\infty} \frac{i_c}{C}dt$

$$= \quad L\frac{d^2 i_L}{dt^2} + 10\frac{di_L}{dt} + \frac{i_L}{C} = 0$$

$$\frac{d^2 i_L}{dt^2} + \frac{10}{L}\frac{di_L}{dt} + \frac{1}{LC} i_L = 0$$

$$\vee$$

Same response as RLC series input

$$\Rightarrow \quad \frac{d^2 i_L}{dt^2} + \frac{5}{2}\frac{di_L}{dt} + i_L = 0$$

$$= \quad s^2 + \frac{5}{2}s + 1 = 0 \longrightarrow (1)$$

$$b^2 - 4ac = \left(\frac{5}{2}\right)^2 - 4(1)(1) = \frac{9}{4}$$

as $\frac{9}{4} > 0 \rightarrow$ Overdamped

$$\alpha = \frac{R}{2L} \Rightarrow \frac{10}{2(4)} = \frac{10}{8} = \frac{5}{4} \quad //$$

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4\left(\frac{1}{4}\right)}} = 1 \quad //$$

response $= \quad A_1 e^{s_1 t} + A_2 e^{s_2 t}$

from (1) $\quad \frac{-\frac{5}{2} \pm \sqrt{\left(\frac{5}{2}\right)^2 - 4(1)(1)}}{2(1)} \Rightarrow -\frac{5}{4} \pm \frac{3}{4}$

$$s_1 = -\frac{1}{2} \; ; \; s_2 = -2$$

$$i(t) = A_1 e^{-\frac{1}{2}t} + A_2 e^{-2t}$$

from $\rightarrow \frac{di_L}{dt} \longrightarrow$ $$L\frac{di_L}{dt} + 10 i_L + v_C(t) = 0$$

(2) $\qquad v_C(0^+) = v_C(0^-) = 0$

$$L\frac{di_L}{dt} = -20$$

$$\frac{di_L}{dt} = \frac{-20}{4} = -5 \; A/s$$

$$i'(t)_{0} = -5$$

$$i'(t) \Rightarrow -\frac{1}{2}A_1 e^{-\frac{1}{2}t} - 2A_2 e^{-2t} =$$

$$i'(0) = -\frac{1}{2}A_1 - 2A_2 = -5$$

$i'(0) = 2 \rightarrow$ $$A_1 + A_2 = 2$$

$$A_1 = -\frac{2}{3} \; ; \; A_2 = \frac{8}{3}$$

$$i(t) \Rightarrow -\frac{2}{3}e^{-\frac{1}{2}t} + \frac{8}{3}e^{-2t}$$

$$i_L(t) \Rightarrow C\frac{dv}{dt}$$

$$\frac{i_L(t)}{C}dt \Rightarrow dv$$

$$v = \frac{1}{C}\int_0^t i_L(t)\,dt$$

$$= \frac{1}{0.25}\left[+\frac{4}{3}e^{-\frac{1}{2}t} - \frac{4}{3}e^{-2t}\right]\Bigg|_0^t$$

$$= \frac{16}{3}e^{-\frac{1}{2}t} - \frac{16}{3}e^{-2t}$$

#

7.10 For the circuit in Fig. 7.90, find $v_o(t)$ for $t > 0$. Determine the time necessary for the capacitor voltage to decay to one-third of its value at $t = 0$.

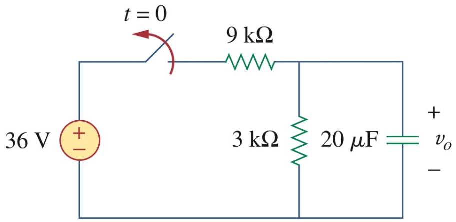
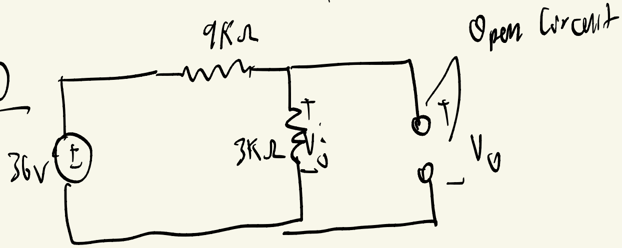


Figure 7.90
For Prob. 7.10.

RC Source free circuit

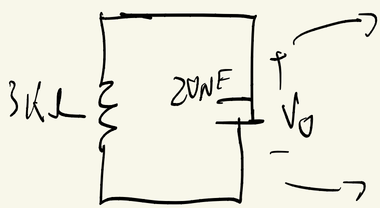
When $t<0$



$$V_o(0^+) = V_o(0^-) \Rightarrow \left(\frac{3k}{3+9}\right)36 = \frac{3}{\cancel{12}}\times\cancel{36}^{\,3} = 9\,V$$

When $t>0$

$$i_c = C\frac{dv}{dt}$$



$$\frac{V}{R} + C\frac{dv}{dt} = 0$$

Response → Same RC circuit →

$$\tau = 3K(20\times10^{-6})$$

$$= 3000\,(20\times10^{-6}) = \frac{3}{50}$$

$$v = V_o e^{-\frac{t}{\tau}} = 9e^{-\frac{t}{\frac{3}{50}}}$$

$$= 9e^{-\frac{50}{3}t}$$

as $v(0) = 9$

$$3 = 9e^{-\frac{50}{3}t}$$

$$\frac{1}{3} = e^{-\frac{50}{3}t}$$

$$\ln\left|\frac{1}{3}\right| = -\frac{50}{3}t$$

$$\underline{\underline{t = 65.92\,ms}}$$

6.62 Consider the circuit in Fig. 6.84. Given that $v(t) = 12e^{-3t}$ mV for $t > 0$ and $i_1(0) = -10$ mA, find: (a) $i_2(0)$, (b) $i_1(t)$ and $i_2(t)$.

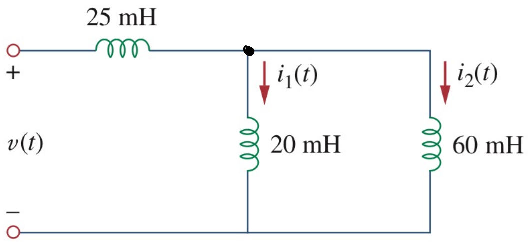


Figure 6.84
For Prob. 6.62.

find $i(t) \rightarrow$ $i(t) = \frac{1}{L_{eq}} \int_{t_0}^{t} v\,dt + i(t_0)$

$= \frac{1}{L_{eq}} \int_{0}^{t} 12e^{-3t}\,mV\; i(0)$

$= \frac{1}{L_{eq}} \left[-4e^{-3t} \Big|_0^t + i(0) \right]$

$L_{eq} \Rightarrow 20 // 60 + 25$

$= \frac{20 \times 60}{20+60} + 25$

$= \frac{120\cancel{0}}{8\cancel{0}} + 25\,mH$

$= 40\,mH$

$= \frac{1}{40}\left(-4e^{-3t} + 4\right) + i(0)$

$i(t) \Rightarrow \left(-\frac{1}{10}e^{-3t} + \frac{1}{10}\right) + i(0)$

→ from current division.

$i_1(t) \Rightarrow i(t)\left(\frac{60}{20+60}\right)$

$i_1(t) \Rightarrow \frac{3}{4} i(t)$

$i_1(0) = \frac{3}{4} i(0)$

$-10\,mA = \frac{3}{4}(i(0))$

$-\frac{40}{3}\,mA = i(0)$

$i(0) = -0.013\overline{3}$ #

(a) $i_2(0) \Rightarrow i(t)\left(\frac{20}{20+60}\right) = \frac{1}{4} i(0)$

$= \frac{1}{4}(-0.0133)$

$\Rightarrow -3.33 \times 10^{-3}\,A$

✓

(b) / (c) $\longrightarrow$ as $i_1(0) = \frac{3}{4} i(0)$

$\Rightarrow i_1(t) = \frac{3}{4} i(t)$

$\Rightarrow \frac{3}{4}\left[\left(\frac{-1}{10mn} e^{-3t} + \frac{1}{10mn}\right) + i_1(0)\right]$

$= -0.075e^{-3t} + 0.075 + \frac{3}{4} i(0)$

$= -0.075e^{-3t} + 0.065$

6.63 In the circuit of Fig. 6.85, sketch v_o.

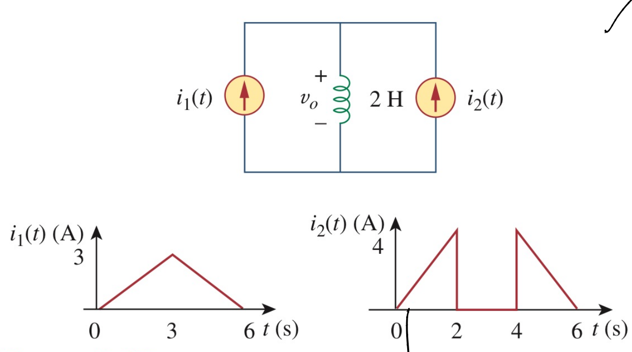


Figure 6.85
For Prob. 6.63.

$i_1(t)\ (0-3)$

$\Rightarrow i_1(t) = t$

$i_2(t)$ for $0<t<2$

$= 2t$

$i_2(t)$ for $4<t<6$

$\Rightarrow -2$

Use superposition: $V_0 = V_1 + V_2$

$V_1 = L \frac{di(t)}{d(t)} \to (0<3)$

$V_1 = L1 = 2 \quad (0<t<3)$

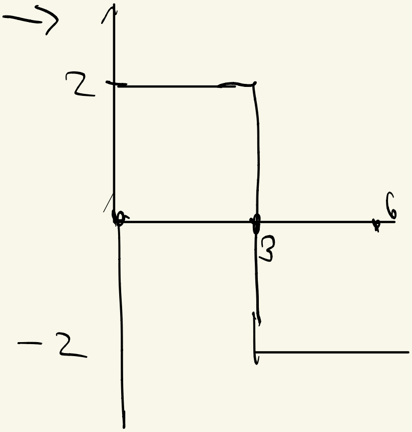
$V_1 = L(-1) = -2 \quad (3<t<6)$

$V_2 = L(2) = 4 \quad (0<t<2)$

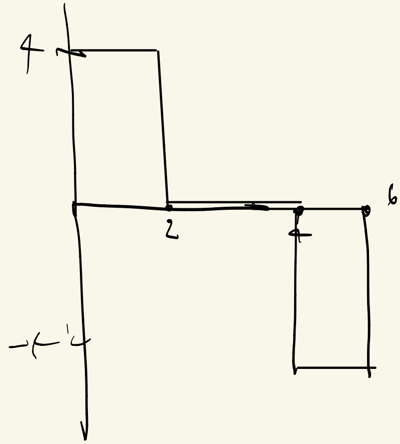
$V_2 = L(0) = 0 \quad (2<t<4)$

$= L(-2) = -4 \quad (4<t<6)$

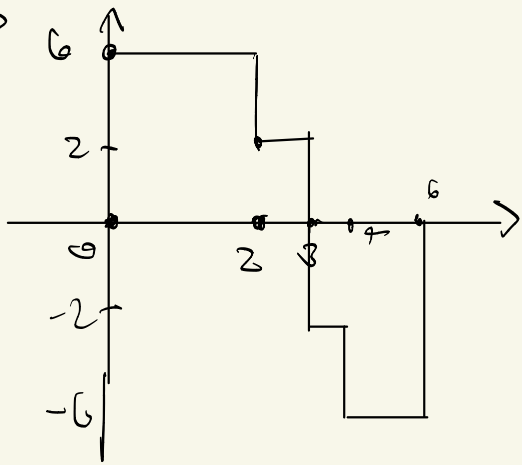
graph V_1 →



V_2



$V(t)$ →



*6.57 Determine L_{eq} that may be used to represent the inductive network of Fig. 6.79 at the terminals.

Super position

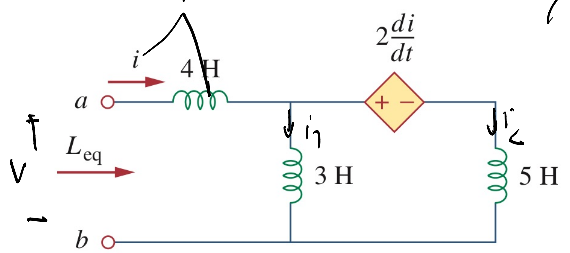


Figure 6.79
For Prob. 6.57.

Final want

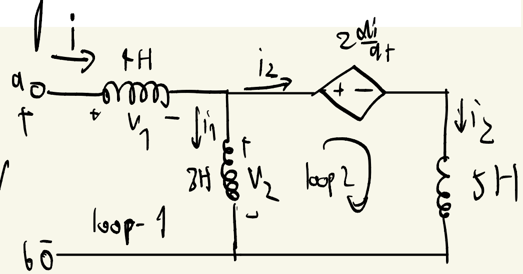
$$V(t) = L_{eq}\frac{di}{dt}$$

Hence need to find V_1 and V_2

we know

$$V(t) = L\frac{di}{dt}$$

redrawn



The total Current $i \rightarrow$ is $\quad i = i_1 + i_2 \rightarrow (2)$

$i_2 = i - i_1$

apply KVL at loop 1

$$V = V_1 + V_2 \rightarrow (1)$$

also $V_1 = 4\dfrac{di}{dt}$; $V_2 = 3\dfrac{di_1}{dt}$

$$\frac{di_1}{dt} = \frac{V_2}{3}$$

from (1) $\rightarrow$ $V = 4\dfrac{di}{dt} + V_2 \rightarrow (3)$

<u>KVL loop 2</u> $\quad 2\frac{di}{dt} + 5\frac{di_2}{dt} - V_2 = 0$

$$V_2 = 2\frac{di}{dt} + 5\frac{di_2}{dt} \longrightarrow \text{from } 2 \rightarrow i - i_1 = i_2$$

$$= 2\frac{di}{dt} + 5\frac{di}{dt} - 5\frac{di_1}{dt}$$

$$V_2 = 7\frac{di}{dt} - 5\frac{di_1}{dt} \quad \text{also } \frac{di_1}{dt} = \frac{V_2}{3}$$

$$V_2 = 7\frac{di}{dt} - 5\frac{V_2}{3}$$

$$V_2 = \frac{21}{8}\frac{di}{dt}$$

sub to (3) $\longrightarrow$ $V = 4\frac{di}{dt} + \frac{21}{8}\frac{di}{dt}$

$$V = \frac{53}{8}\frac{di}{dt}$$

$\downarrow$

Hence from $v(t) = L_{eq}\frac{di}{dt} = \frac{53}{8}\cancel{\frac{di}{dt}} = L_{eq}\cancel{\frac{di}{dt}}$

$\downarrow$

$L_{eq} = \frac{53}{8}$ #

6.61 Consider the circuit in Fig. 6.83. Find: (a) L_{eq}, $i_1(t)$, and $i_2(t)$ if $i_s = 3e^{-t}$ mA, (b) $v_o(t)$, (c) energy stored in the 20-mH inductor at $t = 1$ s.

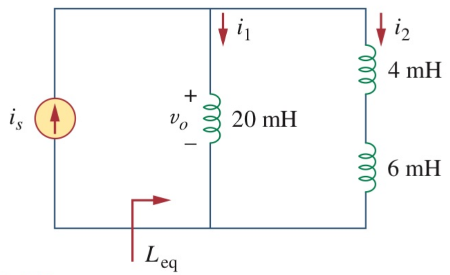


Figure 6.83
For Prob. 6.61.

$L_{eq} \Rightarrow 10 // 20 = \frac{20\cancel{0}}{3\cancel{0}} = \frac{20}{3}$ mH

$i_1(t) = \frac{10}{(20+10)}(3e^{-t}) = e^{-t}$

$i_2(t) \Rightarrow \frac{20}{(30)}(3e^{-t}) = 2e^{-t}$

b.) $v_o = L_{eq}\frac{di_s}{dt} = \frac{20}{3} \times (-3e^{-t}\ \text{mA})$

$$= \frac{20}{3} \times 10^{-3} \left(-3e^{-t} \times 10^{-3}\right)$$

$$w = \frac{1}{2}(20\times10^{-3})(e^{-t})^2 \qquad = -20\, e^{-t}\, nV$$

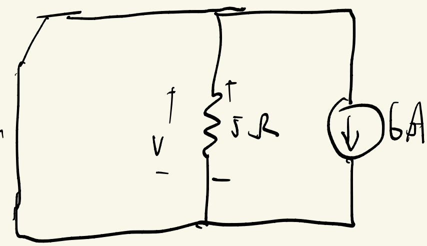
6.64 The switch in Fig. 6.86 has been in position A for a long time. At $t = 0$, the switch moves from position A to B. The switch is a make-before-break type so that there is no interruption in the inductor current. Find:

(a) $i(t)$ for $t > 0$,

(b) v *just after* the switch has been moved to position B,

(c) $v(t)$ long after the switch is in position B.

when $t<0$



Short circuit

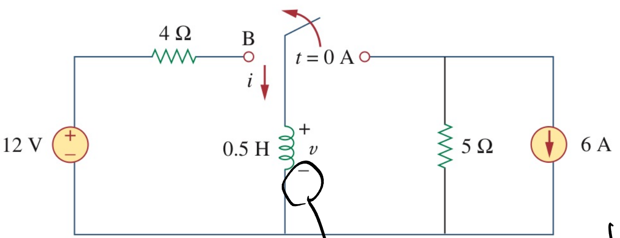
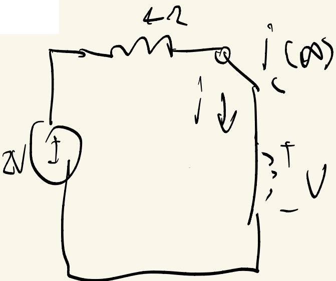


Figure 6.86
For Prob. 6.64.

i เข้ามาด้านลบ => -6

$i(0^+) = i(0^-) = -6\,A$

when $t>0 \Rightarrow i \to \infty$



$i(\infty) \Rightarrow \frac{12}{4} = 3\,A$

Step response

$$i(t) \Rightarrow i(0) + (i(\infty) - i(0))e^{-\frac{t}{\tau}}$$

$$\tau = \frac{L}{R} = \frac{0.5}{4} = \frac{1}{8}$$

$$i(t) \Rightarrow 3 + (-6-3)e^{-8t}$$

$$i(t) = 3 - 9e^{-8t}$$ #

(b) when $t>0$ $\quad v = L\dfrac{di(t)}{dt}$

$$v(0) = 0.5\left(72e^{-8(0)}\right) = \frac{1}{2}(72)$$

$$= 36V$$

(c) = Given $v(t) \to \infty$ inductor become short circuit

hence $V=0$

7.46 For the circuit in Fig. 7.113, $i_s(t) = 5u(t)$. Find $v(t)$.

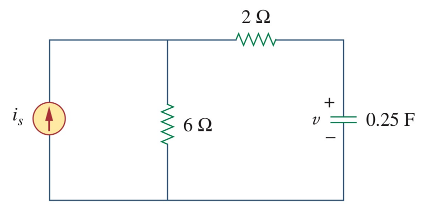
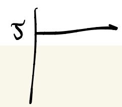
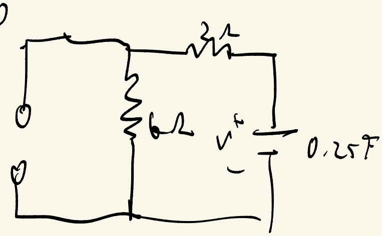


Figure 7.113
For Prob. 7.46.

$i_s(t) = 5u(t) \to$

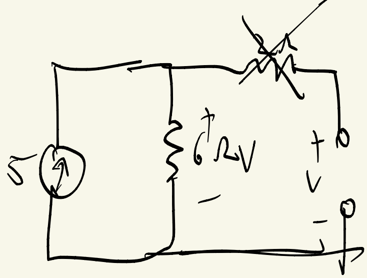


when $t<0$



$$v(0^+) = v(0^-) = 0$$

when $t>0$

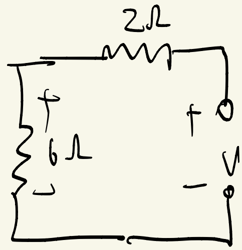


$v \to \infty \to \quad v = iR \Rightarrow 5(6) = 30$

$$v(t) = 30 + (0-30)e^{-\frac{t}{\tau}}$$

$\tau = RC \Rightarrow$

$R = R_{TH} \to$



$R_{TH} \to$ Close independent source $\to$ Current become open circuit

Voltage become short circuit

$R = R_{TH} = 8 \to \tau = RC = 8 \times 0.25 = 2$

$$v(t) = 30 - 30\,e^{-\frac{t}{2}}$$

7.47 Determine $v(t)$ for $t > 0$ in the circuit of Fig. 7.114 if $v(0) = 0$.

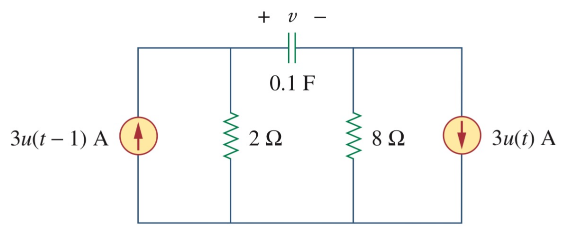


Figure 7.114
For Prob. 7.47.

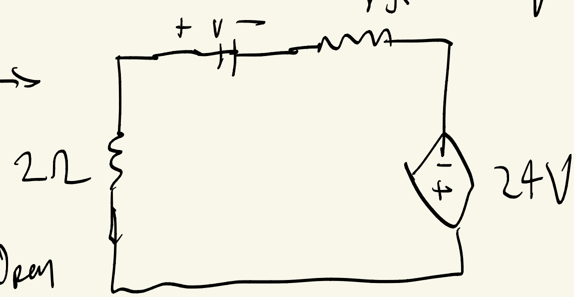
given $V(0) \rightarrow 3u(t-1) \rightarrow$ when $t < 1 = 0$

When $t > 1 = 3$

when $t < 0 \rightarrow V(0) = 0$

$V = IR$

When $0 < t < 1 \longrightarrow$



↓

as $v(\infty) \rightarrow$

Capacitor ⇒ Open Circuit and $V(\infty) = 24V$

$\Rightarrow V(t) = 24 + (0 - 24)e^{-\frac{t}{\tau}}$

$\tau = RC \Rightarrow R_{TH} = 10 \rightarrow \tau = 10(0.1) = 1$

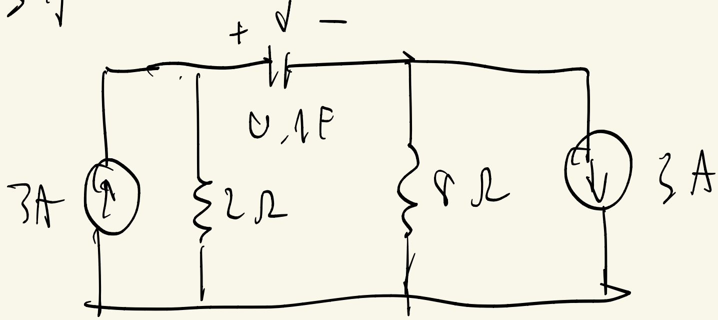
$v(t) = 24 - 24e^{-t}$ V for $0 < t < 1$

$v(1) = 24 - 24e^{-1} \Rightarrow 15.17V$

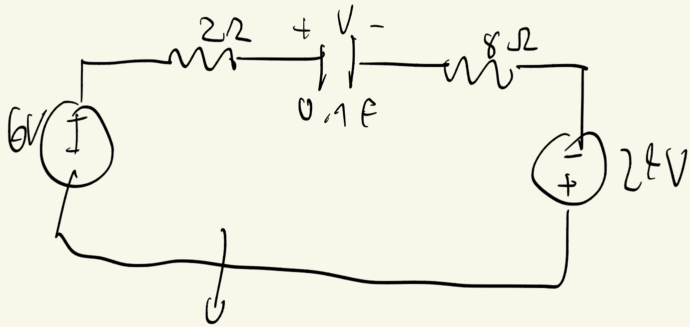
↓

Now when $t > 1$

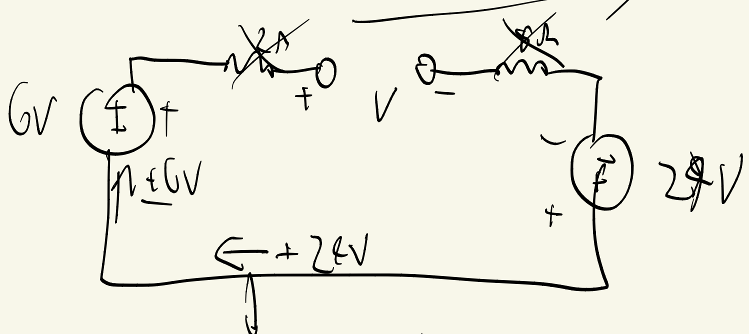
$v(0) = 15.97\ V$



$V(\infty) =$



Capacitor become open circuit no current though



$v(\infty) = 30V$

$T = RC = 10(0.1) = 1$

$= v(t) \Rightarrow 30 + (15.97 - 30)e^{-t}$

$30 - 18.83\, e^{-(t-1)}$ → when $t > 1$

7.48 Find $v(t)$ and $i(t)$ in the circuit of Fig. 7.115.

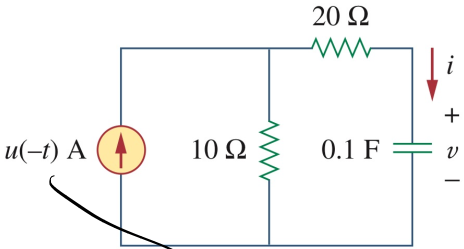
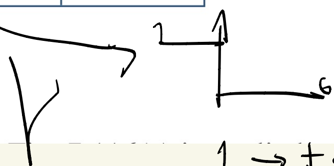


Figure 7.115
For Prob. 7.48.

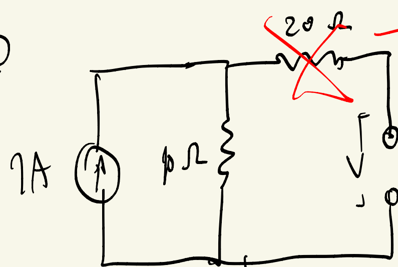


$1 \rightarrow t < 0$

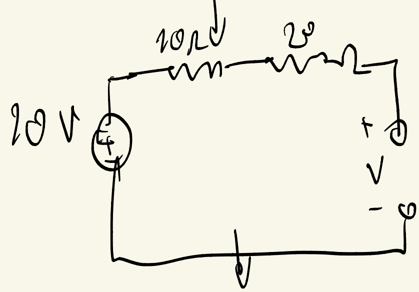
$0 \rightarrow t > 0$

When $t < 0$

Open circuit ; No current flow in

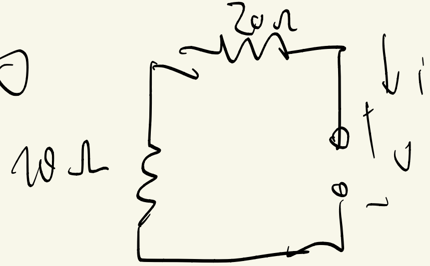


→ become open circuit



$V(0) = 10V$
$= V_0$

when $t>0$



$v(\infty) = 0 \qquad \tau = RC \rightarrow = 30(0.1)$

$$= 3 \ s$$

$$v(t) = 0 + (10 - 0)e^{-\frac{t}{3}}$$

$$v(t) = 10e^{-\frac{t}{3}}$$

$$i(t) = C\frac{dv}{dt} = 0.1\left(-\frac{10}{3}e^{-\frac{t}{3}}\right)$$

$$i(t) = -\frac{1}{3}e^{-\frac{t}{3}}$$

\#

***7.50** In the circuit of Fig. 7.117, find i_x for $t > 0$. Let $R_1 = R_2 = 1\ \text{k}\Omega$, $R_3 = 2\ \text{k}\Omega$, and $C = 0.25$ mF.

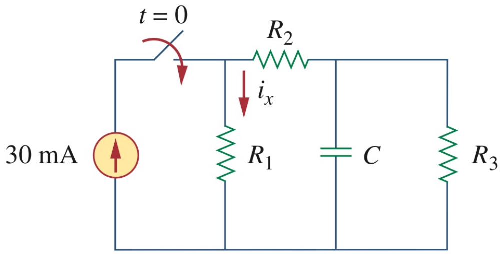
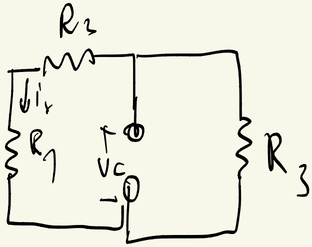


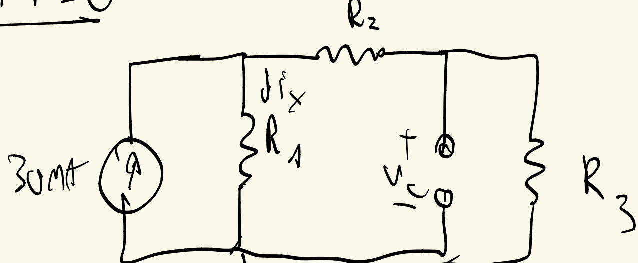
Figure 7.117
For Prob. 7.50.

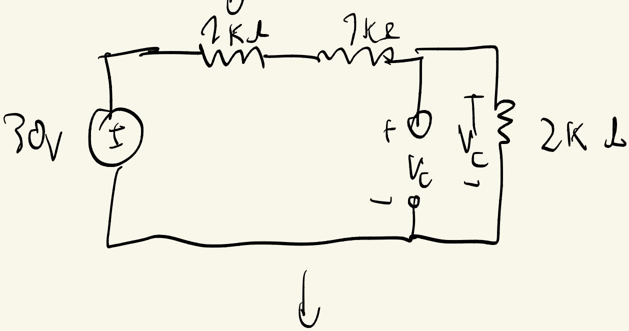
When $t<0$



$v(0^+) = v(0^-) = V_0 = 0$

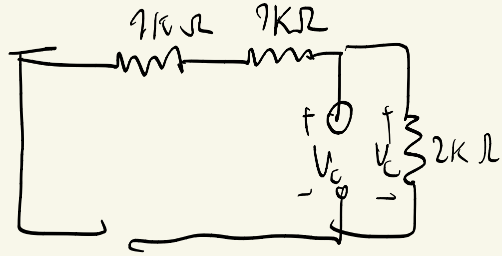
When $t>0$





$$V_C(\infty) = \left(\frac{2}{2+2}\right) 30 = \frac{60}{4} = 15\,V$$

$\tau = R_{TH} C \Rightarrow R_{TH} \to$ Close all independent source



$$= 1+1 // 2$$

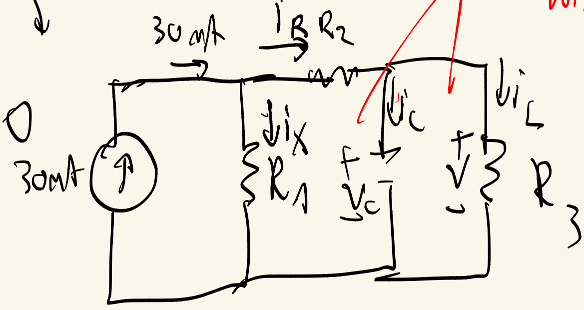
$$= \frac{2\times 2}{2+2} = 1\,k\Omega$$

$$\tau = 1\,k\Omega \times 0.25\,mF \Rightarrow 0.25\,s$$

$$V_C(t) = 15 + (0 - 15)e^{-4t}$$

$I(x) \to$ when $t > 0$

Parallel have have the same Voltage.



V

$$30 = i_x + i_R$$

$$\downarrow$$

$$i_R = 30 - i_x$$

$$i_R = i_C + i_L$$

$$\downarrow$$

$$i_C = C\frac{dv}{dt} + \frac{V}{R_3}$$

$$20.25x.$$

$$\downarrow$$

solve then $I_x = 30 - i_R$

***7.57** Find $i_1(t)$ and $i_2(t)$ for $t > 0$ in the circuit of Fig. 7.123.

first step is to find the whole $v(t)$ or $i(t)$ first

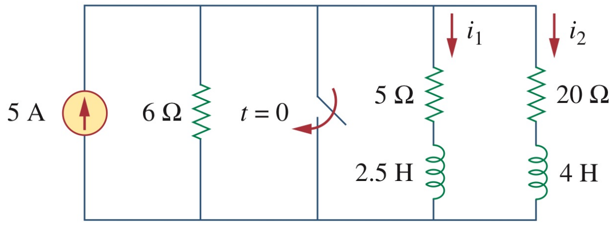
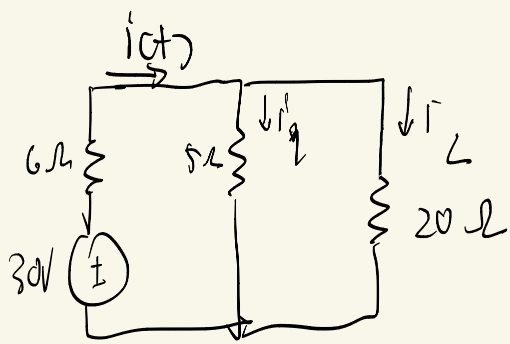


Figure 7.123
For Prob. 7.57.

When $t < 0$ → inductor become short circuit



$$i_{total} \Rightarrow \frac{V}{R_{eq}} = \frac{30}{10} = 3$$

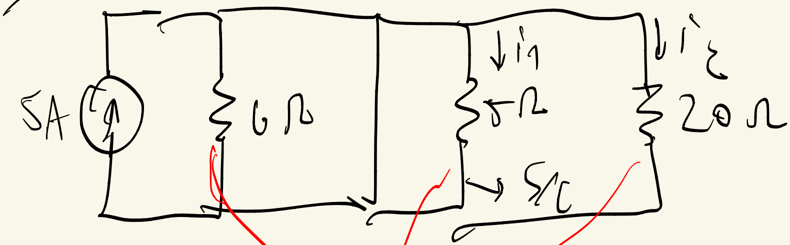
$i(t) = 3$

$i_t(0) = 3$

Current division find $i_1(0)$ & $i_2(0)$

$$i_1(0) = \left(\frac{20}{25}\right)3 = 2.4 \; ; \; i_2(0) = 0.6$$

$\underline{\text{for } t>0}$ $\to t \to \infty$



Pararull to short circuit hence no

$i_L = 0$;

$i_1(\infty) = 0$; $i_L(\infty) = 0$

8.5 Refer to the circuit in Fig. 8.66. Determine:

(a) $i(0^+)$ and $v(0^+)$,

(b) $di(0^+)/dt$ and $dv(0^+)/dt$,

(c) $i(\infty)$ and $v(\infty)$.

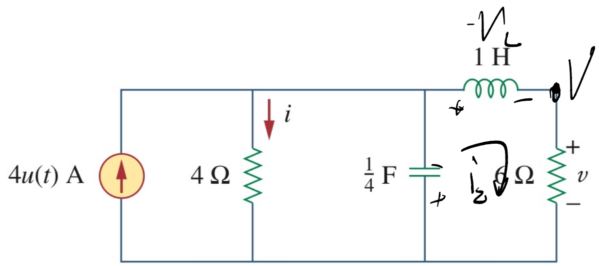
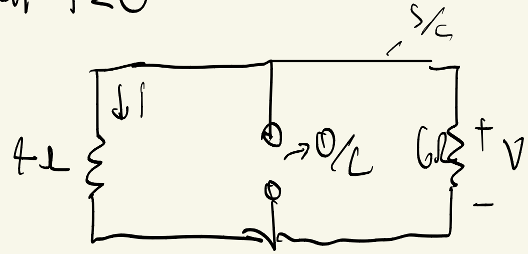


Figure 8.66
For Prob. 8.5.

(a). When $t<0$

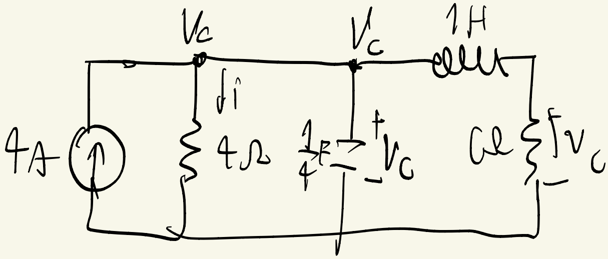


$V_c(0^+) = V_c(0^-) = 0$

$I_L(0^+) = I_L(0^-) = 0$ → as $I_L = 0$

$V_L = 0$

when $t>0$



$$i = \frac{v_C}{R}$$

$$\frac{di_C}{dt} = \frac{1}{R}\frac{dv_C}{dt}$$

KCL at V_C

$$-4 + \frac{V_C}{4} + I_L + C\frac{dv_C}{dt}$$

$$C\frac{dv_C}{dt} = 4 - \cancel{\frac{V_C}{4}} - \cancel{I_L} \to 0$$

$$\frac{dv_C}{dt} = \frac{4}{C} \qquad V_C(0) = 0$$

$$\frac{4}{\frac{1}{4}} = 16$$

$$\frac{di}{dt} = \frac{1}{4} \times 16$$

$$\frac{di}{dt} = 4\ A/S$$

KVL at loop 2.

$$+V_L + V + \frac{1}{4} = 0$$

$$0 + V + \frac{1}{4} = 0$$

$$V = -\frac{1}{4} \to \frac{dv(0^+)}{dt} = 0$$

$$V_C = L\frac{di_L}{dt}$$

$$\int \frac{V_C}{L}\,dt = i_L$$

$$\frac{1}{L}\int v_C\,dt$$

***8.21** Calculate $v(t)$ for $t > 0$ in the circuit of Fig. 8.75.

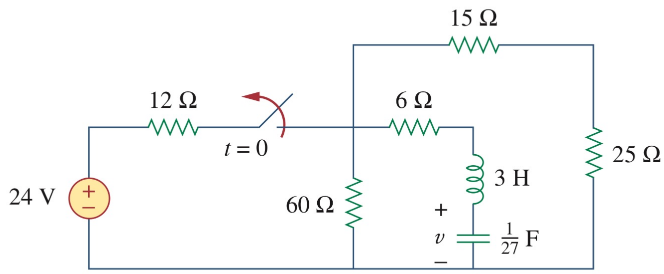
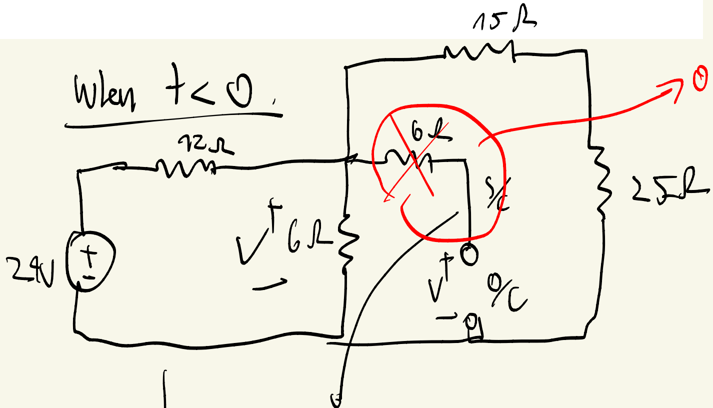


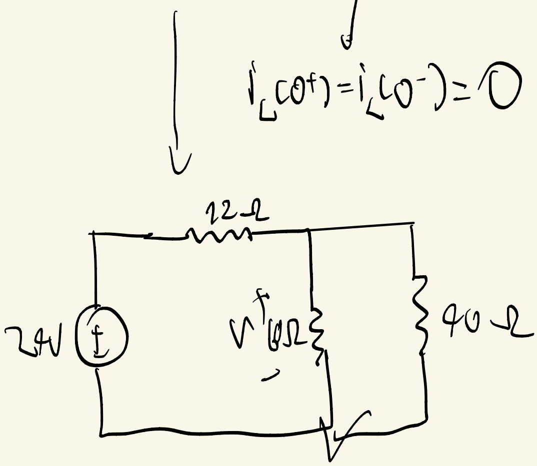
Figure 8.75
For Prob. 8.21.

When $t<0$.

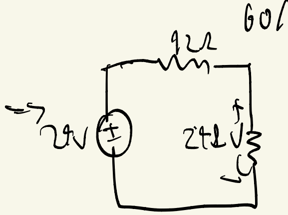


→ Open Circuit ; No current source go through it. R = 0

$i_L(0^+) = i_L(0^-) = 0$



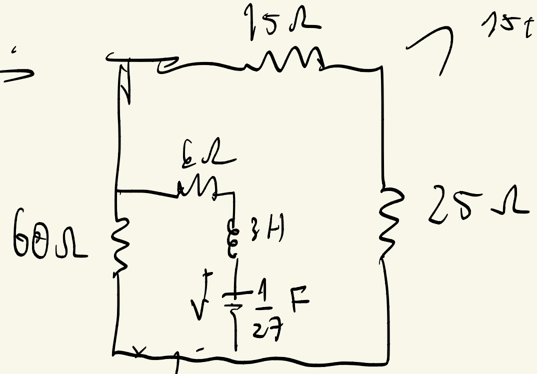
60 // 40



$\rightarrow V_c = \left(\frac{24}{12+24}\right) 24 = 16V$

$V_c(0^+) = V_c(0^-) = 16V$

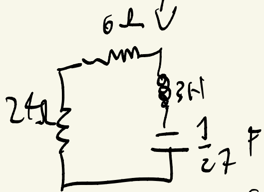
when $t > 0$.



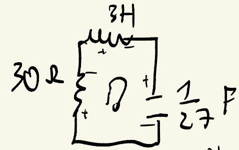
$15 + 25 = 40$

$40 \,//\, 60$

$= 24$



$\rightarrow$



$i = C\frac{dv}{dt}$

$\frac{i}{C}dt = dv$

$v = \frac{1}{C}\int_{-\infty}^{\infty} i\,dt$

Source free RLC circuit

$$L\frac{di_L}{dt} + \frac{1}{C}\int_{-\infty}^{\infty} i\,dt + 30i = 0$$

$$\frac{L\frac{d^2 i_L}{dt^2} + \frac{i}{C} + 30\frac{di_L}{dt} = 0}{L}$$

$$\frac{d^2 i_L}{dt^2} + \frac{30}{L}\frac{di_L}{dt} + \frac{1}{LC}i = 0$$

$$s^2 + 10s + 9 = 0$$

use $b^2 - 4ac \rightarrow 10^2 - 4(1)(9)$

$100 - 36 > 0$

Overdamped.

$$\frac{-10 \pm \sqrt{100 - 4(1)(9)}}{2} = 0$$

$\downarrow$

$$s = -5 \pm 4$$

$$s_1 = -1 \; ; \; s_2 = -9 \quad \#$$

response $\rightarrow$ $i(t) \Rightarrow A_1 e^{-t} + A_2 e^{-9t}$

from $\rightarrow$ $L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{\infty} i\,dt + 30i = 0$

$$L\frac{di_L}{dt} + V_C + 30i = 0$$

$i_L(0) = 0$

$V_C(0^+) = 16$

$$L\frac{di_L}{dt} = -16$$

$$\frac{di_L}{dt} = \frac{-16}{3} = -\frac{16}{3}$$

$i(0) = 0 \rightarrow A_1 + A_2 = 0$

$i'(0) = -\frac{16}{3} \rightarrow i'(t) = -A_1 e^{-t} - 9A_2 e^{-9t}$

$i'(0) = -\frac{16}{3} \rightarrow -A_1 - 9A_2 = -\frac{16}{3}$

$$A_1 = -\frac{2}{3} \quad ; \quad A_2 = \frac{2}{3}$$

$$i(t) \Rightarrow -\frac{2}{3}e^{-t} + \frac{2}{3}e^{-9t}$$

$$V = L\frac{di}{dt} = 3\left[\frac{2}{3}e^{-t} - 6e^{-9t}\right]$$

$$V(t) = 2e^{-t} - 18e^{-9t} \; \text{V}$$