

Chapter 1-3. Zenth law: if 2 bodies are in thermal equilibrium with a 3rd body, then they will both have the same temperature.

$T_A = T_C; T_B = T_C \rightarrow$ then $T_A = T_B$. Unit. $\rightarrow$ Energy: Joules $\Rightarrow N \cdot m$; Work $\Rightarrow F \cdot d \Rightarrow N \cdot m = J$; P.E: $mgh = Mgh \Rightarrow (kg)(\frac{m}{s^2})(m)$

KE $\Rightarrow \frac{1}{2}mv^2 \Rightarrow (kg)(\frac{m}{s})^2 = J$; Heat Emby: J ; Density $\Rightarrow \rho = \frac{CG}{(cm^3)}$; Pressure $\Rightarrow$ unit $\Rightarrow [\frac{N}{m^2}] = [Pa]$; $[\frac{J}{m^3}] = [Pa]$

Abs pressure $\Rightarrow$ Actual pressure relative to 0. Gauge pressure $\Rightarrow$ pressure relative to local atmospheric pressure. Vacuum Pressure $\rightarrow$ a negative gauge pressure

$\Delta P = \rho gh$ Change in pressure when fluid goes down (same fluid) $\Rightarrow P_1 + \rho gh = P_2$ Flow Work $\Rightarrow Fd = P \Delta V$ PV ENERGY $\Rightarrow E = U + PE + KE + FW [kJ]$

Mass flow rate: $\dot{M} = \rho \dot{V} \Rightarrow \rho VA$; Energy flow rate: $\dot{E} = \dot{M}e \rightarrow kW = kJ/s$ Mechanical Energy $\Rightarrow \Delta e_{mech} = \frac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1) + \frac{P_2 - P_1}{\rho}$

Heat $\rightarrow Q [kJ]$; Work $\rightarrow F \cdot d \rightarrow \dot{W} = FV$ Electrical Work: $\dot{W} = VI$ First Law of thermodynamics $Q_{in} + Q_{out} = \Delta KE$ Efficiency $\eta = \frac{\text{desired output}}{\text{Required input}}$

$\eta = \frac{\dot{W}_{out}}{\dot{W}_{in}}$ efficiency of motor $\rightarrow \eta_{motor} = \frac{\dot{W}_{shaft}}{\dot{W}_{electric}}$ efficiency of a pump $\rightarrow \eta_{pump} = \frac{\dot{W}_{actual}}{\dot{W}_{shaft}}$ Small efficiency of pump $\rightarrow \eta = (\eta_{pump})(\eta_{motor})$

Pure Substance For a substance that constant on freezing. P vs T diagram showing solid, liquid, gas regions. $P_v = \phi P_{sat}(T)$ Partial pressure of water vapor. ϕ Relative humidity.

Energy Analysis for Closed System for closed system $Q + W = \Delta E$ Boundary Work $\rightarrow$ Expansion $\rightarrow W_b = \text{"Work out"}$ Compression $\rightarrow W_b = \text{"Work in"}$

Ideal Gas Eq. $Pv = RT$ $R = \frac{R_u}{M}$ $Pv = NR_u T$ $P = \rho RT$ Boundary Work $W_b = \int P dv$ for "cycle" If constant volume $W_b = P \Delta V$

For a substance that expand with compressibility factor $P_r = ZRT$ $P_r = \frac{P}{P^*}$ $T_r = \frac{T}{T^*}$ $P_r = \frac{P}{P^*}$ $T_r = \frac{T}{T^*}$ $P_r = \frac{P}{P^*}$ $T_r = \frac{T}{T^*}$

Chapter 5 Mass and Energy of Control Volume $\dot{M}_{in} - \dot{M}_{out} = \Delta \dot{M}$ $\dot{Q} + \dot{W} = \dot{M}e_{out} - \dot{M}e_{in}$ $\dot{Q} + \dot{W} = \dot{M}e_{out} - \dot{M}e_{in}$

Steady Flow Process $\dot{Q} + \dot{W} = \dot{M}e_{out} - \dot{M}e_{in}$ $\dot{Q} + \dot{W} = \dot{M}e_{out} - \dot{M}e_{in}$ $\dot{Q} + \dot{W} = \dot{M}e_{out} - \dot{M}e_{in}$

Energy of a Non Flowing Fluid $E = Pv$ Energy of a Flowing Fluid $e = u + \frac{V^2}{2} + gz$ $e = u + \frac{V^2}{2} + gz$

Chapter 6 2nd Law $\Rightarrow$ Process Occur at a certain direction and Energy has a quality/quantity. $\Delta S = \frac{\delta Q}{T}$ $\Delta S = \frac{\delta Q}{T}$

Heat engine $\rightarrow$ its not easy to convert heat to work $K \rightarrow Q_{in} - Q_{out} = W_{net}$ $Q_{in} - Q_{out} = W_{net}$

Thermo efficiency and heat engines $\eta_{th} = \frac{W_{net}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$ $\eta_{th} = 1 - \frac{Q_{out}}{Q_{in}}$

Refrigerator/Heat Pump $COP = \frac{Q_{in}}{W_{net}}$ $COP = \frac{Q_{in}}{W_{net}}$

Refrigerators transfer heat from low temp to high temp $COP = \frac{Q_{in}}{W_{net}}$ $COP = \frac{Q_{in}}{W_{net}}$

Heat Pump $COP = \frac{Q_{out}}{W_{net}}$ $COP = \frac{Q_{out}}{W_{net}}$

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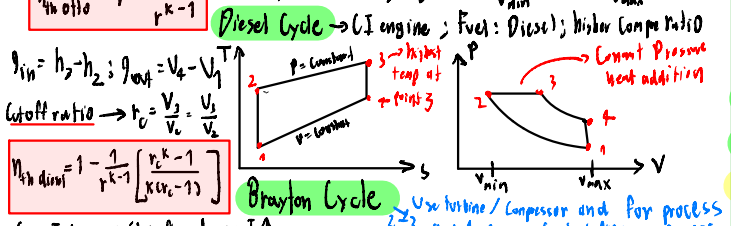
Chapter 9. Carnot Cycle → 4 totally reversible process.

$\eta_{Carnot} \Rightarrow 1 - \frac{T_L}{T_H}$ Carnot is impractical as reversible isothermal heat transfer is quite difficult to achieve in reality. Compression ratio $\rightarrow r_c = \frac{V_{max}}{V_{min}}$ bound to achieve.

MED $\rightarrow \frac{W_{net}}{V_{max} - V_{min}}$ Otto cycle $\rightarrow$ 1) Isentropic compression, 2) Constant volume heat addition, 3) Isentropic expansion, 4) Constant volume heat rejection.

$W_{out, 0-1} = P_0(V_1 - V_0)$
 $W_{in, 1-2} = P_0(V_2 - V_1)$
 $q_{in} = V_3 - V_2$; $q_{out} = V_4 - V_1$

$\eta_{Otto} = 1 - \frac{1}{r_c^{k-1}}$



Diesel Cycle $\rightarrow$ CI engine; Fuel: Diesel; higher compression ratio

$q_{in} = h_3 - h_2$; $q_{out} = h_4 - h_1$

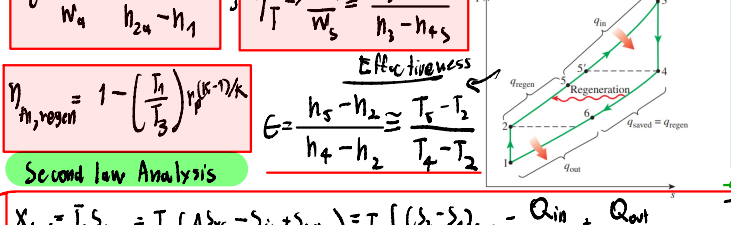
Cutoff ratio $\rightarrow r_c = \frac{V_3}{V_2}$

$\eta_{Diesel} = 1 - \frac{1}{r_c^{k-1}} \left(\frac{r_c^k - 1}{k(r_c - 1)} \right)$

Brayton Cycle Use turbine/compressor and for process 2-3 and 4-1 use constant pressure process

Pressure ratio $r_p = \frac{P_2}{P_1}$ where $P_2 > P_1$ usually $\frac{P_2}{P_1} = \frac{1}{r_p}$

Brayton cycle with regeneration



Isentropic efficiencies

$\eta_c = \frac{W_s}{W_a} \approx \frac{h_{2s} - h_1}{h_{2a} - h_1}$

$\eta_T = \frac{W_a}{W_s} \approx \frac{h_3 - h_{4a}}{h_3 - h_{4s}}$

$\eta_{regen} = 1 - \left(\frac{T_1}{T_3} \right)^{\frac{1}{k-1}}$

Effectiveness $\epsilon = \frac{h_5 - h_2}{h_5 - h_2} = \frac{T_5 - T_2}{T_5 - T_2}$

Second law Analysis

$X_{dest} = T_0 S_{gen} = T_0 (S_{out} - S_{in}) = T_0 \left(\sum \dot{m}_s \ln \frac{T_{out}}{T_{in}} + \frac{Q_{out}}{T_{out}} - \frac{Q_{in}}{T_{in}} \right)$

for closed system exergy destruction

rate form for steady flow system

Chapter 11. Refrigerator. 2) The transfer of heat from a low temp medium to a high temp one is the heat pump.

Carnot cycle $\rightarrow \text{COP}_R = \frac{1}{\frac{T_H}{T_L} - 1}$; $\text{COP}_H = \frac{1}{1 - \frac{T_L}{T_H}}$

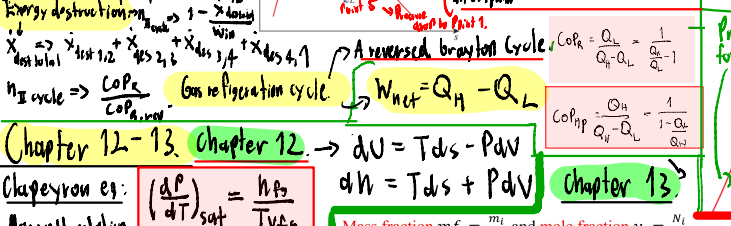
Second Law Analysis

Compressor $\rightarrow \dot{X}_{dest, 1-2} \rightarrow \dot{m}(h_2 - h_1)$

Condenser $\rightarrow \dot{X}_{in, 2-3} \rightarrow T_0 \ln \left(\frac{h_2 - h_3}{h_1 - h_3} \right)$

Expansion Valve $\rightarrow \dot{X}_{out, 3-4} \rightarrow \dot{m}(h_3 - h_4)$

Evaporator $\rightarrow \dot{X}_{in, 4-1} \rightarrow T_0 \ln \left(\frac{h_4 - h_1}{h_2 - h_1} \right)$



Chapter 12-13. Chapter 12 $\rightarrow dU = Tds - PdV$

Chapter 13 $\rightarrow dH = Tds + PdV$

Maxwell relation $\left(\frac{\partial P}{\partial T} \right)_{sat} = \frac{h_{fg}}{T v_{fg}}$

Average Molar mass $M_m = \frac{m_m}{N_m} = \frac{\sum N_i M_i}{\sum N_i}$

Mass and Molar fraction relation $m_i = \frac{M_i}{M_m} y_i$

Dalton's law of additive pressures: The pressure of a gas mixture is equal to the sum of the pressures each gas would exert if it existed alone at the mixture temperature and pressure.

Chapter 8. Exergy $\rightarrow$ Availability or available energy; for exergy $\rightarrow$ initial state is specified.

Useful Work $\rightarrow W_u = W - W_{surr}$ (Useful work potential) useful work that can be produced

Inversibility $\rightarrow I = W_{rev, out} - W_{u, out}$ Second law efficiency $\eta_{II} = \frac{W_u}{W_{rev}}$

For electric resistance $\eta_{II} \approx 1 - \frac{T_0}{T_H}$

Exergy of a close system $X = (U - U_0) + P_0(V - V_0) - T_0(S - S_0) + \frac{mV^2}{2} + mgz$

For unit mass $x = m \phi$

is the same except that $(U - U_0) + \frac{mV^2}{2} + mgz$ can be combine to $(U - U_0)$

Flow exergy $\psi = (h - h_0) - T_0(s - s_0) + \frac{V^2}{2} + gz$

Exergy transfer by heat $X_{heat} = \left(1 - \frac{T_0}{T} \right) Q$

Exergy transfer by mass $X_{mass} = M \psi$

Exergy balance for close system $X_{in} - X_{out} - X_{destroyed} = \Delta X_{system}$

!!! We can cancel $X_{in, out}$ as we only consider the boundary temp.

Exergy balance for steady flow $\sum \dot{X}_{in} - \sum \dot{X}_{out} - \dot{X}_{destroyed} = 0$

for single stream $\Rightarrow \left(1 - \frac{T_0}{T} \right) \dot{Q} - \dot{W} + \dot{m}(\psi_1 - \psi_2) - \dot{X}_{destroyed} = 0$

Second law efficiency of steady flow device

Adiabatic turbine $\eta_{II} = \frac{W_{act}}{W_{rev}} = \frac{h_1 - h_2}{h_1 - h_{2s}}$

Adiabatic compressor $\eta_{II} = \frac{W_{rev}}{W_{act}} = \frac{h_2 - h_{2s}}{h_2 - h_1}$

Maximum power output $\dot{W}_{max} = \dot{m}(h_1 - h_{2s})$

Reversible work $\dot{W}_{rev} = \dot{m}(h_1 - h_{2s})$

Exergy destroyed $\dot{X}_{dest} = T_0 S_{gen}$

Reversible work $\dot{W}_{rev} = \dot{m}(h_1 - h_{2s})$

Vapor and Combined Power Cycle.

Rankine Cycle $\rightarrow$ Working fluid = Water

for Carnot Vapor cycle $\rightarrow$ steam as working fluid is impractical.

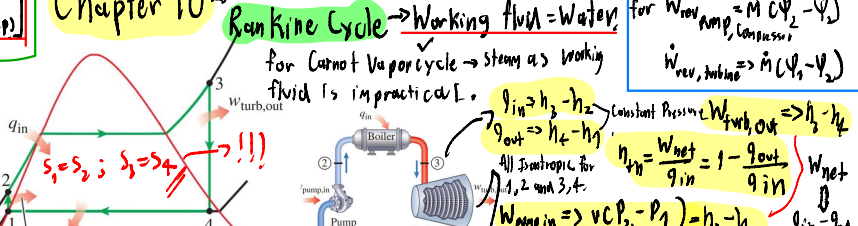
for Rankine cycle $\rightarrow$ steam as working fluid is impractical.

To increase thermal efficiency

1) Lower the condenser pressure.

2) Super heating the steam to high temp.

3) increase the boiler pressure.



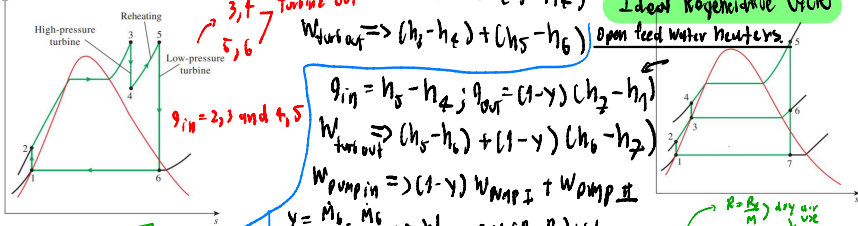
Ideal Reheat Rankine Cycle.

$q_{in} = (h_3 - h_2) + (h_5 - h_4)$

$W_{turb, out} = (h_3 - h_4) + (h_5 - h_6)$

$W_{pump, in} = (1 - y) W_{pump, I} + y W_{pump, II}$

$y = \frac{m_2}{m_1} \rightarrow W_{pump, I} = v_1(p_2 - p_1); W_{pump, II} = v_2(p_4 - p_2)$



Chapter 1-7. Ideal gas eq.

Energy for Control Volume

Steady flow $\rightarrow \sum \dot{m}_i h_i = \sum \dot{m}_e h_e$

for control volume with heat and work $\dot{Q} + \dot{W} = \dot{m}(h_{in} + \frac{V^2}{2} + gz) - \dot{m}(h_{out} + \frac{V^2}{2} + gz)$

for unsteady $\dot{Q} + \dot{W} + \sum \dot{m}_i h_i - \sum \dot{m}_e h_e = \frac{dU_{cv}}{dt}$

Carnot Engine $\eta_{Carnot} = \frac{W_{net}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$

For Turbine $\eta_{Carnot} = \frac{1}{\frac{T_H}{T_L} - 1}$

Exergy balance $\Delta E = \sum \dot{Q} \left(1 - \frac{T_0}{T} \right) + \sum \dot{m} (e_{in} - e_{out})$

Chapter 13. Relation of P_i, V_i to y_i

$\frac{P_i}{P_m} = \frac{V_i}{V_m} = \frac{N_i}{N_m} = y_i$

Dalton's law of additive pressures: The pressure of a gas mixture is equal to the sum of the pressures each gas would exert if it existed alone at the mixture temperature and pressure.

Amagat's law of additive volumes: The volume of a gas mixture is equal to the sum of the volumes each gas would occupy if it existed alone at the mixture temperature and pressure.

Important Point: Internal energy and enthalpy depend on Temp.

Chapter 14: Air in the atmosphere usually contains water vapor referred to as atmospheric air. Air that contains no water vapor is called dry air. If water vapor can be treated as ideal gas, it is isobaric compressed then $w_1 = w_2 \rightarrow$ For heat pump, the ratio of heat supply to the room is Q_H meaning $1m^3 = 1000L$

Absolute or specific humidity $\Rightarrow W = \frac{M_v}{M_a} \rightarrow$ dry air $W = 0.622 \frac{P_v}{P - P_v}$
Saturated air $\rightarrow$ Air cannot hold more moisture
Relative humidity $\Rightarrow \phi = \frac{P_v}{P_g} = \frac{P_v}{P_{sat}(T)}$
 $\phi = \frac{W P}{(0.622 + W) P}$ and $W = \frac{0.622 P_g}{P - P_g}$
Even if absolute humidity remain the same, relative humidity can change as it at the saturation temp.

Enthalpy of Atmospheric air $\rightarrow h = h_a + W h_g$
dry-bulb temp $\rightarrow$ Ordinary temp of atmospheric air.
Dew-point temp $\rightarrow$ Temp which condensation begin when the air is cooled at a constant pressure.
 $\omega_1 = \frac{h_{g1} - h_{f1}}{h_{g1} - h_{f2}}$
 $\omega_2 = \frac{P_2 - P_{s2}}{P_2 - P_{s1}}$
 $P_4 = P_{s2}$

Actual humidity ratio of incoming air $\rightarrow$ Humidity ratio of saturated air at the wet bulb.
Wet-bulb temp $\rightarrow$ A thermometer whose bulb is covered with a cotton wick saturated with water.

Chapter 15: In combustion $\rightarrow$ dry air $\Rightarrow$ 21% oxygen and 79% nitrogen.
Reactant $\rightarrow$ monovalent; **Product** $\rightarrow$ multivalent.
Air-fuel ratio $\rightarrow \frac{M_a}{M_f}$
Complete if Carbon in the fuel burn to CO_2 and all hydrogen burns to H_2O .
 $\rightarrow$ denote as standard reference state $\rightarrow 25^\circ C$ and 1 atm.
Enthalpy of Reaction $\rightarrow h_R = h_P - h_{R,ref}$
Enthalpy of formation h_f° $\rightarrow$ Chemical energy at some reference state.
Heating Value $\rightarrow$ The amount of heat released when a fuel is burned completely.
First Law analysis $\rightarrow$ Steady flow system $\rightarrow$ Enthalpy $\Rightarrow \dot{h}_p^\circ - (\dot{h}_R^\circ - \dot{h}_O^\circ)$

for Combustion Chamber $\rightarrow Q_{in} - W_{in} + \sum N_r (\dot{h}_f^\circ + (\dot{h} - \dot{h}^\circ)_r) = Q_{out} - W_{out} + \sum N_p (\dot{h}_f^\circ + (\dot{h} - \dot{h}^\circ)_p)$
Adiabatic Flame Temp $\rightarrow$ when there's no heat loss to the surrounding the temp of product reaches a maximum.
 $\sum N_p (\dot{h}_f^\circ + (\dot{h} - \dot{h}^\circ)_p) = \sum N_r (\dot{h}_f^\circ + (\dot{h} - \dot{h}^\circ)_r)$
Close or steady-flow system $\rightarrow \frac{Q_{in}}{T_{in}} + S_{gen} = S_{out} - S_{react}$
Second Law Analysis $\rightarrow$ Exergy destroyed $X_{destroyed} = T_0 S_{gen}$
Energy Analysis $\rightarrow \dot{E}_{in} - \dot{E}_{out} = \dot{A}E \rightarrow Q + W = \Delta U + \Delta PE + \Delta KE$

!!! Average Molar mass of Air is equal to 29 kg/kmol
air $\rightarrow O_2 + 3.76 N_2$ Also for entropy $S_i = N_i \bar{s}_i(T, P_i) = N_i (\bar{s}_i^\circ(T, P_0) - R \ln(y_i P_m))$
1 atm $\Rightarrow$ 101.325 kPa final answer if want for unit mass then divide by Molar mass $\frac{KJ}{kg}$

In Chemical Reaction it's irreversible hence the total entropy change $\rightarrow$ entropy generation
14-9C Consider a tank that contains moist air at 3 atm and whose walls are permeable to water vapor. The surrounding air at 1 atm pressure also contains some moisture. Is it possible for the water vapor to flow into the tank from surroundings? Explain. $\rightarrow$ Yes, if the vapor pressure in the surrounding is greater than the vapor pressure in the tank.
14-18 Humid air at 100 kPa, $20^\circ C$, and 90 percent relative humidity is compressed in a steady-flow, isentropic compressor to 800 kPa. What is the relative humidity of the air at the compressor outlet?
 $P_{v1} = \phi P_{s1} = \phi P_{sat}(20^\circ C) = (0.90)(2.3392 \text{ kPa}) = 2.105 \text{ kPa}$
 $\omega_2 = \omega_1 = \frac{0.622 P_{v1}}{P - P_{v1}} = \frac{(0.622)(2.105 \text{ kPa})}{(100 - 2.105) \text{ kPa}} = 0.0134 \text{ kg H}_2\text{O/kg dry air}$

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 $\omega_2 = \omega_1 = \frac{0.622 P_{v1}}{P - P_{v1}} = \frac{(0.622)(2.105 \text{ kPa})}{(100 - 2.105) \text{ kPa}} = 0.0134 \text{ kg H}_2\text{O/kg dry air}$

14-19C Consider a tank that contains moist air at 3 atm and whose walls are permeable to water vapor. The surrounding air at 1 atm pressure also contains some moisture. Is it possible for the water vapor to flow into the tank from surroundings? Explain. $\rightarrow$ Yes, if the vapor pressure in the surrounding is greater than the vapor pressure in the tank.
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Rate of Degree Subcool $\rightarrow \Delta T_{subcool} = T_{sat} - T_{actual}$ (water = 1.18 $\frac{K}{kg \cdot K}$)
2nd Law of Thermodynamics $\rightarrow$ The entropy of the universe never decrease.
 $\eta = 1 - \frac{T_c}{T_h}$ The increase in Work produce will equal to the work consume by devil.
6-30C A refrigerator, heat transfer from a lower-temperature medium (the refrigerated space) to a higher-temperature medium (the kitchen air). Is this a violation of the second law of thermodynamics? Explain.
6-31C A heat pump is a device that absorbs energy from the cold outdoor air and transfers it to the warmer indoors. Is this a violation of the second law of thermodynamics? Explain.
6-32C Define the coefficient of performance of a refrigerator in words. Can it be greater than unity?
6-33C Define the coefficient of performance of a heat pump in words. Can it be greater than unity?
6-34C A heat pump is used to heat a house has a COP of 2.5. That is, the heat pump delivers 2.5 kW of energy to the house for each 1 kWh of electricity it consumes. Is this a violation of the first law of thermodynamics? Explain.
6-35C What is the Clausius expression of the second law of thermodynamics?

6-36C A refrigerator has a COP of 1.5. That is, the refrigerator removes 1.5 kWh of energy from the refrigerated space for each 1 kWh of electricity it consumes. Is this a violation of the first law of thermodynamics? Explain.
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Heat pump using refrigerant R-134a maintains a house at $20^\circ C$ in cold winter. The cold outdoor air at $-10^\circ C$ infiltrates into the house through cracks of windows, doors and walls while the warm air inside leaves the house at a rate of 0.06 kg/s. The house is also losing heat at a rate of 20 kW through conduction. The refrigerant enters the compressor of the heat pump at 280 kPa and $0^\circ C$, and leaves at 0.8 MPa, at a rate of 0.11 kg/s. The isentropic efficiency of the compressor is 66%. A heat exchanger is used to transfer the energy from nearby lake water to the heat pump. The water temperature decreases by $3^\circ C$ as it flows through the heat exchanger. Assuming constant specific heats for air and water. (air: $C_p = 1.005 \text{ kJ/kg}\cdot K$; water: $C_p = 4.18 \text{ kJ/kg}\cdot K$). Determine
(a) the total heat supply to the house by this heat pump, (5 points)
(b) the power input to the heat pump, (5 points)
(c) the temperature at the exit of the condenser, (3 points)
(d) the minimum mass flow rate of lake water, (5 points)

Energy Balance
 $Q_{HP} + \dot{m}_{H_2O} h_{f,10} = Q_{loss} + \dot{m}_{H_2O} h_{f,20}$
 $Q_{HP} = Q_{loss} + \dot{m}_{H_2O} (h_{f,20} - h_{f,10})$
 $Q_{HP} = 20 \text{ kW} + (0.06 \text{ kg/s})(253.17 - 253.17) \text{ kJ/kg} = 20 \text{ kW}$

Process 1 $\rightarrow$ 280 kPa and $0^\circ C$
 $h_1 = 253.17 \text{ kJ/kg}$
 $s_1 = 0.9362 \text{ kJ/kg}\cdot K$
 $s_2 = s_1 = 0.9362 \text{ kJ/kg}\cdot K$
 $h_2 = 289.85 \text{ kJ/kg}$
 $s_2 = 0.9362 \text{ kJ/kg}\cdot K$

Process 2 $\rightarrow$ 0.8 MPa $\rightarrow$ $s_3 = s_2 = 0.9362 \text{ kJ/kg}\cdot K$
 $h_3 = 289.85 \text{ kJ/kg}$
 $s_3 = 0.9362 \text{ kJ/kg}\cdot K$
 $h_4 = 289.85 \text{ kJ/kg}$
 $s_4 = 0.9362 \text{ kJ/kg}\cdot K$

Process 3 $\rightarrow$ 280 kPa and $0^\circ C$
 $h_5 = 253.17 \text{ kJ/kg}$
 $s_5 = 0.9362 \text{ kJ/kg}\cdot K$
 $h_6 = 253.17 \text{ kJ/kg}$
 $s_6 = 0.9362 \text{ kJ/kg}\cdot K$

Process 4 $\rightarrow$ 280 kPa and $0^\circ C$
 $h_7 = 253.17 \text{ kJ/kg}$
 $s_7 = 0.9362 \text{ kJ/kg}\cdot K$
 $h_8 = 253.17 \text{ kJ/kg}$
 $s_8 = 0.9362 \text{ kJ/kg}\cdot K$

Process 5 $\rightarrow$ 280 kPa and $0^\circ C$
 $h_9 = 253.17 \text{ kJ/kg}$
 $s_9 = 0.9362 \text{ kJ/kg}\cdot K$
 $h_{10} = 253.17 \text{ kJ/kg}$
 $s_{10} = 0.9362 \text{ kJ/kg}\cdot K$

Process 6 $\rightarrow$ 280 kPa and $0^\circ C$
 $h_{11} = 253.17 \text{ kJ/kg}$
 $s_{11} = 0.9362 \text{ kJ/kg}\cdot K$
 $h_{12} = 253.17 \text{ kJ/kg}$
 $s_{12} = 0.9362 \text{ kJ/kg}\cdot K$

Process 7 $\rightarrow$ 280 kPa and $0^\circ C$
 $h_{13} = 253.17 \text{ kJ/kg}$
 $s_{13} = 0.9362 \text{ kJ/kg}\cdot K$
 $h_{14} = 253.17 \text{ kJ/kg}$
 $s_{14} = 0.9362 \text{ kJ/kg}\cdot K$

Process 8 $\rightarrow$ 280 kPa and $0^\circ C$
 $h_{15} = 253.17 \text{ kJ/kg}$
 $s_{15} = 0.9362 \text{ kJ/kg}\cdot K$
 $h_{16} = 253.17 \text{ kJ/kg}$
 $s_{16} = 0.9362 \text{ kJ/kg}\cdot K$

Process 9 $\rightarrow$ 280 kPa and $0^\circ C$
 $h_{17} = 253.17 \text{ kJ/kg}$
 $s_{17} = 0.9362 \text{ kJ/kg}\cdot K$
 $h_{18} = 253.17 \text{ kJ/kg}$
 $s_{18} = 0.9362 \text{ kJ/kg}\cdot K$