Lecture 1-2

Current ← $\boxed{i = \frac{dq}{dt}}$

Charge = $\boxed{Q = \int_{t_0}^{t} i\,dt}$

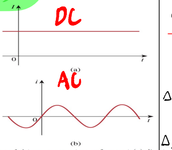


Figure 1.4 two common types of current (a) direct current (dc), (b) alternating current (ac).

Power → $p = \frac{dW}{dt} = \frac{dW}{dq} \cdot \frac{dq}{dt} = vi$

$P = +vi$ → absorbing power ; $P = -vi$ → Supplying power

!! $\Sigma p = 0$ The algebraic sum of power in a circuit, at any instant of time must be zero.

Energy → $w = \int_{t_0}^{t} p\,dt = \int_{t_0}^{t} vi\,dt$ → Capacity to do work Measured in Joule.

high resistance → lead to lower current

Short Vs Open Circuit

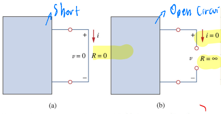


Figure 2.2 Two extreme possible values of R: (a) short circuit ($R = 0$), (b) open circuit ($R = \infty$).

Conductance → (S) or (℧)

$\boxed{G = \frac{1}{R}}$ → Resistance

Power dissipated by a resistor can be expressed in terms of R or G.

as $\boxed{p = vi = i^2R = \frac{v^2}{R}}$ $\boxed{p = vi = \frac{i^2}{G} = v^2G}$

ohms law → $V = iR$ → $i = \frac{V}{R}$

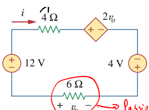
Fundamental theorem of network topology;

$\boxed{b = m + n - 1}$

branches mesh nodes

KCL: Note → Circuit Cannot Contain two different Current unless $i_1 = i_2$ ※

KVL



Ohm law

$4i + 2V_0 - 4 - V_0 - 12 = 0$

$V = iR \rightarrow -V_0 = iR \rightarrow i = \frac{-V_0}{6}$

!! → If the branch are parallel to each, then it have the Same Voltage.

Delta to Wye Conversion Each resistance in the Y network is the product of the resistances in the two adjacent Δ branches, divided by the sum of the three Δ resistances.

$$\begin{cases} R_1 = \dfrac{R_bR_c}{R_a + R_b + R_c} \\ R_2 = \dfrac{R_cR_a}{R_a + R_b + R_c} \\ R_3 = \dfrac{R_aR_b}{R_a + R_b + R_c} \end{cases}$$

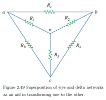


Figure 2.49 Superposition of wye and delta networks as an aid in transforming one to the other.

Wye to Delta Conversion Each resistance in the Δ network is the sum of all possible products of Y resistances taken two at a time, divided by the opposite Y resistance.

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

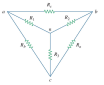
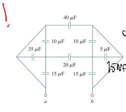


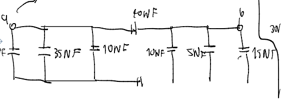
Figure 2.49 Superposition of wye and delta networks as an aid in transforming one to the other.

57

!!!



Can be converted as



Lecture 3-4

Solve Using Cramer Rule

$$\begin{bmatrix} 3 & -1 \\ -6 & 10 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 120 \end{bmatrix}$$

$\Delta = \begin{bmatrix} 3 & -1 \\ -6 & 10 \end{bmatrix} = 30 - 6 = 24$

$\Delta_1 = \begin{bmatrix} 20 & -1 \\ 120 & 10 \end{bmatrix} = 200 - (-120) = 320$

$\Delta_2 = \begin{bmatrix} 3 & 20 \\ -6 & 120 \end{bmatrix} = 360 + 120 = 480$

$V_2 = \frac{\Delta_2}{\Delta}$; $V_1 = \frac{\Delta_1}{\Delta}$

Example 3.8 Write the node-voltage matrix equations for the circuit in Fig. 3.27 by inspection.

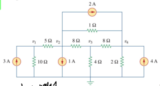
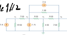


Figure 3.27

V_{11} → Conductance node 1

V_{12} → Conductance node 1//2 → negative



Solution :

$$\begin{bmatrix} \frac{1}{5}+\frac{1}{10} & -\frac{1}{5} & 0 & 0 \\ -\frac{1}{5} & \frac{1}{5}+\frac{1}{8}+1 & -\frac{1}{8} & -1 \\ 0 & -\frac{1}{8} & \frac{1}{4}+\frac{1}{8}+\frac{1}{8} & -\frac{1}{8} \\ 0 & -1 & -\frac{1}{8} & 1+\frac{1}{8}+\frac{1}{2} \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 3 \\ -1-2 \\ 0 \\ 2+4 \end{bmatrix}$$

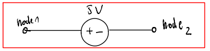
current to node 1 ยิงเข้า => positive

20

$V_{ij} = V_{ji}$ ※

Supernode

→ Voltage Source Connected between 2 node



Mesh Analysis → Use KVL

Can only be apply with planar circuit.

$\boxed{b = l + n - 1}$

branch, independent loop, node

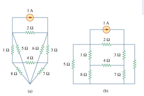


Figure 3.15 (a) A planar circuit with crossing branches, (b) the same circuit redrawn with no crossing branches.

Mesh Analysis by Inspection

Current between loop i,j

Voltage in loop = +V

$$\begin{bmatrix} 10+12 & -10 & -12 \\ -10 & 4+24+10 & -4 \\ -12 & -4 & 4+12 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \\ -4(i_1-i_2) \end{bmatrix}$$

Sum of Current in loop i

Example 3.6 Use mesh analysis to find the current I_o in the circuit of Fig. 3.20.

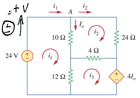
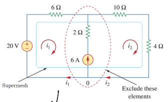


Figure 3.20

Super Mesh



exclude current source between 2 mesh

Nodal Versus Mesh Analysis

Nodal Analysis	Mesh Analysis
Contain many parallel-connected elements, current sources, or super-nodes	Contain many series-connected elements, voltage sources, or super-meshes
Fewer nodes than meshes	Fewer meshes than nodes
If node voltages are required	If branch or mesh currents are required

→ Nodal Analys by Inspection

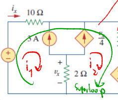
Can Apply when there is no voltage source.

Mesh Analysis by Inspection

Can Apply when there is no Current source.

Mesh Analysis ; We also know

$V_x = (i_1 - i_2) * 2$



additive example

Let's say:

- You give input x_1 → it gives output y_1
- You give input x_2 → it gives output y_2

Then we test:

- If we give input $x_1 + x_2$, do we get output $y_1 + y_2$?

If YES → ✅ Additive

If NO → ❌ Not additive

4.2 Linearity

Linear if both additive / homogeneous

Homogeneous

→ Linear Circuit where one output is linearly related to its input

=> If you multiply the input by a number, the output also multiplied by that same number. E.g., y=ax → a(kx)=k(ax)=ky

!!! Test homogeneity

If $V_s = 12V$ jump to $24V$ by a constant of 2. Then the I_0 also need to be multiply by 2 for $I_{0(V_s=12)}$ to $I_{0(V_s=24)}$ in order to prove homogeneous.

Cramer Rule with Consant

$$\begin{bmatrix} 12 & -4 \\ -10 & 16 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} -v_s \\ v_s \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 12 & -4 \\ -10 & 16 \end{vmatrix} = 152$$

$$\Delta_2 = \begin{vmatrix} 12 & -v_s \\ -10 & v_s \end{vmatrix} = 2v_s$$

$$I_o = i_2 = \frac{\Delta_2}{\Delta} = \frac{v_s}{76}$$

Superposition

- The principle of superposition helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately and adding algebraically all the contributions to find the total contribution.

Step → 1. Consider One independent Source at a time and close other independent source.

2, dependent source are left intact.

3, plus all the individual response to

Source Transform

- A source transformation is the process of replacing a voltage source in series with a resistor by a current source in parallel with a resistor, or vice versa.
- Source transformation is a tool for simplifying circuits.

Resistance Stay the Same.

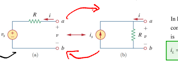


Figure 4.15 Transformation of independent sources.

$\boxed{v_s = i_sR}$ or $\boxed{i_s = \frac{v_s}{R}}$

Thevenin theorem

STEP

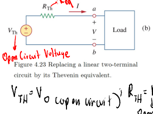


Figure 4.23 Replacing a linear two-terminal circuit by its Thevenin equivalent.

To apply the idea in finding the Thevenin resistance R_{Th}, we need to consider two cases.

1. If the network has no dependent sources, we turn off all independent sources. R_{Th} is the equivalent resistance looking between terminals a and b, as shown in Fig. 4.24 (b).
2. If the network has dependent sources, we turn off all independent sources. And we apply a voltage source v_o (or current source i_o) at terminals a and b and determine the resulting current i_o (or voltage v_o). Then $R_{Th} = v_o / i_o$, as shown in Fig. 4.25.

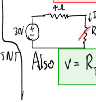
$R_{Th} = R_{eq}$ with all independent sources off

$V_{TH} = V_0$ (open circuit) ; $R_{TH} = R_{in}$

open circuit the wanted terminal

!!! For Thevenin/Norton → We replace the Voltage Source with short Circuit and Current Source with open circuit.

Independent



Find current through RL ?

Also $\boxed{v = R_{TH}i + V_{TH}}$

R_{TH} - negative → Supply Power

✓ Technique → Try to see if we can do source transformation.

In Thevenin ⇒ No Independent Source $\boxed{V_{TH} = 0}$

Norton

Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and R_N is the equivalent resistance at the terminals when the independent sources are turned off.

→ $\boxed{R_n = R_{TH}}$

$\boxed{I_N = \frac{V_{TH}}{R_{TH}}}$

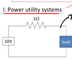
$\boxed{R_{TH} = \frac{V_{oc}}{I_{sc}}}$ → $I_N = I_{sc}$

Power Transfer

Circuit types	I. Power utility systems	II. Communication & instrumentation systems
Power scale	Large quantities of electric power (generation, etc.)	Small amount of power is being transferred
Primary concern of power	Efficiency of the power transfer	Transmit as much of power as possible to the load

Assuming the same source for easy comparison

I. Power utility systems



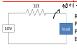
$10 + i + i(9) = 0$ → higher resistance load

$i = 1$

Care electricity bills

R_L=9Ω
Power generated from the source = 10W
Power delivered to the load = 9W
Efficiency of power transfer 90% (loss=10%)

II. Communication & instrumentation systems → lower resistance



$10 + i + i = 0$

$i = 5$

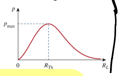
Care supplying sufficient power

R_L=1Ω
Power generated from the source = 50W
Power delivered to the load = 25W
Efficiency of power transfer 50% (loss=50%)

The power delivered to the load is

$$p = i^2R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

as sketched in Fig. 4.49.



Max Power Theorem

Max Power is transfer to the load when $R_{TH} = R_L$.

The maximum power transferred

$$p_{max} = \frac{V_{Th}^2}{4R_{Th}}$$

$P_{max} = \frac{V_{TH}^2}{4R_{TH}}$

It should be noted that delivering maximum power to the ... significant internal losse

Ideal Voltage Source

Source Modeling (Section 4.10)

A practical voltage source is modeled by an ideal voltage source v_s in series with an internal (or a source) resistance R_s, a practical current source is modeled by an ideal current source i_s in parallel with an internal resistance R_p, as shown in Fig. 4.58.

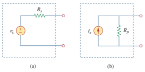
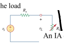


Figure 4.58 (a) Practical voltage source. (b) practical current source.

In Fig. 4.59(a), a practical voltage source is connected to a variable load. The load voltage is

$$\boxed{v_L = \frac{R_L}{R_s + R_L}v_s}$$

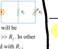


This equation tells us that (1) v_L will be constant if $R_s = 0$ or at least $R_s << R_L$. In other words, the smaller R_s is compared with R_L, the closer the voltage source is to being ideal.

(2) When the voltage source is open-circuited, $\boxed{v_{oc} = v_s}$. Thus, v_s may be regarded as the *unloaded source voltage*. The connection of R_L causes the terminal voltage to drop in magnitude; this is known as the *loading effect*.

In Fig. 4.60(a), a practical current source is connected to a variable load. The load current is

$$\boxed{i_L = \frac{R_p}{R_p + R_L}i_s}$$



This equation tells us that (1) i_L will be constant if $R_p = \infty$ or at least $R_p >> R_L$. In other words, the larger R_p is compared with R_L, the closer the current source is to being ideal.

(2) When the current source is short-circuited, $\boxed{i_{sc} = i_s}$. Thus, i_s may be regarded as the *unloaded source current*. The connection of R_L causes the terminal current to drop in magnitude; this is known as the *loading effect*.

First, we measure the open-circuit voltage v_{oc} and set

$v_s = v_{oc}$

Then, we connect a variable load R_L across the terminals. We adjust R_L until $v_L = v_{oc} / 2$. At this point, we disconnect R_L and measure it. We set

$R_s = R_L$

Example 4.16 The terminal voltage of a voltage source is 12 V when connected to a 2-W load. When the load is disconnected, the terminal voltage rises to 12.4 V. (a) Calculate the source voltage v_s and i... resistance R_s. (b) determine the volta... an 8-Ω load is connected to the sour... → V_s

Solution :

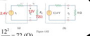


Figure 4.62

(a) $v_s = v_{oc} = 12.4$ V

$p_L = \frac{v_L^2}{R_L} \Rightarrow R_L = \frac{v_L^2}{p_L} = \frac{12^2}{2} = 72\ (\Omega)$

from $p = i^2R_L$, $i = \frac{V}{R_L}$, $P = \frac{V^2}{R_L^2} \times R_L$, $P = \frac{V_L^2}{R_L}$

$v_L = v_s\frac{R_L}{R_s + R_L} \Rightarrow R_s = R_L\left(\frac{v_s}{v_L} - 1\right) = 72 \times \left(\frac{12.4}{12} - 1\right) = 2.4\ (\Omega)$

(b) $v_L = v_s\frac{R_L}{...} = 12.4 \times \frac{8}{...} \approx 9.54$ (V)

Lecture 5

Op-Amp → behave like a Voltage-Controlled Voltage Source. Multiply ; Addition ; Subtraction.

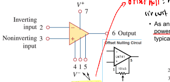
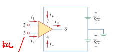


Figure 5.2(b) A typical op amp: circuit symbol.

offset null : null out any error in the circuit

- As an active element, the op amp must be powered by one or two voltage supplies as typically shown in Fig. 5.3.



KCL ↓ $i_0 = i_1 + i_2 + i_+ + i_-$

→ In Op Amp → magnitude of output Voltage cannot exceed $|V_{cc}|$ → Supply Voltage.

5.4 Inverting Amplifier (Inverter)

An inverting amplifier reverses the polarity of the input signal while amplifying it. The *closed - loop gain* of the inverting amplifier is

$$\boxed{A_v = \frac{v_o}{v_i} = -\frac{R_f}{R_i}}$$

or $V_o = -V_i\left(\frac{R_f}{R_i}\right)$

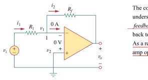
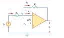


Figure 5.10 the inverting amplifier.

The concept of *feedback* is crucial to our understanding of op amp circuits. A *negative feedback* is achieved when the output is fed back to the inverting terminal of the op amp. As a result of the negative feedback, the op amp operates in the linear region.



5.5 Noninverting Amplifier

The noninverting amplifier is a circ... designed to provide positive voltage... The closed-loop gain for the noninverting amplifier is

$$\boxed{A_v = \frac{v_o}{v_i} = 1 + \frac{R_f}{R_1}}$$

→ $V_0 = V_i\left(1 + \frac{R_f}{R_1}\right)$

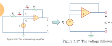
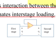


Figure 5.17 The voltage follower

portrayed in Fig.5.18. The voltage follower minimizes interaction between the two stages and eliminates interstage loading.



5.6 Summing Amplifier (Ac

A summing amplifier is an op amp ... that combines more than one inpu... produces an output that is the wei... sum of the inputs, with all weights ... the same sign. For example, in Fig

$$\boxed{v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right)}$$

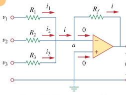


Figure 5.21 The summing amplifier.

5.7 Difference Amplifier (Subtractor)

A difference (or differential) amplifier is an op amp circuit that combines two inputs and produces an output that is a weighted sum of the two inputs, with the two weights having different signs. For example, in Fig. 5.24,

$$\boxed{v_o = (1 + R_2/R_1)\frac{R_4/R_3}{1 + R_4/R_3}v_2 - \frac{R_2}{R_1}v_1}$$

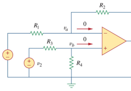


Figure 5.24 Difference amplifier.

Cascaded OP-Amp Circuit

Figure 4.28 displays the block diagram of a three-stage cascaded op amp circuits. The overall gain is the product of the gains of the individual stages:

→ arrange of two or more Op amp Circuit. The output of 1 is the input of the next.

→ each circuit in the string is called a stage

$$\boxed{A = \frac{v_o}{v_1} = \frac{v_2}{v_1} \cdot \frac{v_3}{v_2} \cdot \frac{v_o}{v_3} = A_1A_2A_3}$$

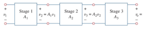


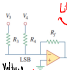
Figure 5.28 A three-stage cascaded connection.

Digital to Analog Converter

Example 5.12

In the op amp circuit of Fig. 5.36(b), let R_f = 10 kΩ, R_1 = 10 ... R_2 = 20 kΩ, R_3 = 40 kΩ, and R_4 = 80 kΩ. Obtain the analog out... for binary inputs [0000], [0001], [0010],…, [1111].

Like Summing Amplifier



Voltage Output is V_o

$$\boxed{-V_o = \frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3 + \frac{R_f}{R_4}V_4}$$

$= V_1 + 0.5V_2 + 0.25V_3 + 0.125V_4$

2. digital input $[V_1V_2V_3V_4]$ = [0000] produces an analog output of $-V_o$ = 0 V

[0001] → $-V_o$ = 0.125 V

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Instrummtation Amp

An IA circuit is shown in Fig. 5.26. Show that

$$v_o = \frac{R_2}{R_1}\left(1 + \frac{2R_3}{R_4}\right)(v_2 - v_1)$$

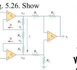


Figure 5.26 Instrumentation amplifier.

Proof : $V_a = V_1$; $V_b = V_2$

$$\begin{cases} v_a = v_1, v_b = v_2 \\ i = \dfrac{v_a - v_b}{R_4} = \dfrac{v_{o1} - v_{o2}}{R_3 + R_4 + R_3} \end{cases}$$

$V_2 - V_0 = \left(1 + \frac{2R_3}{R_4}\right)V_2 - V_1$

$V_0 = \frac{R_2}{R_1}(V_{o2} - V_{o1})$

$= \frac{R_2}{R_1}\left(1 + \frac{2R_3}{R_4}\right)(V_2 - V_1)$ ※

Lecture 6

Capacitor / Inductors → Storage Element.; Capacity → Store energy Electric field

$\boxed{q = Cv}$ ← charge

If $V \cdot i > 0$ → Capacitor charge

If $V \cdot i < 0$ → Capacitor discharging

$\boxed{i = C\frac{dv}{dt}}$

$$\boxed{v = \frac{1}{C}\int_{-\infty}^{t} i\,dt = \frac{1}{C}\int_{t_0}^{t} i\,dt + v(t_0)}$$

where $v(t_0) = \frac{1}{C}\int_{-\infty}^{t_0} i\,dt = \frac{q(t_0)}{C}$ is the

Instantaneous Power

$$\boxed{p = vi = v\left(C\frac{dv}{dt}\right)}$$

Energy Store in C

$w = \int_{-\infty}^{t} p\,dt = \int_{-\infty}^{t} v\left(C\frac{dv}{dt}\right)dt = C\int_{v(-\infty)}^{v} v\,dv = \frac{1}{2}Cv^2\Big|_{v(-\infty)}^{v}$ $\boxed{= \frac{1}{2}Cv^2}$ → energy

$v(-\infty) = 0$ → Voltage in Capacitor

→ When it is a DC Voltage, Current through Capacitor = 0.

Ceq Equivalence

Capacitor in Parallel ⇒ $C_{eq} = C_1 + C_2 + C_n$

Capacitor In Series = $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_n}$.

Inductor

$\boxed{V = L\frac{di}{dt}}$ for inductor

Magnetic Flux $\boxed{\Psi = Li}$

The current-voltage relation of the ... is

$$\boxed{i = \frac{1}{L}\int_{-\infty}^{t} v\,dt = \frac{1}{L}\int_{t_0}^{t} v\,dt + i(t_0)}$$

where $i(t_0) = \frac{1}{L}\int_{-\infty}^{t_0} v\,dt = \frac{\psi(t_0)}{L}$ is the current through the inductor at tim...

instantaneous Power

$$\boxed{p = vi = \left(L\frac{di}{dt}\right)i}$$

Energy $\boxed{W = \frac{1}{2}Li^2}$

If the Current across the inductor → is DC then Hence the Voltage across the inductor = 0

for $L_{eq} = R_{eq}$ > Work the same way

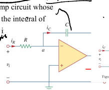
Integrator Circuit

An *integrator* is an op amp circuit whose output is proportional to the integral of the input. For the circuit in the figure

$$v_o = -\frac{1}{RC}\int_0^t v_i\,dt + v_o(0)$$

If $v_o(0) = 0$, then

$$\boxed{v_o = -\frac{1}{RC}\int_0^t v_i\,dt}$$



Differentiator → Op Amp Circuit where output is proportional to the derivative of input

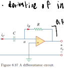


Figure 6.37 A differentiator circuit.

$\boxed{V_0 = -R_C\frac{dV_i}{dt}}$

Lecture 7.

Source free RC Circuit

at time $t=0$; $v(0) = V_0$

energy store $= W_c(0) = \frac{1}{2}CV_0^2$

Circuit Response $\rightarrow i_C + i_R = 0$

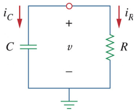


Figure 7.1 A source-free RC circuit. 8

$i_C = C\frac{dv}{dt}$; $i_R = \frac{V}{R}$ $\rightarrow$ give $\boxed{C\frac{dv}{dt} + \frac{V}{R} = 0}$ First Order dif eq. for RC circuit

or $\frac{dv}{dt} + \frac{V}{CR} = 0$

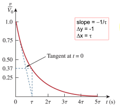


Figure 7.3 Graphical determination of the time constant from the response curve.

Response

$\boxed{v = V_0 e^{-\frac{t}{T}}}$ $\rightarrow$ $T = RC$

$\boxed{i_R = \frac{V_0}{R}e^{-\frac{t}{T}}}$

$\boxed{P = Vi_R = \frac{V_0^2}{R}e^{-\frac{2t}{T}}}$

Energy

$\boxed{W = \frac{1}{2}CV_0^2(1-e^{-\frac{2t}{T}})}$

STEP

1.) find $v(0) = V_0$ across capacitor.

2.) The time constant $T = RC$ where R is R_{Th}) and use $v = V_0 e^{-\frac{t}{T}}$

First Order Op-Amp Circuit

Find V_0 for $t>0$

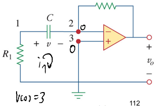


Figure 7.55(a)

$V(0) = 3$

$R_f = 80k$, $R_1 = 20\Omega$; $C = 5nF$

KCL node 2.

$-C\frac{dv}{dt} + \frac{0-V_0}{R_f} = 0$

$C\frac{dV}{dt} = -\frac{V_0}{R_f} \rightarrow (1)$

KVL at loop 1.

$R_1 i_1 + V = 0$

$R_1(C\frac{dv}{dt}) + V = 0$

$\frac{dv}{dt} + \frac{1}{RC}V = 0$ Same response as RC free source circuit

Hence $V = V_0 e^{-\frac{t}{T}} \rightarrow T = R_1 \times C$ $\Rightarrow 20k \times 5nF = 0.1s$

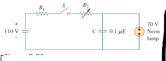
$V(t) = 3e^{-10t}$ $\rightarrow$ To find V_0 from (1)

Plug in $V_0 = -CR_f(\frac{d}{dt}3e^{-10t})$

$V_0 = 12e^{-10t}$

Delay Circuit

Example 7.19 Consider the circuit in Fig. 7.73, and assume that $R_1 = 1.5$ MΩ, $0 < R_2 < 2.5$ MΩ. (a) Calculate the extreme limits of the time constant of the circuit. (b) How long does it take for the lamp to glow for the first time after the switch is closed? Let R_2 assume its largest value.



(b) $v_C(0^+) = 0$, $v_C(\infty) = 110$ V, $\tau = 0.4$ s

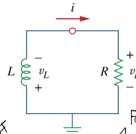
$$v_C(t) = v_C(\infty) + \left[v_C(0^+) - v_C(\infty)\right]e^{-t/\tau}$$

$$t = \tau \ln\left(\frac{v_C(0^+) - v_C(\infty)}{v_C(t) - v_C(\infty)}\right)$$

$$= 0.4\ln\left(\frac{0-110}{70-110}\right) \approx 0.4046 \text{ (s)}$$

The lamp glows after the switch is closed for 0.4046 secomds.

Source Free RL Circuit



The Inductor has initial Current

$i(0) = I(0) \rightarrow$ Energy $\Rightarrow W_L(0) = \frac{1}{2}LI_0^2$ #

$V_L + V_R = 0 \rightarrow V_R = iR \rightarrow L\frac{di}{dt} + iR = 0$ Hence $\boxed{\frac{di}{dt} + \frac{R}{L}i = 0}$ First Order dif eq for RL circuit

Response $\rightarrow \boxed{i = I_0 e^{-\frac{t}{T}}}$ $\rightarrow T = \frac{L}{R}$

with this we can find resistor Voltage.

$\boxed{V_R = I_0 R e^{-\frac{t}{T}}} \rightarrow \boxed{P = V_R i = I_0^2 R e^{-\frac{2t}{T}}}$

$\boxed{W_R \Rightarrow \frac{1}{2}LI_0^2(1-e^{-\frac{2t}{T}})}$

STEP

Like RC Circuit but find $i(0) = I_0$ Instead of $V(0)$.

!! look at sign for i_0 and V_0

Unit DipShit !!

$\boxed{\delta(t) = \frac{d}{dt}u(t)}$; $\boxed{u(t) = \int_{-\infty}^{t}\delta(T)\,dt}$

impulse

Unit step function

$\Rightarrow \boxed{f(t_0)\int_{-\infty}^{\infty}\delta(t-t_0)\,dt = f(t_0)}$

Unit impulse function.

$\boxed{r(t) = \int_{-\infty}^{t}u(T)\,dT = tu(t)}$

$$\begin{tikzcd} \delta(t) \arrow[r, shift left, "\int\sim dt"] & u(t) \arrow[l, shift left, "d/dt"] \arrow[r, shift left, "\int\sim dt"] & r(t) \arrow[l, shift left, "d/dt"] \end{tikzcd}$$

An RC Circuit with Step input.

Initial Condition $\rightarrow v(0^+) = v(0^-) = V_0$; Start with $(C\frac{dv}{dt})R + v = V_s$

first Order response

$\boxed{\frac{dv}{dt} + \frac{1}{T}v = \frac{V_s}{T}}$ $T = RC$

The Complete response for Method 2.

$\boxed{V(t) = V_n(t) + V_f(t) = Ae^{-\frac{t}{T}} + V_s}$

$A = V_0 - V_s$

Complete / final response $\rightarrow v(t) = \boxed{V(\infty) + [V(0^+) - V(\infty)]e^{-\frac{t}{T}}}$

STep 1. find initial Capacitor Voltage $V(0^+)$

2. Final capacitor voltage $V(\infty)$; 3. The time constant $T = RC$.

Source free RC circuit is when $V(\infty) = 0$.

An RL Circuit with Step input

Initial Condition $i(0^+) = i(0^-) = I_0$ for $t>0 \rightarrow$ First Order Step response

$\boxed{\frac{di}{dt} + \frac{1}{T}i = \frac{V_s}{RT}}$ where $T = \frac{L}{R}$

$i(t) = \frac{V_s}{R} + (I_0 - \frac{V_s}{R})e^{-\frac{t}{T}}$, $i(\infty) = \frac{V_s}{R}$

Complete response $\boxed{i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-\frac{t}{T}}}$

Step $\rightarrow$ The same ; but need to find initial inductor Current first $i(0^+)$ instead of $V(0^+)$

Lecture 8.

$\rightarrow$ To find initial Value for $\frac{di(0^+)}{dt}$, $\frac{dv(0^+)}{dt}$; Use either KCL or KVL . Then Apply the properties of $i = C\frac{dv}{dt}$ or $V = L\frac{di}{dt}$ to Solve for it.

Source free series RLC circuit.

$\rightarrow$ The circuit is being excited by the energy stored in the capacitor and inductor. at $t = 0$; $V(0) = V_0$; $i(0) = I_0$

Voltage across Capacitor at the moment after switching

Current through Inductor at the moment right after switching.

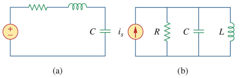


Figure 8.1 Typical examples of second-order circuits: (a) series RLC circuit, (b) parallel RLC circuit.

Apply KVL, We get $i(0^+) = i(0^-) = I_0$ and

$i'(0^+) = -\frac{1}{L}(I_0 R + V_0)$ Solve $\boxed{S = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}}$

eq response $\Rightarrow \boxed{\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0}$ $\boxed{\alpha = \frac{R}{2L}}$ $\rightarrow$ neper frequency

$\boxed{W_0 = \frac{1}{\sqrt{LC}}}$ $\rightarrow$ Undamp natural frequency

Solution $\rightarrow$ 1. Overdamped If $\boxed{\alpha > W_0}$ $\boxed{S_{1,2} \Rightarrow -\alpha \pm \sqrt{\alpha^2 - W_0^2}}$

Hence We have $\boxed{i(t) \Rightarrow A_1 e^{s_1 t} + A_2 e^{s_2 t}}$

where $A_1 \Rightarrow \frac{i'(0^+) - s_2 i(0^+)}{s_1 - s_2}$; $A_2 \Rightarrow \frac{s_1 i(0^+) - i'(0^+)}{s_1 - s_2}$

2. Critically Damped.

If $\boxed{\alpha = W_0}$; $\boxed{S_1 = S_2 = -\alpha}$

$\boxed{i(t) \Rightarrow (B_1 t + B_2)e^{-\alpha t}}$ $\rightarrow$ Where $B_1 = i'(0^+) + \alpha i(0^+)$; $B_2 = i(0^+)$ } $s_1 < 0, s_2 < 0$; $s_1 = s_2$

3. Under Damped $\rightarrow$ If $\boxed{\alpha < W_0}$, $\boxed{S_{1,2} = -\alpha \pm iW_d}$ $\boxed{W_d = \sqrt{W_0^2 - \alpha^2}}$ Damping frequency

$\boxed{i(t) \Rightarrow e^{-\alpha t}(C_1 \cos W_d t + C_2 \sin W_d t)}$ $\rightarrow C_1 = i(0^+)$; $C_2 \Rightarrow \frac{i'(0^+) + \alpha i(0^+)}{W_d}$

Once $i(t)$ is found $\Rightarrow$ other circuit quantities can be found such as $V_R(t) = i(t)R$

$V_L(t) = L\frac{di(t)}{dt}$; $V_c(t) = \frac{1}{C}\int_0^t i(t)\,dt + V_c(0)$ } i.e. $\Rightarrow$ Voltage across Resistor, Capacitor, Inductor

Source free Parallel RLC Circuit

$\rightarrow$ Same ; Energy initially stored at $t = 0$

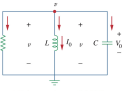


Figure 8.13 A source-free parallel RLC circuit.

Do KCL. $\rightarrow \frac{V}{R} + i + C\frac{dv}{dt}$

$V(0) = V_0$, $i(0) = I_0$ $\boxed{S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - W_0^2}}$

$\boxed{\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = 0}$ $V'(0^+) \Rightarrow -\frac{1}{C}\left(\frac{V(0^+)}{R} + i(0^+)\right) \rightarrow -\frac{1}{C}(\frac{V_0}{R} + I_0)$

!!! $\boxed{\alpha = \frac{1}{2RC}}$; $\boxed{W_0 = \frac{1}{\sqrt{LC}}}$

Solution

Same as Source free Series RLC circuit just change from $i(t)$ to $v(t)$

once the Capacitor voltage $v(t)$ is found other quantities can also be found $\rightarrow$ $i_R(t) = \frac{v(t)}{R}$; $i_L(t) = \frac{1}{L}\int_0^t v(t)\,dt + i_L(0)$; $i_C(t) \Rightarrow C\frac{dv(t)}{dt}$.

Series RLC circuit with Step input.

When $t>0$; We can see that $V_s = iR + L\frac{di}{dt} + v$; $i = C\frac{dv}{dt}$

Second Order Response $\rightarrow \boxed{\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{1}{LC}v = \frac{1}{LC}V_s}$

Solution $\rightarrow$ 1. Overdamped $\Rightarrow \boxed{v(t) \Rightarrow A_1 e^{s_1 t} + A_2 e^{s_2 t} + V_s}$

2.) Critically damped $\rightarrow \boxed{v(t) \Rightarrow (A_1 + A_2 t)e^{-\alpha t} + V_s}$

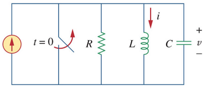
3.) Under damped $\Rightarrow \boxed{v(t) \Rightarrow e^{-\alpha t}(A_1 \cos W_d t \; A_2 \sin W_d t) + V_s}$

Pararell RLC Circuit with step input

$\boxed{\frac{d^2i}{dt^2} + \frac{1}{RC}\frac{di}{dt} + \frac{1}{LC}i = \frac{1}{LC}I_s}$

$\rightarrow$ We want to find i due to sudden Application of a DC current. $I_s = \frac{V}{R} + i + C\frac{dv}{dt}$; $V = L\frac{di}{dt}$

Use as $i(t)$



Solution $\rightarrow$ Same as RLC Series but change from $+V_s$ to $+I_s$!!!

General Second Order Circuit

for Series RLC step input we know $\rightarrow$ $\boxed{V(t) = V_t(t) + V_{ss}(t)}$

transient

steady state

for Pararell RLC circuit Step input $\boxed{i(t) = i_t(t) + i_{ss}(t)}$

natural response $\rightarrow V_n(t) = Ae^{s_1 t} + Be^{s_2 t}$

When counter RCC - RLL circuit or one that have Combination of pararell / series

Duality

$\rightarrow$ Dual if they are described by the same characteristic equation with dual pair interchange.

STEP $\rightarrow$ 1.) Place a Node at the center of each mesh of the given circuit, then place the reference node of the dual circuit outside the given circuit.

2.) Draw lines between nodes such that each line across an element, Replace the element by its dual.

3. To determine the polarity of voltage sources and direction of current sources, follow this rule: A voltage source that produces a positive (clockwise) mesh current has as its dual a current source whose reference direction is from the ground to the nonreference node. !!!

For $a\frac{d^2x}{dt^2} + b\frac{dx}{dt} + cx = 0$ (homogeneous),

- No root:

$$r = -\frac{b}{2a}, \omega = \frac{\sqrt{4ac - b^2}}{2a}$$

$$x(t) = e^{rt}(C_1 \sin \omega t + C_2 \cos \omega t)$$

- One root s:

$$x(t) = (C_1 + C_2 t)e^{st}$$

- To roots s_1, s_2:

$$x(t) = C_1 e^{s_1 t} + C_2 e^{s_2 t}$$

For $a\frac{d^2x}{dt^2} + b\frac{dx}{dt} + cx = X_0$ (non-homogeneous), adopt

$$x(t) = x_{homogeneous}(t) + x_{particular}(t)$$

TABLE 8.1 Dual Pairs

Resistance	Conductance
Inductance	Capacitance
Voltage	Current
Voltage source	Current source
Node	Mesh
Series path	Parallel path
Open circuit	Short circuit
KVL	KCL
Thevenin	Norton

Underdamped	Critically damped	Overdamped
$b^2 - 4ac < 0$ No root	$b^2 - 4ac = 0$ One root s	$b^2 - 4ac > 0$ Two roots s_1, s_2

Chapter 9.

(Time domain represent) → Can be sine or cosine function

→ Sinusoid signal → $V(t) = V_m \sin(wt + \phi)$ + φ เลื่อนไปทางซ้าย → -φ เลื่อนไปทางขวา

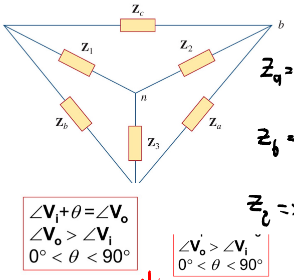
V_m = Amplitude ; W: Angular frequency ; φ = initial phase.

Phasor form → $\tilde{V} = V_m e^{j\phi} = V_m \angle\phi$ // Admittance Y(s) => $Y = \frac{1}{Z}$ / Impendance

Inpedence → $Z = R + jX$ or $|z|\angle\theta$ → if X > 0 → impedance lagging ; X<0 = leading.

Also Admitance → $Y = G + jB$; G = Conductance , B = susceptance

Phasor Diagram



Y-Δ

$$Z_a \Rightarrow \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_1}$$

$$Z_b \Rightarrow \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_2}$$

$$Z_c \Rightarrow \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_3}$$

Δ-Y

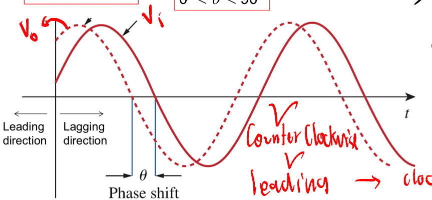
$$Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c}$$

$$Z_2 = \frac{Z_a Z_c}{Z_a + Z_b + Z_c}$$

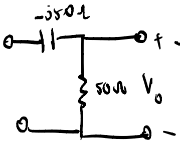
$$Z_3 \Rightarrow \frac{Z_b Z_a}{Z_a + Z_b + Z_c}$$

∠V$_i$+θ=∠V$_o$
∠V$_o$ > ∠V$_i$
0° < θ < 90°

∠V$_o$ > ∠V$_i$
0° < θ < 90°



Counter Clockwise → leading → clockwise => lagging

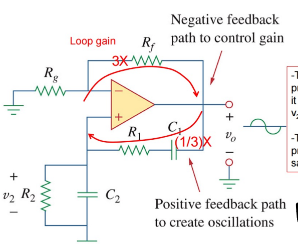


If the reactance and resis tor got the same value ; then its ∠θ will be 45° //

Chapter 10.

Application / SineWave oscillator → for different frequency need to use superposition

For R$_1$=R$_2$, C$_1$=C$_2$



-The positive feedback: to produce zero phase shift, it imposes a constraint of v$_2$/v$_o$=1/3

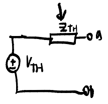
-The negative feedback: provide a gain of 3 to satisfy this constraint!

!!! If we got 2 source in a circuit like a cos and sin function in 1 circuit. We can Convert sin to Cos function

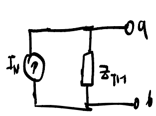
for ex → $\cos\theta = \sin(\theta - \frac{\pi}{2})$

ex => $2\sin 5t \rightarrow 2\cos(5t - \frac{\pi}{2}) \approx 2\angle{-90°}$ #

Thevenin,

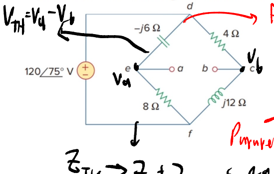


Norton

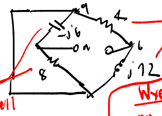


Obtain the Thevenin equivalent at terminals *a-b* of the circuit in Fig.

$V_{TH} = V_a - V_b$



Find Z_{TH} → close 120∠75°



$Z_{TH} \rightarrow Z_1 + Z_2 = 6.48 - j2.64$

Chapter 11 AC Power Analysis.

The average power (Average of instantaneous Power)

$$P = \frac{1}{T}\int_0^T p(t)\,dt \Rightarrow \frac{1}{2} V_M I_M \cos(\theta_v - \theta_i)$$

Average Power in phasor

$$P = \frac{1}{2}\operatorname{Re}(\tilde{V}\tilde{I}^*)$$

When $\theta_v = \theta_i$ → Purely resistive load → $P = \frac{1}{2}V_M I_M = \frac{1}{2}I_M^2 R = \frac{1}{2}\frac{V_m^2}{R}$

Inphase Absorb power all the time

When $\theta_v - \theta_i = \pm 90°$ → Purely reactive load → $P = \frac{1}{2}V_M I_M \cos(\pm 90) = 0$

Average Power Absorb by Impedence → $Z = R + jX$

$$P = \frac{1}{2}I^2R$$ → Only resistance will contribute to average power

Power Conservation

The average power supplied by the voltage source is the same as the average power absorb.

Maximum Average Power → $P = \frac{1}{2}I^2R_L = \frac{1}{2}\frac{V_{TH}^2 R_L}{(R_{TH}+R_L)^2 + (X_{TH}+X_L)^2}$

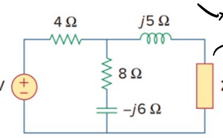
Note → $Z_{TH} = R_{TH} + jX_{TH}$

$Z_L \Rightarrow R_L + jX_L$

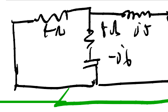
$R_L = R_{TH}$
$X_L = -X_{TH}$
$Z_L = Z_{TH}^*$

for Purely resistive give $$P_{MAX} = \frac{V_{TH}^2}{8R_{TH}}$$ → only real part used.

P_{max} for $R_L = \sqrt{R_{TH}^2 + X_{TH}^2} = |Z_{TH}|$; $X_L = 0$ → use same eq



→ หา Z_L ที่ให้ Max Power Open circuit (Z_{TH})



Effective / RMs Value.

The *effective value* of an ac current *i* is the dc current I_{eff} that delivers the same average power to a resistor as the ac current .

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt}$$

$$V_{eff} = \sqrt{\frac{1}{T}\int_0^T v^2\,dt}$$

T = period.

For the sinusoid $i(t) = I_m \cos wt$ → $I_{rms} = \frac{I_M}{\sqrt{2}}$ // ; $V_{rms} = \frac{V_M}{\sqrt{2}}$ // → The average power → $$P = V_{rms}I_{rms}\cos(\theta_v - \theta_i)$$

We also define → $\tilde{V}_{rms} = V_{rms}\angle\theta_v = \frac{V_M}{\sqrt{2}}\angle\theta_v = \frac{\tilde{V}}{\sqrt{2}}$; $\tilde{I}_{rms} = I_{rms}\angle\theta_i = \frac{I_M}{\sqrt{2}}\angle\theta_i = \frac{\tilde{I}}{\sqrt{2}}$

load Impedence → $Z = \frac{\tilde{V}}{\tilde{I}} = \frac{\tilde{V}_{rms}}{\tilde{I}_{rms}}$

Apparent power / Power factor => from $P = \underbrace{V_{rms}I_{rms}}_{|S|}\cos(\theta_v - \theta_i)$

Apparent power |S| ; pf = cos(θ$_v$−θ$_i$) ; $\theta_v - \theta_i$ = P.f angle.

$$P_f = \cos\theta = \frac{P}{|S|}$$

Note that P.f angle is equal to the angle of load impedence pf = ∠Z = θ$_v$−θ$_i$

Power by Resistor

$$P = I_{eff}^2 R$$

$I_{eff} = I_{rms}$
$V_{eff} = V_{rms}$

DC Formula.

$$P = \frac{V_{RMS}^2}{R}$$

$$P = \frac{V_{rms}^2}{R}$$

$P = \operatorname{Re}(V \cdot I^*)$

Chapter 11 Continued

for purely resistive load → $\theta_v - \theta_i = 0$ → $P_f = 1$ → $P = |S|$

for purely reactive load → $\theta_v - \theta_i = \pm 90°$; $P_f = 0$ → $P = 0$; VAR

Also Apparent power $|S| = V_{rms}I_{rms} \Rightarrow \frac{1}{2}V_m I_M$

∠S = ∠Z = Pf angle = $\theta_v - \theta_i$

Complex power $S = P + jQ$

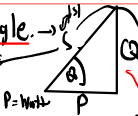
$$S = \frac{1}{2}\tilde{V}\tilde{I}^* = \tilde{V}_{rms}\tilde{I}_{rms}^* = V_{rms}I_{rms}\angle\theta_v - \theta_i = |S|\angle\theta_v - \theta_i$$ → $S = V_{rms}I_{rms}\angle\theta_v - \theta_i$ main eq.

The average power (real power) => $P = I_{rms}^2 R = V_{rms}I_{rms}\cos(\theta_v - \theta_i) = |S|\cos(\theta_v - \theta_i)$ (VA)

reactive power => VAR $Q = ImS = I_{rms}^2 X = V_{rms}I_{rms}\sin(\theta_v - \theta_i) = |S|\sin(\theta_v - \theta_i)$; Z L

Q<0 ⇔ X<0 ⇔ Negative pf angle (θ<0) ⇔ θ$_i$>θ$_v$ ⇔ leading
Q>0 ⇔ X>0 ⇔ Positive pf angle (θ>0) ⇔ θ$_i$<θ$_v$ ⇔ lagging

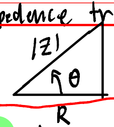
Power triangle.



Complex Power → unit => VA ; P = Watt

Can also j = |S|

Impedence triangle



X ; Capacitor $Q < 0$; $Q > 0$ → Inductive

Relations

$S = \frac{V_{rms}^2}{Z^*}$; $S = I_{rms}^2 Z$

$S = I_{rms}^2 R + jI_{rms}^2 X$

To find Complex power Phasor

Conservation of AC Power → The complex, real and reactive power of the source equal the respective sums of the complex, real and reactive powers of the individual load.

Power factor Correction → Add Capacitance ; Phase angle reduce → Power factor increase

The value of required shunt Capacitance.

$$C = \frac{P(\tan\theta_1 - \tan\theta_2)}{wV^2}$$

→ Q_c or KVAR rating = $P(\tan\theta_1 - \tan\theta_2)$

→ $w = 2\pi \times$ Value of omega W, $V = V_L$ also

Chapter 12

→ from Chapter 11 → given voltage or current in phasor form ; then it's a peak value I_M and V_M

To find $\tilde{V}_{rms}$ and $\tilde{I}_{rms}$ → just divide by $\sqrt{2}$

Three phase Circuit. → Voltage balance ; Same frequency with phase different $\tilde{V}_{cn} = V_p\angle{+120°}$

abc Sequence (Positive) => $\tilde{V}_{an} = V_p\angle 0°$; $\tilde{V}_{bn} = V_p\angle{-120°}$, $\tilde{V}_{cn} = V_p\angle{-240}$

acb sequence (negative) => $\tilde{V}_{an} = V_p\angle 0°$; $\tilde{V}_{bn} = V_p\angle{-240}$; $V_p\angle{+120}$; $\tilde{V}_{cn} = V_p\angle{-120}$

for Inpedence Z → Balance if same in magnitude / phase → $Z_1 = Z_2 = Z_3 = Z_y$; $Z_A = Z_B = Z_C = Z_\Delta$

A wye-connected load / Delta → $Z_\Delta = 3Z_y$; $Z_y = \frac{1}{3}Z_\Delta$

Note that for Z_y → $Z_y = Z_s + Z_l + Z_L$

Instantaneous Power → $P = P_a + P_b + P_c$ => $V_{AN}i_a + V_{BN}i_b + V_{CN}i_c$; $P = 3V_pI_p\cos\theta$ → Total instantaneous power is Constant and do not change with time.

Average Power per phase → (for Y or Δ) → $P_p = V_pI_p\cos\theta$; Reactive Power per phase → $Q_p = V_pI_p\sin\theta$

!!! for a Y-Connected lod → $I_L = I_p$ but $V_L = \sqrt{3}V_p$; for a Δ-Connected load → $I_L = \sqrt{3}I_p$; $V_L = V_p$

$P = 3V_pI_p\cos\theta = \sqrt{3}V_LI_L\cos\theta$ → total power ; $Q = 3Q_1 = 3V_pI_p\sin\theta = \sqrt{3}V_LI_L\sin\theta$ The total reactive power

V_p, I_p Is the Voltage/I for load.

Note that V_p, I_p, V_L and I_L are all RMS value.

Recall $Z_y = Z\angle\theta$; $\theta = \angle V_p - \angle I_p$

Wye Connection, from the load only

Complex Power S → $S = 3S_p = 3\tilde{V}_p\tilde{I}_p^* = 3I_p^2 Z_p = \frac{3V_p^2}{Z_p^*}$ → unit = KVA

Power loss in the two wire. $P_{loss} = 2I_L^2R = 2\left(\frac{P_L}{V_L}\right)^2R$

Unbalance 3 Phase Source Voltage are not equal in magnitude or differ by phase angle that are unequal, → Unbalance System can be solve directly by Mesh or Nodal analysis.

Note that → $\tilde{I}_n = -(\tilde{I}_a + \tilde{I}_b + \tilde{I}_c)$

Applications: Watt Meter

Instrument for Measuring the average Power. → for three Watmeter → $P_T = P_1 + P_2 + P_3$

Two wattmeter → $P_T = P_1 + P_2$; P_1 = Reading (Power) from W_1 ; P_2 = Reading (Power) from W_2.

Consider a balance Y-load → $P_1 = V_LI_L\cos(\theta + 30)$; $P_2 = V_LI_L\cos(\theta - 30)$.

Hence $P_1 + P_2 = \sqrt{3}V_LI_L\cos\theta = P_T$; $P_2 - P_1 \Rightarrow V_LI_L\sin\theta = \frac{Q_T}{\sqrt{3}}$; $\tan\theta = \frac{Q_T}{P_T}$ → $\sqrt{3}(P_2 - P_1)$; → $P_2 + P_1$

Chapter 13.

→ $V = L\frac{di}{dt}$ → Self-inductance → $L = N\frac{d\phi}{di}$; For two coin where i_1 Only carry the current

Energy in a Coupled Circuit → energy store in the circuit is

Hence $V_1 = L_1\frac{di}{dt}$; $V_2 = M_{21}\frac{di_1}{dt}$

$W_1 = \int P_1\,dt = L_1\int_0^{I_1} i_1\,di_1 = \frac{1}{2}L_1 i_1^2$; $P_1 = V_1 i_1 = L_1\frac{di_1}{dt}i_1$; $W_2 = M_{12}I_1I_2 + \frac{1}{2}L_2I_2^2$; $M_{21} = \frac{N_2 d\phi_{12}}{di_1}$

M_{21} is the Mutual inductance of Coil 2 respect to Coil 1.

Total energy when i_1/i_2 reach constant → $W = W_1 + W_2$

Total energy → $W = W_1 + W_2 = \frac{1}{2}L_1I_1^2 + \frac{1}{2}L_2I_2^2 + M_{12}I_1I_2$ → !!! $M_{12} = M_{21} = M$

!! If one Current leave and one Current exit then → $W = \frac{1}{2}L_1I_1^2 + \frac{1}{2}L_2I_2^2 - MI_1I_2$ Total energy stor can never be negative

Coefficient of Coupling → $K = \frac{M}{\sqrt{L_1L_2}}$ loosely couple if K < 0.5 ; Tight if if K > 0.5.

Linear Transformer → $Z_{in} = R_1 + jwL_1 + \frac{w^2M^2}{R_2 + jwL_2 + Z_L}$ → Z_R = reflected impedence due to the coupling.

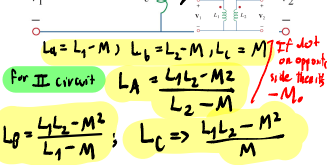
T circuit

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} jw(L_a+L_c) & jwL_c \\ jwL_c & jw(L_b+L_c) \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Π Circuit

for T

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{j\omega L_A} + \frac{1}{j\omega L_C} & -\frac{1}{j\omega L_C} \\ -\frac{1}{j\omega L_C} & \frac{1}{j\omega L_B} + \frac{1}{j\omega L_C} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$



$L_a = L_1 - M$, $L_b = L_2 - M$, $L_c = M$; If dot on opposite side then it's −M.

For Π circuit $L_A = \frac{L_1L_2 - M^2}{L_2 - M}$

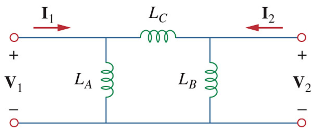
$L_B = \frac{L_1L_2 - M^2}{L_1 - M}$; $L_C \Rightarrow \frac{L_1L_2 - M^2}{M}$

	Phase voltages/cu	Line voltages/currents
Y-Y	$V_{an} = V_p\angle 0°$ $V_{bn} = V_p\angle -120°$ $V_{cn} = V_p\angle +120°$ Same as line curr	$V_{ab} = \sqrt{3}V_p\angle 30°$ $V_{bc} = V_{ab}\angle -120°$ $V_{ca} = V_{ab}\angle +120°$ $\tilde{I}_a = \tilde{V}_{an}/Z_Y$ $I_b = I_a\angle -120°$ $I_c = I_a\angle +120°$
Y-Δ	$V_{an} = V_p\angle 0°$ $V_{bn} = V_p\angle -120°$ $V_{cn} = V_p\angle +120°$ $\tilde{I}_{AB} = \tilde{V}_{AB}/Z_\Delta$ $\tilde{I}_{BC} = \tilde{V}_{BC}/Z_\Delta$ $\tilde{I}_{CA} = \tilde{V}_{CA}/Z_\Delta$	$V_{ab} = V_{AB} = \sqrt{3}V_p\angle 30°$ $V_{bc} = V_{BC} = V_{ab}\angle -120°$ $V_{ca} = V_{CA} = V_{ab}\angle +120°$ $I_a = I_{AB}\sqrt{3}\angle -30°$ $I_b = I_a\angle -120°$ $I_c = I_a\angle +120°$ Same as phase voltages
Δ-Δ	$V_{ab} = V_p\angle 0°$ $V_{bc} = V_p\angle -120°$ $V_{ca} = V_p\angle +120°$ $\tilde{I}_{AB} = \tilde{V}_{ab}/Z_\Delta$ $\tilde{I}_{BC} = \tilde{V}_{bc}/Z_\Delta$ $\tilde{I}_{CA} = \tilde{V}_{ca}/Z_\Delta$	$I_a = I_{AB}\sqrt{3}\angle -30°$ $I_b = I_a\angle -120°$ $I_c = I_a\angle +120°$ Same as phase voltages
Δ-Y	$V_{ab} = V_p\angle 0°$ $V_{bc} = V_p\angle -120°$ $V_{ca} = V_p\angle +120°$ Same as line curr	$I_a = \dfrac{V_p\angle -30°}{\sqrt{3}Z_Y}$ $I_b = I_a\angle -120°$ $I_c = I_a\angle +120°$

→ Positive Sequence

Chapter 13 Continued

Pic for Π Circuit →



If n=1 => isolation transformer.

Ideal transformer → $K=1$; $R_1=R_2=0$; $L_1, L_2, M \to \infty$ → n= turn ratio

$\boxed{\frac{V_2}{V_1} = \frac{N_2}{N_1} = n}$ Apply the same as phasor voltages $\frac{\tilde{V}_2}{\tilde{V}_1}$

for current → $\boxed{\frac{i_1}{i_2} = \frac{V_2}{V_1} = n}$ $\boxed{\frac{\tilde{I}_1}{\tilde{I}_2} = \frac{\tilde{V}_2}{\tilde{V}_1} = n}$

Step-up transformer ; $n > 1$: $V_2 > V_1$

Step-down → $V_2 < V_1$; $n < 1$

Rules => If V_1/V_2 are both positive or negative at the dotted terminal then $\boxed{n = \frac{\tilde{V}_2}{\tilde{V}_1} \text{ else } \frac{\tilde{V}_2}{\tilde{V}_1} = -n.}$

2.) If I_1/I_2 both enter into or both leave the dotted terminal then $\boxed{\frac{\tilde{I}_1}{\tilde{I}_2} = -n \text{ else } \frac{\tilde{I}_1}{\tilde{I}_2} = n}$

Ideal Transformer absorb no power → $\boxed{S_1 = S_2}$ same apparent Power $|S|$

Complex Power

Input Impedance → $Z_{in} = \frac{\tilde{V}_1}{\tilde{I}_1}$ → $\boxed{\frac{1}{n^2}Z_L}$

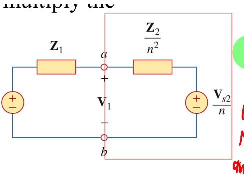
KVA rating → $|S| = V_1 I_1$

Power delivere to load → $P = |I_L|^2 Z_L$

Thevenin equivalent

reflecting the Secondary Circuit to primary side

divide second Z by n^2 and divide second V by n and Multiply second I by n.

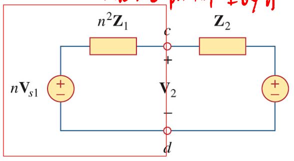


reflecting the primary circuit to the secondary side is

Multiply the primary Z by n^2

Multiply the primary V by n

and divide the primary I by n



Autotransformer → transfer large apparent Power

for Step-down. → $\boxed{\frac{V_1}{V_2} = \frac{N_1+N_2}{N_2} = \frac{I_2}{I_1}}$ also apply for phasor

Step-up

$\boxed{\frac{\tilde{V}_1}{\tilde{V}_2} = \frac{N_1}{N_1+N_2} = \frac{\tilde{I}_2}{\tilde{I}_1}}$

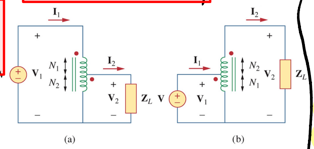


Figure 13.42 (a) Step-down autotransformer, (b) step-~~down~~ up autotransformer.

Application. → Transformer as an Isolation device: => To provide Isolation → Turn ratio $n=1$. Cits Prevent any dc voltage in one stage from effecting the dc bias of the next stage.) Impedence Matching → (Max Power transfer)

$\boxed{R_s = \frac{R_L}{n^2} \Rightarrow n = \sqrt{\frac{R_L}{R_s}}}$

Chapter 14

Frequency Response => Amplitude / Phase angle remain constant and Vary the frequency.

$\boxed{H(j\omega) = \frac{\tilde{Y}(j\omega)}{\tilde{X}(j\omega)}}$

Four type of frequency Response

1.) $H(j\omega) = \frac{\tilde{V}_o(j\omega)}{\tilde{V}_i(j\omega)}$ Voltage gain.

2.) $H(j\omega) \Rightarrow \frac{\tilde{I}_o(j\omega)}{\tilde{I}_i(j\omega)}$ Current gain.

3.) $H(j\omega) = \frac{\tilde{V}_o(j\omega)}{\tilde{I}_i(j\omega)}$ transfer Impedence.

4.) $H(j\omega) \Rightarrow \frac{\tilde{I}_o(j\omega)}{\tilde{V}_i(j\omega)}$

The Gain G

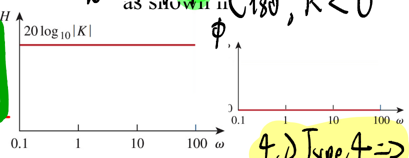
$G_{DB} = 10\log_{10}\frac{P_2}{P_1} = 10\log_{10}\left(\frac{V_2^2/R_2}{V_1^2/R_1}\right)$ → lineal scale → 10^{G_bel} → 0bel = 1 time

If $R_1 = R_2$ → then $\boxed{G_{DB} = 20\log_{10}\frac{V_2}{V_1} = 20\log_{10}\frac{I_2}{I_1}}$

Bode Plot

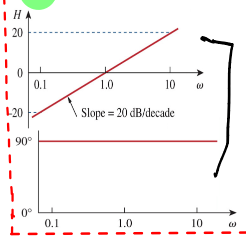
1st facttor → Constant B. → (1). For the gain K

$H_{dB} = 20\log_{10}|K|$; $\phi = \begin{cases} 0, & K>0 \\ 180, & K<0 \end{cases}$



2.) for jw

$H_{DB} = 20\log_{10}|j\omega|$; $\phi = 90°$



for $j\omega^{-1}$ or $\frac{1}{j\omega}$

Slope $H_{DB} = -20$

$\phi = -90°$

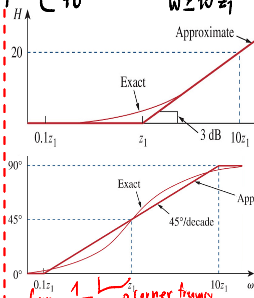
H_{DB}

$(j\omega)^N = \pm 20N$

$\phi = 90N$

3.) Type 3 → $\left(1 + \frac{j\omega}{z_1}\right)$, $\frac{1}{\left(1+\frac{j\omega}{z_1}\right)}$ Pole

$H_{DB} = \begin{cases} 0 \to \omega \le z_1 \\ 20 \to \omega \ge z_1 \end{cases}$ $\phi \begin{cases} 0 & \omega \le 0.1 z_1 \\ 45 + 45\log_{10}\left(\frac{\omega}{z_1}\right) & 0.1z_1 \le \omega \le 10z_1 \\ 90° & \omega \ge 10z_1 \end{cases}$



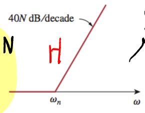
for $\frac{1}{\left(1+\frac{j\omega}{p_1}\right)}$ → corner frequency is equal to p_1 and slope $H = -20$ and $\phi = -45°$ per decade. for case $\left(1+\frac{1}{j\omega}\right)^N$ just multiply the slope and phase by N.

4.) Type 4 =>

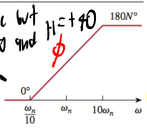
4. z

4. z_m=c.c.; p_n=c.c.

$\left[1 + \frac{2j\omega\xi}{\omega_n} + \left(\frac{j\omega}{\omega_n}\right)^2\right]^N$



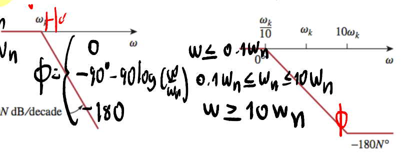
Same as Pole but $\phi = +90$ and $H = +40$



$\frac{1}{1 + 2j\omega\zeta/\omega_n + (j\omega/\omega_n)^2]^N}$

$H_{DB} = \begin{cases} 0 & \omega \le \omega_n \\ -40\log_{10}\left(\frac{\omega}{\omega_n}\right) & \omega \ge \omega_n \end{cases}$

$\phi = \begin{cases} 0 & \omega \le 0.1\omega_n \\ -90° - 90\log\left(\frac{\omega}{\omega_n}\right) & 0.1\omega_n \le \omega_n \le 10\omega_n \\ -180 & \omega \ge 10\omega_n \end{cases}$



Series Resonance → Resonance Occur when $Im(Z) = 0$ → Resonance frequency ω_0

The reactive and Capacitive Reactance are equal. $\boxed{\omega_0 L = \frac{1}{\omega_0 C}}$ $\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$

In resonance → It's Purely resistive → $Z = R$, The Voltage $\tilde{V}_s$ and $\tilde{I}$ are in phase

highest Power dissipated at Resonance. → $P(\omega_0) = \boxed{\frac{1}{2}I_{max}^2 R = \frac{1}{2}\frac{V_m^2}{R}}$

Half Power frequency. → $I(\omega_1) = I(\omega_2) = \frac{I_{max}}{\sqrt{2}}$ → $\boxed{P(\omega_1) = P(\omega_2) = \frac{1}{2}P(\omega_0) = \frac{V_m^2}{4R}}$

ω_1 and ω_2

$\boxed{\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \ ; \ \omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}}$ → $\omega_0 = \sqrt{\omega_1\omega_2}$

Half-Power Bandwidth → $BW = \omega_2 - \omega_1 = \frac{R}{L}$

Measured the sharpness of resonance.

; Quality factor Q => $= \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$

high Q circuit when $Q \ge 10$

Relation Between B/Q → $\boxed{BW = \frac{R}{L} = \frac{\omega_0}{Q} \Rightarrow Q = \frac{\omega_0}{BW}}$

$\boxed{V_L = \frac{V_m}{R}\omega_0 L = \frac{V_m}{R}\frac{1}{\omega_0 C} = V_C}$

for Resonance → The inductor / Capacitance can be much more than the source voltage.

Passive filter. → Contain only passive elements R, L and C. pass signal with desire frequency.

Lowpass filter → Pass low frequencies and reject high frequencies

$\boxed{H(j\omega) = \frac{1}{1+j\omega RC} \ ; \ \text{Cut off frequency} \to \omega_c = \frac{1}{RC} \text{ or set } H(j\omega) = \frac{1}{\sqrt{2}}}$

Highpass filter. → Passes high frequencies and rejected low frequencies.

$\boxed{H(j\omega) = \frac{j\omega RC}{1+j\omega RC} \ ; \ \omega_c = \frac{1}{RC}}$

Band Pass filter.

Passes frequency within a frequency band and block frequency outside the band.

$\boxed{H(j\omega) \Rightarrow \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}}$

ω_0 Center frequency

ω_1 → lower Cut off frequency; ω_2 → Upper Cut off frequency

Band stop filter → Passes frequency outside a frequency band and block frequency within the band.

$\boxed{H(j\omega) = \frac{j[\omega L - 1/(\omega C)]}{R + j[\omega L - 1/(\omega C)]}}$

STEP → Solve for $H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$ and try to manipulate to get the same as frequency response of the filter

Highpass filter → $\boxed{\frac{j\omega/\omega_c}{1 + j\omega/\omega_c}}$

Active filter. → Combination of resistor capacitor and op-Amps Smaller and less expensive than passive filter.

First Order Low Pass filter → $\boxed{H(j\omega) = -\frac{R_f}{R_i}\left(\frac{1}{1+\frac{j\omega}{\omega_c}}\right) ; \ \omega_c = \frac{1}{R_f C_f}}$

First Order high pass filter. → $\boxed{H(j\omega) \Rightarrow -\frac{R_f}{R_i}\left(\frac{j\omega/\omega_c}{1 + j\omega/\omega_c}\right) ; \ \omega_c = \frac{1}{R_i C_i}}$

$\boxed{\omega_c = \frac{1}{(R_i C_i)}}$

Pararell Resonance → Resonance Occur when Y admidtance $Im(Y) = 0$; $\omega_0 = \frac{1}{\sqrt{LC}}$

Current instead of Voltage

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$BW = \omega_2 - \omega_1 = \frac{1}{RC}$$

$$Q = \frac{\omega_0}{BW} = \omega_0 RC = \frac{R}{\omega_0 L}$$

$\boxed{\text{Energy In Inductor} \to \frac{1}{2}LI^2}$

Note → $\boxed{\omega_1 \Rightarrow 2\pi f_1 \ ; \ \omega_2 \Rightarrow 2\pi f_2}$

for Band Reject $\omega_1 = \frac{1}{(RC_1)}$ lower cutoff frequency

$\omega_2 = \frac{1}{(RC_2)}$ Upper Cutoff Frequency.

Band Reject Filter

Pararell Combination of a low pass filter and a high pass filter.

$$H(j\omega) = K\frac{1 + 2\zeta(j\omega)/\omega_0 + (j\omega/\omega_0)^2}{(1 + j\omega/\omega_1)(1 + j\omega/\omega_2)}$$

$$K = \frac{R_f}{R_i}, \ \omega_0 = \sqrt{\omega_1\omega_2}, \ \zeta = \sqrt{\frac{\omega_1}{\omega_2}}$$

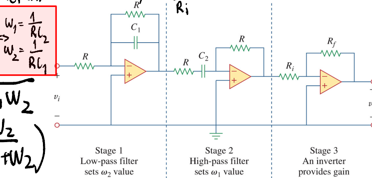
Band pass filter. → Cascading low pass filter and high pass filter and an inverter gain $-\frac{R_f}{R_i}$

$\boxed{H(j\omega) = -\frac{R_f}{R_i}\left(\frac{j\omega/\omega_1}{(1 + j\omega/\omega_1)(1 + j\omega/\omega_2)}\right) \Rightarrow \omega_1 = \frac{1}{RC_2} \ \omega_2 = \frac{1}{RC_1}}$

center frequency → $\omega_0 = \sqrt{\omega_1\omega_2}$

$BW = \omega_2 - \omega_1$

Pass ban gain K → $\frac{R_f}{R_i}\left(\frac{\omega_2}{\omega_1 + \omega_2}\right)$



Multiple Choice.

for Time domain and not Phasor

The sum of balance current = 0.

The sum of balance voltage not 0.

How to locate on a log Scale. ex → 0.5

for 0.1 to 1 $\log_{10}(0.1) = -1$

$\log_{(10)} 1 = 0$; $\log_{(10)}(0.5) = -0.301$

Hence $\frac{-0.301-(-1)}{0-(-1)} = \frac{0.699}{1} = 69.9\%$ on the scale 0.1 to 1

Bode Plot Continued for φ

for $(1+\frac{j\omega}{z_1}) \to \phi = \tan^{-1}(\frac{\omega}{z_1})$

fo type 4 → $\phi = -\tan^{-1}\left(\frac{2\delta_2 \omega/\omega_n}{1-\omega^2/\omega_n^2}\right)$

How to Know if Value is RMS

→ If given phasor like $120\angle 30^\circ$ then its likely a RMS Value. However if given $v(t) = V_m\cos(\omega t+\theta)$ → then V_m is a peak Value and we find V_{rms} → by $\frac{V_m}{\sqrt{2}}$!!

Z_{in} for Magnetically Couple Circuit

$Z_{in} = j\omega L_{eq}$

$Z_{in} = \frac{V}{I}$; $L_{eq} = \frac{1}{jI_1}$ assume $V=1$, $\omega=1$

10.62 Using Thevenin's theorem, find v_o in the circuit of Fig. 10.105.

$\omega = 1$; $2H = 2j$

$\frac{1}{4}F = -4j$

$\frac{1}{8}F = -8j$

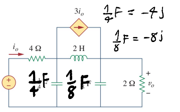
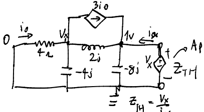


Figure 10.105
For Prob. 10.62.

Open circuit at V_0 as we want V_0

find Z_{TH} → close all independent Source



$Z_{TH} = \frac{V_x}{i_x}$

At Node 1. → $\frac{V_x}{4} + \frac{V_x}{-4j} + \frac{V_x-1}{2} + 3i_0 = 0$

V_x

$\frac{V_x}{-j4} - \frac{2V_x}{4} = \frac{1-V_x}{2j}$

$V_x = 0.4 + j0.8$

At Node 1V → $\frac{1-V_x}{2} - 3i_0 + \frac{1}{-8j} - i_X = 0$

$i_0 = \frac{-V_x}{4}$

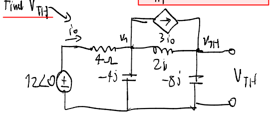
$I_X = (0.75 + j0.5)V_x - j\frac{3}{8}$

$I_X = -0.1 + 0.425j$

$Z_{TH} = \frac{V}{I_X} = \frac{1}{(-0.1+0.425j)} = -0.5246 - j2.229$

$\boxed{Z_{TH} \Rightarrow 2.29\angle -103.24^\circ}$

Find V_{TH}



KCL at V_1 $\frac{V_1-12}{4} + \frac{V_1}{-4j} + 3i_0 + \frac{V_1-V_{TH}}{2j} = 0$

$(\frac{1}{4} - \frac{1}{4}j)V_1 - \frac{1}{2j}V_{TH} = 3 - 3i_0 \to (1)$

$i_0 = \frac{12-V_1}{4}$

$3-3i_0 \Rightarrow 3 - 3\left(\frac{12-V_1}{4}\right)$

$\Rightarrow 3 - \left(\frac{36-3V_1}{4}\right) = -6 + \frac{3}{4}V_1$

$(-\frac{1}{2} - \frac{1}{4}j)V_1 - \frac{1}{2j}V_{TH} = -6 \to (1)$

KCL at V_{TH}

$\frac{V_{TH}-V_1}{2j} - 3i_0 + \frac{V_{TH}}{-8j} = 0$

$\frac{V_{TH}-V_1}{2j} - \frac{36}{4} + \frac{3V_1}{4} + \frac{V_{TH}}{-8j} = 0$

$(\frac{3}{4} + \frac{1}{2}j)V_1 - \frac{3}{8}jV_{TH} = 9 \to (2)$

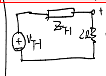
Find V_{TH} → Δ2

$$\begin{vmatrix} -\frac{1}{2}-\frac{1}{4}j & -\frac{1}{2j} \\ \frac{3}{4}+\frac{1}{2}j & -\frac{3}{8}j \end{vmatrix} \begin{vmatrix} V_1 \\ V_{TH} \end{vmatrix} = \begin{vmatrix} -6 \\ 9 \end{vmatrix}$$

$\Delta \Rightarrow \frac{5}{32} - \frac{3}{16}j$

$\Delta_2 \Rightarrow \begin{vmatrix} -\frac{1}{2}-\frac{1}{4}j & -6 \\ \frac{3}{4}+\frac{1}{2}j & 9 \end{vmatrix} \Rightarrow \frac{3}{4}j$

$\boxed{V_{TH} = \frac{\Delta_2}{\Delta} = \frac{-144}{61} + \frac{120}{61}j \approx 3.073\angle 140.194^\circ}$



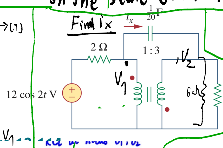
$V_0 \to V_0 = V_{TH}\left(\frac{2}{2+Z_{TH}}\right)$

$V_0 = \frac{2(3.073\angle -219.8^\circ)}{1.4754 - j2.229}$

$V_0 \to 2.3\angle -163.3^\circ$

$\boxed{V_0 = 2.3\cos(t - 163.3)V.}$

Find Ix



KCL at nodes V1, V2

$\frac{12-\tilde{V}_1}{2} = \frac{\tilde{V}_1-\tilde{V}_2}{-j10} + \tilde{I}_1 = 0$

$\frac{\tilde{V}_2}{6} + \tilde{I}_2 = \frac{\tilde{V}_1-\tilde{V}_2}{-j10}$

Transformer

$\frac{\tilde{V}_2}{\tilde{V}_1} = -3$ $\frac{\tilde{I}_2}{\tilde{I}_1} = \frac{1}{3}$

Hence

$\tilde{V}_1 = 1.8 - j1.5$

$\tilde{I}_x = \frac{\tilde{V}_1-\tilde{V}_2}{-j10} = 0.9\angle 51.3^\circ$

$i_x = 0.9\cos(2t + 51.3^\circ)$ A

Find Current I_0

$i_3 = i_0$

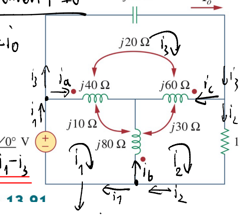
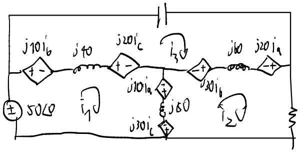


Figure 13.91

$i_1 = i_3 + i_a$; $i_a = i_1 - i_3$

$i_b = i_2 - i_1$

$i_c = i_3 - i_2$



KVL Mesh i_1 $-50 + j10i_b + j40(i_1 - i_3) + j20i_c - j30(i_c) + j80(i_1 - i_2) - j10i_a$

Sub $i_a = i_1 - i_3$; $i_b = i_2 - i_1$; $i_c = i_3 - i_2$

Solve give $10I_1 - 6I_2 - 4I_3 = -5j \to (1)$ $+30j(i_3 - i_2)$

KVL Mesh i_2 → $-30j(i_2 - i_1) + 60j(i_2 - i_3) - 20j(i_1 - i_3) + 100i_2 + 80j(i_2 - i_1) + 10j(i_1 - i_3) = 0$

solve give $-60jI_1 + (100+80j)I_2 - 20jI_3 = 0 \to (2)$

KVL Mesh i_3 → $-j50i_3 + j20(I_1 - i_3) + 60j(i_3 - i_2) + 30j(i_2 - i_1) - j10(i_2 - i_1) + j40(i_3 - i_1) - j20(i_3 - i_2) = 0$

solve give → $-j40i_1 - j20i_2 + 10ji_3 = 0 \to (3)$

Solve give $i_2 = \frac{-7i_3}{-22+20j} \Rightarrow 0.2355\angle 42.3 \Rightarrow (0.174 + 0.158j)i_3$

$i_3 = j3.333 + (-1.2273 - j1.1623)i_3 \longrightarrow \boxed{I_0 = I_3 = 1.3040\angle 63^\circ \text{ A}}$

Exercise 7.5 Find the current I_1, I_2 and I_0 in the circuit below, and the average power delivered to the load $10 + j40\Omega$.

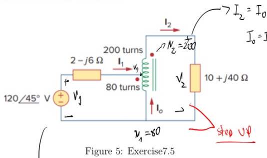


Figure 5: Exercise7.5

$I_2 = I_0 + I_1$

$I_0 = I_2 - I_1$

Auto transformer → single continuous winding.

Can use formula.

for step up:

for stepup → $\frac{\tilde{V}_1}{\tilde{V}_2} = \frac{N_1}{N_1+N_2} = \frac{\tilde{I}_2}{\tilde{I}_1}$ or $\frac{V_1}{V_2} = \frac{N_1}{N_1+N_2} = \frac{I_2}{I_1}$

for KVL at V_1 $120\angle 45 = (2 - j6)I_1 + V_1$

for V_2 → We known $V_2 = I_2(10 + j40)$

from $\frac{V_1}{V_2} = \frac{N_1}{N_1+N_2} \Rightarrow \frac{V_1}{V_2} = \frac{80}{280}$

Sub V_2 and I_2 into $V_2 = I_2(10+j40)$ $V_2 = \frac{7}{2}V_1$

$\frac{7}{2}V_1 = \frac{2}{7}I_1(10+j40)$

$\frac{7}{2}V_1 = \frac{20}{7}I_1 + \frac{80}{7}jI_1$ also $\frac{I_2}{I_1} = \frac{2}{7}$

$V_1 = \frac{40}{49}I_1 + \frac{160}{49}jI_1$ $I_2 = \frac{2}{7}I_1$

Hence $120\angle 45 = (2-j6)I_1 + (\frac{40}{49}I_1 + \frac{160}{49}jI_1)$

$120\angle 45 = (\frac{138}{49} - \frac{134}{49}j)I_1$

$I_1 = 30.6\angle 89.7$ $I_2 = \frac{2}{7}(30.6\angle 89.8)$, $I_2 = 8.74\angle 89.8^\circ$

$I_0 = I_2 - I_1 = 21.86\angle -90.2$ → Average Power → $|I_2|^2(Z_L) = (8.74)^2(10+j40)$

⇒ 763.8 W

$M_1 = 80$; $N_2 = 200$

$n > 0$

$\frac{V_1}{V_2} = \frac{N_1}{N_2}$

$\frac{V_2}{V_1} = \frac{N_2}{N_1}$

Bode Plot

for Type 3. $(1 + \frac{j\omega}{1}) \to z_n = 1$

$H_{dB} = \begin{cases} 0 \to \omega \leq 1 \\ 20 \to \omega \geq 1 \end{cases}$

$\phi = \begin{cases} 0 & \omega \leq 0.1 \\ 45 + 45\log_{10}(\frac{\omega}{z_1}) & 0.1 \leq \omega_1 \leq 10 \\ 90 & \omega \geq 10 \end{cases}$

for Type 4. $\omega_n = 5$ $-20\log_{10}|1 + j\omega\frac{2}{5} + (\frac{j\omega}{5})^2|$

H_{dB} → magnitude

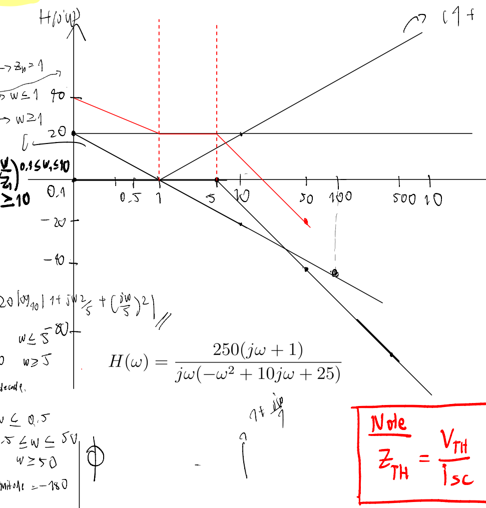
$\begin{cases} 0 & \omega \leq 5 \\ -40 & \omega \geq 5 \end{cases}$

slope/decade.

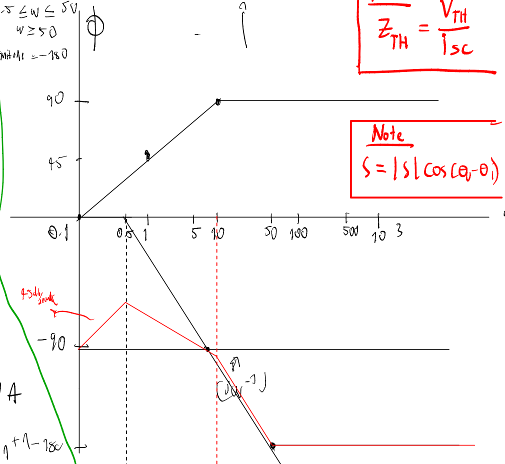
$\phi = \begin{cases} 0 & \omega \leq 0.5 \\ -90 & 0.5 \leq \omega \leq 50 \\ -180^\circ & \omega \geq 50 \end{cases}$

slope/decade

slope = 0, magnitude = −180



$$H(\omega) = \frac{250(j\omega+1)}{j\omega(-\omega^2 + 10j\omega + 25)}$$

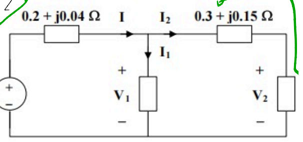


Note

$Z_{TH} = \frac{V_{TH}}{I_{sc}}$

Note

$S = |S|\cos(\theta_v - \theta_i)$



$S_2 = 15 - j\frac{15}{0.8}\sin(\cos^{-1}(0.8)) = 15 - j11.25$

$S_2 = V_2 I_2^*$

$I_2^* = \frac{S_2}{V_2} = \frac{15 - j11.25}{120}$

$I_2 = 0.125 + j0.09375$

$V_1 = V_2 + I_2(0.3 + j0.15)$

$V_1 = 120 + (0.125 + j0.09375)(0.3 + j0.15)$

$V_1 = 120.02 + j0.0469$

$S_1 = 10 + j\frac{10}{0.9}\sin(\cos^{-1}(0.9)) = 10 + j4.843$

$S_1 = V_1 I_1^*$

$I_1^* = \frac{S_1}{V_1} = \frac{11.111\angle 25.84^\circ}{120.02\angle 0.02^\circ}$

$I_1 = 0.093\angle -25.82^\circ = 0.0837 - j0.0405$

$I = I_1 + I_2 = 0.2087 + j0.053$

$V_s = V_1 + I(0.2 + j0.04)$

$V_s = (120.02 + j0.0469) + (0.2087 + j0.053)(0.2 + j0.04)$

$V_s = 120.06 + j0.0658$

$\boxed{T = \frac{2\pi}{\omega}}$

Chapter 11 Qs → AC Power Conservation

10.64 For the circuit shown in Fig. 10.107, find the Norton equivalent circuit at terminals *a-b*.

Mesh Analysis

for Z_{TH}

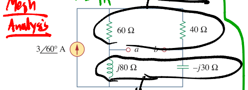


Figure 10.107
For Prob. 10.64.

$Z_{TH} = (60+40) \,//\, (j80 - j30)$

10.68 Find the Thevenin equivalent at terminals *a-b* in the circuit of Fig. 10.111.

ML

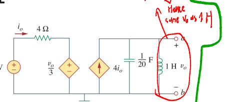
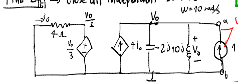


Figure 10.111
For Prob. 10.68.

Find Z_{TH} → Close all independent Source

ω = 10 rad/s



KCL at $\frac{V_0}{3}$ → $i_0 = \frac{-\frac{V_0}{3}}{4} = -\frac{V_0}{12}$

KCL at V_0 $-4i_0 - 1 + \frac{V_0}{-2j} + \frac{V_0}{10j} = 0$

$\frac{V_0}{-2j} + \frac{V_0}{10j} = 1 + 4(\frac{-V_0}{12})$

$\frac{V_0}{-2j} + \frac{V_0}{10j} = 1 + \frac{-V_0}{3}$

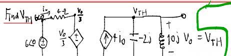
$\frac{V_0}{-2j} + \frac{V_0}{10j} + \frac{V_0}{3} = 1$

$(\frac{1}{3} + \frac{2}{5}j)V_0 = 1$

$V_0 = \frac{75}{61} - \frac{90}{61}j \approx 1.92\angle -50.194^\circ$

$\boxed{Z_{TH} = \frac{V_0}{1A} = 1.2291 - 1.475j \approx 1.92\angle -50.194^\circ}$

Find V_{TH}



KCL at $\frac{V_0}{3}$ → $i_0 = \frac{6 - \frac{V_0}{3}}{4}$ → $V_{TH} = V_0$

KCL at V_{TH} → $-4i_0 + \frac{V_{TH}}{-2j} + \frac{V_{TH}}{10j} = 0$

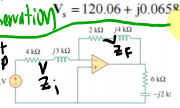
$-4\left(\frac{6-\frac{V_0}{3}}{4}\right) + \frac{V_{TH}}{-2j} + \frac{V_{TH}}{10j} = 0$

$-6 + \frac{V_0}{3} + \frac{V_{TH}}{-2j} + \frac{V_{TH}}{10j} = 0$

$(\frac{1}{3} + \frac{2}{5}j)V_{TH} = 6$

$\boxed{V_{TH} = \left(\frac{450}{61} - \frac{540}{61}j\right) \approx 11.525\angle -50.194^\circ \text{ V}}$

Invert Opamp



As an inverter,

$V_o = \frac{-Z_f}{Z_i}V_s = \frac{-(2+j4)}{4+j3}\cdot(4\angle 45^\circ)$

$I_o = \frac{V_o}{6-j2}$ mA $= \frac{-(2+j4)(4\angle 45^\circ)}{(6-j2)(4+j3)}$ mA

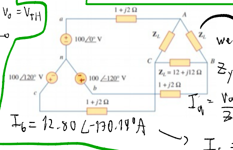
The power absorbed by the 6-kΩ resistor is

$P = |I_o|^2 R = \left(\frac{\sqrt{20}\times 4}{\sqrt{40}\times 5}\right)^2 \times 10^{-6} \times 6\times 10^3$

P = 1.92 mW

For Non-Invert.

$\boxed{V_0 = V_{in}\cdot\left(1 + \frac{R_2}{R_1}\right)}$



Convert load from Δ to Y

we know $Z_\Delta = 3Z_Y \to Z_Y = \frac{1}{3}Z_\Delta$

$Z_Y = 4 + 4j \to Z_y = Z_\ell + Z_L + Z/3 = 5 + 6j$

$I_a = \frac{V_a}{Z_y} = \frac{100\angle 0}{5+6j} = 12.80\angle -50.19$

$I_b = 12.80\angle -170.19^\circ A$ → $I_c = 12.80\angle 69.81^\circ A$

Convert the Δ-connected source to a Y-connected source.

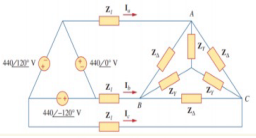
$V_{an} = \frac{V_p}{\sqrt{3}}\angle -30^\circ = \frac{440}{\sqrt{3}}\angle -30^\circ = 254\angle -30^\circ$

Convert Δ to Y connect source

Convert the Δ-connected load to a Y-connected load.

$Z = Z_Y \,||\, \frac{Z_\Delta}{3} = (4 + j6) \,||\, (4 - j5) = \frac{(4+j6)(4-j5)}{8+j}$

$Z = 5.723 - j0.2153$



$I_a = \frac{V_{an}}{Z_L + Z} = \frac{254\angle -30^\circ}{7.723 - j0.2153} = 32.88\angle -28.4^\circ$ A

$I_b = I_a\angle -120^\circ = 32.88\angle -148.4^\circ$ A

$I_c = I_a\angle 120^\circ = 32.88\angle 91.6^\circ$ A

13.14 Obtain the Thevenin equivalent circuit for the circuit in Fig. 13.83 at terminals *a-b*.

$Z_{TH} = \frac{V_0}{1}$ Apply 1 A

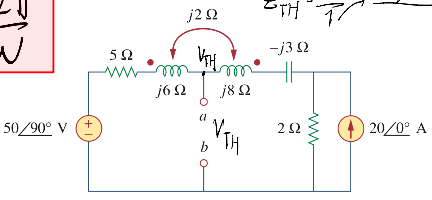
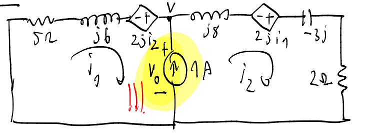


Figure 13.83

find Z_{TH} → Close all independent Source apply current 1A



$i_2 = i_1 + 1 - 180$

$1 = i_2 - i_1$

Super mesh i_1, i_2 → $\cancel{5i_1} + \cancel{6ji_1} - 2ji_2 + 8ji_2 - \cancel{2ji_1} - 3ji_2 + 2i_2 = 0$

$(5+4j)i_1 + (3j+2)i_2 = 0 \to (1)$

also $I_2 - i_1 = 1 \to i_2 = i_1 - 1 \longrightarrow (2)$

sub (2) to (1) give $i_1 = -(2+j3)/7+j7$

also from left loop KVL → $(5+6j)i_1 - 2ji_2 + V_0 = 0$

$(5+6j)i_1 - 2j(i_1 + 1) + V_0 = 0$

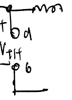
$(5+4i)i_1 + 2j + V_0 = 0$

$V_0 = +2j - \left[(5+4j)\left(\frac{-(2+3j)}{7+7j}\right)\right]$

$V_0 = 1.5 + 3.7857j \approx 4.07\angle 68.385^\circ$

$Z_H = \frac{V_0}{1}$

find V_{TH} Apply KVL Whole loop →



$-50j + (5 + 10j - 3j + 2)I + 40 = 0$

give $I \to (-40 + 50j)/7 + 7j)$

also from left loop → KVL

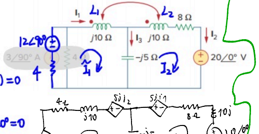
$-j50 + (5 + j6)I - 2jI + V_{TH} = 0$

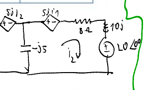
$V_{TH} = j50 - (5+4j)I$ → $V_{TH} = 26.74\angle 34.99$

Exercise 7.2 (20%) Find the currents I_1, I_2 and I_3 in the circuit below, and the energy stored in the coupled coils at $t = 2$ ms. Assume $\omega = 1000$ rad/s.

$L_1 = \frac{X_1}{j\omega} = 10\,mH$ $L_2 = \frac{X_2}{j\omega} = 10\,mH$

$M = k\sqrt{L_1L_2} = 5\,mH$





Left mesh

$4\tilde{I}_1 - 12\angle 90^\circ + j5\tilde{I}_2 + j10\tilde{I}_1 - j5(\tilde{I}_1 - \tilde{I}_2) = 0$

Right mesh

$-j5(\tilde{I}_2 - \tilde{I}_1) + j5\tilde{I}_1 + j10\tilde{I}_2 + 8\tilde{I}_2 + 20\angle 0^\circ = 0$

$\tilde{I}_1 = 2.5\angle 72.2^\circ$ $\tilde{I}_2 = 0.9\angle -97.5^\circ$

$\tilde{I}_3 = \tilde{I}_1 - \tilde{I}_2 = 3.3\angle 74.90$

$i_1(t) = 2.5\sqrt{2}\cos(1000t + 72.2^\circ) = -3.45$

$i_2(t) = 0.9\sqrt{2}\cos(1000t - 97.5^\circ) = 1.19$

$W = \frac{1}{2}L_1i_1^2 + \frac{1}{2}L_2i_2^2 + Mi_1i_2 = 0.046$ J

☆ In this formula, you need to use instantaneous values. → i(t), v(t)

Average power in inductors are zero.

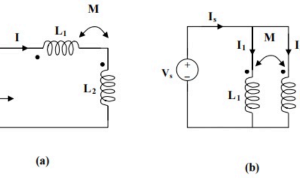
instantaneous Values → Peak Value

$I_{rms} \to \frac{I_m}{\sqrt{2}}$ → Peak Value

$\Delta = 107 + 60j$; $\Delta_1 = -60 + 296j$; $\Delta_2 = 40 - 100j$

(a) For the series connection shown in Figure (a), the current I enters each coil from its dotted terminal. Therefore, the mutually induced voltages have the same sign as the self-induced voltages. Thus,

$$L_{eq} = L_1 + L_2 + 2M$$



(a) (b)

(b) For the parallel coil, consider Figure (b).

$I_s = I_1 + I_2$ and $Z_{eq} = V_s/I_s$

Applying KVL to each branch gives,

$$V_s = j\omega L_1I_1 + j\omega MI_2 \quad (1)$$

$$V_s = j\omega MI_1 + j\omega L_2I_2 \quad (2)$$

or

$$\begin{bmatrix} V_s \\ V_s \end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$\Delta = -\omega^2L_1L_2 + \omega^2M^2$, $\Delta_1 = j\omega V_s(L_2 - M)$, $\Delta_2 = j\omega V_s(L_1 - M)$

$I_1 = \Delta_1/\Delta$, and $I_2 = \Delta_2/\Delta$

$I_s = I_1 + I_2 = (\Delta_1 + \Delta_2)/\Delta = j\omega(L_1 + L_2 - 2M)V_s/(-\omega^2(L_1L_2 - M^2))$
$= (L_1 + L_2 - 2M)V_s/(j\omega(L_1L_2 - M^2))$

$Z_{eq} = V_s/I_s = j\omega(L_1L_2 - M^2)/(L_1 + L_2 - 2M) = j\omega L_{eq}$

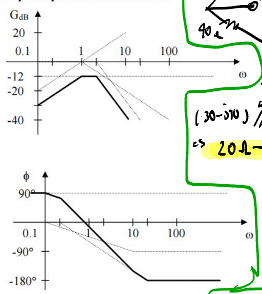
i.e., $L_{eq} = (L_1L_2 - M^2)/(L_1 + L_2 - 2M)$

$$G(\omega) = \frac{(1/4)j\omega}{(1+j\omega)(1+j\omega/2)^2}$$

$$G_{dB} = -20\log_{10}4 + 20\log_{10}|j\omega| - 20\log_{10}|1+j\omega| - 40\log_{10}|1+j\omega/2$$

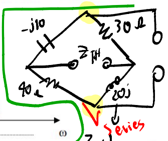
$$\phi = -90° - \tan^{-1}\omega - 2\tan^{-1}\omega/2$$

ude and phase plots are shown below.



Conservation of AC Power. →

11.17 Calculate the value of $\mathbf{Z}_L$ in the circuit of Fig. 11.48 in order for $\mathbf{Z}_L$ to receive maximum average power. What is the maximum average power received by $\mathbf{Z}_L$?



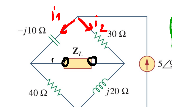


Figure 11.48 For Prob. 11.17.

Z_{TH}

$(30-j10)\,//\,(40+20j)$

$\Rightarrow 20\Omega \rightarrow Z_L = Z_{TH} \rightarrow P_{max} = \frac{V_{TH}^2}{8Z_L} \rightarrow$

$Z_L = Z_{TH}$

find V_{TH}

Use Current Division

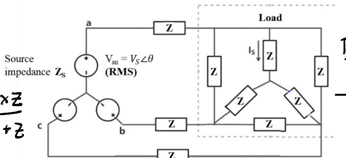
$$i_1 = \frac{(30+j20)}{(40-10j)+(30+20j)}(5\angle 90°)$$

$$i_2 = \frac{(40-j10)}{(40-j10)+(30+j20)}(5\angle 90) = 1.1+2.7j \quad i_1 \Rightarrow -1.1+2.3j$$

$$V_{TH} = V_a - V_b \rightarrow -i_1 + \frac{V_a}{40} = 0 \rightarrow V_a = 40 i_1 \;;\; -i_2 + \frac{V_b}{j20} = 0$$

$$V_{TH} = V_a - V_b \rightarrow 10+70j \rightarrow P_{max} = \frac{(50\sqrt{2})^2}{8(20)} \rightarrow 31.25W \qquad V_b = 20j(i_2)$$

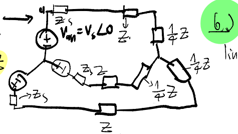
Q5. The circuit below shows a balanced source, a-b-c sequence three phase system. Line impedance ($\mathbf{Z_l}$) is **Z**, load impedance ($\mathbf{Z_L}$) Δ and Y is **Z**, respectively, and source impedance ($\mathbf{Z_S}$) is $\mathbf{Z_S}$. Suppose $\mathbf{V_{an}}$ is $V_S\angle\theta$. Both Δ and Y load impedances are present. (20 points)



a.) → Find equivalent Circuit in Y-Y form.

for load on the right → $Z_Y = \frac{Z_\Delta}{3} \rightarrow Z_t = \frac{Z}{3} // Z = \frac{\frac{Z}{3}\times Z}{\frac{Z}{3}+Z}$

$$Z_t = \frac{Z^2}{Z+3Z} \rightarrow \quad Z_t \Rightarrow \frac{1}{4}Z$$



b.) derive line currents and and the Voltage drop caused by the line Z_l and load impedence Z_L of three line.

$$I_a = \frac{V_s\angle 0}{Z_s + Z + \frac{1}{4}Z} \Rightarrow \frac{V_s\angle 0}{Z_s + \frac{5}{4}Z} \;;\; I_b = \frac{V_s\angle 0-120}{Z_s+\frac{5}{4}Z} \;;\; I_c = \frac{V_s\angle 0-240}{Z_s+\frac{5}{4}Z}$$

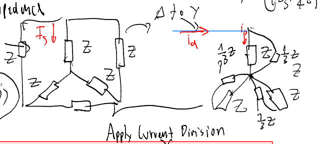
Find Voltage drop by Z_l and Z → as it Connect in Series We can do Voltage division V_{L-a}

$$\text{Hence} \rightarrow V_{L-a} = V_s\angle 0\left(\frac{\frac{1}{4}Z+Z}{Z_s+\frac{5}{4}Z}\right) \Rightarrow \frac{\frac{5}{4}Z V_s\angle\theta}{Z_s+\frac{5}{4}Z} \;;\; V_{L-b} \Rightarrow \frac{\frac{5}{4}Z V_s\angle\theta-120}{Z_s+\frac{5}{4}Z} \;;\; V_{L-c} \Rightarrow \frac{\frac{5}{4}Z V_s\angle\theta-240}{Z_4+\frac{5}{4}Z}$$

c.) Complex Power absorb by the load (Z_L) → $S = I_p^2 Z_p \rightarrow \frac{V_s^2}{(Z_s+\frac{5}{4}Z)^2}\left(\frac{1}{4}Z\right) \rightarrow S = 3I_p^2 Z_p \Rightarrow \frac{3}{4}Z\left(\frac{V_s^2}{(Z_s+\frac{5}{4}Z)^2}\right)$

load current — load impedance

d.) Please derive I_s → To find I_s → We know that → Original



Apply Current Division

$$\boxed{i_s = i_a\left(\frac{\frac{1}{3}Z}{\frac{1}{3}Z+Z}\right) \Rightarrow \frac{V_s\angle 0}{Z_s+\frac{5}{4}Z}\left(\frac{1}{4}\right)}$$

Chapter 11.

To find L → $L = \frac{V^2}{Q^2 W}$

Capacitor → Q = negativ leading

Inductor → Q = positive lagging

Mutual Induc Energy

Convert rad to degree

$= x \times \frac{180}{\pi}$ anynum

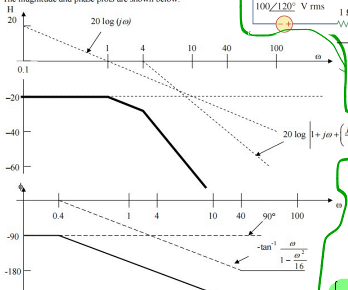
degree to rad ⇒ $\times \frac{\pi}{180}$

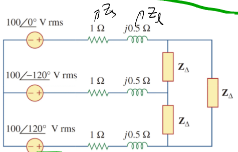
Chapter 14, Solution 16.

$$H(\omega) = \frac{\frac{1.0}{16}}{j\omega\left[1+j\omega+\left(\frac{j\omega}{4}\right)^2\right]} = \frac{0.1}{j\omega\left[1+j\omega+\left(\frac{j\omega}{4}\right)^2\right]}$$

$$H_{db} = 20\log_{10}|0.1| - 20\log_{10}|j\omega| - 20\log_{10}|1+j\omega+(j\omega/4)^2|$$

The magnitude and phase plots are shown below.





Exercise 3.5 (20%) For the three-phase circuit below, find the average power absorbed by the delta-connected load with

$$Z_\Delta = 21 + j24\,\Omega.$$

$$Z_Y = \frac{1}{3}Z_\Delta \Rightarrow 7+8j \;;\; I_a = \frac{V_{an}}{Z_Y} \Rightarrow \frac{100\angle 0 \text{ Vrms}}{1+0.5j+7+8j}$$

$I_a = 8.567\angle -46.736$ We know $S = 3I_p^2 Z_p \rightarrow S = 3(I_a)^2(7+8j)$

Hence → $S = 3(8.567)^2(7+8j)$

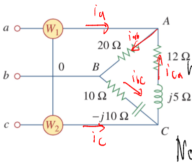
$S = 1541.263 + 1761j$

Average Power → 1541.263 W

Two wattmeters are properly connected to the unbalanced load supplied by a balanced source such that $V_{ab} = 208\angle 0°$ V with positive phase sequence.

1. Determine the reading of each wattmeter.
2. Calculate the total apparent power absorbed by the load.

unbalance → Need to use kcl or kvl to solve



$\rightarrow i_a = i_{AB} - i_{CA} \rightarrow P_1 = V_{AB} i_a \cos(\theta_v - \theta_{ia})$

$V_{ab} = 208\angle 0 \;;\; V_{bn} = 208\angle -120 \;;\; V_{cn} = 208\angle -240$

$I_{CA} \Rightarrow \frac{208\angle -240}{12+j5} = 16\angle 97.38° \;;\; I_{BC} \Rightarrow \frac{208\angle -120}{10-j10} \Rightarrow 14.707\angle -75°$

We know $V_{ab} = 208\angle 0° \;;\; i_{ab} = \frac{208}{20} \Rightarrow 10.4$

Now ⇒ $i_a = i_{ab} - i_{CA} = 10.4 - 16\angle 97.38 \Rightarrow 20.171\angle -51.87° \;;\; I_c = I_{CA} - I_{BC} \Rightarrow 30.64\angle 101.03°$

Now $P_1 = V_{ab} i_a \cos(\theta_v - \theta_i) \Rightarrow 208(20.171)\cos(0-(-51.87)) \Rightarrow 2.540\text{ KW}$; $P_2 = V_{cb} I_c \cos(\theta_v - \theta_i)$; $V_{cb} = -V_{bc}$ ⇒ $208\angle -60$; ± 180

$P_2 = 208(30.64)\cos(60-101.03) \Rightarrow 4.808\text{ KW}$

e.) → $P_T = P_1 + P_2 \Rightarrow 2.540\text{ kW} + 4.808$

$P_T \Rightarrow 7.348\text{ KW}$

Complex power $S \rightarrow P_T + jQ_T$

$Q_T = \sqrt{3}(P_2 - P_1) = \sqrt{3}(4.808\text{ kW} - 2.590\text{ kW}) \Rightarrow 3.84168\text{ kW}$

$\rightarrow S = 7.398\text{ kW} + j3.84168\text{ kW} \rightarrow \boxed{|S| = \sqrt{(7.398)^2 + (3.84168)^2} \rightarrow 8.336\text{ kW}}$

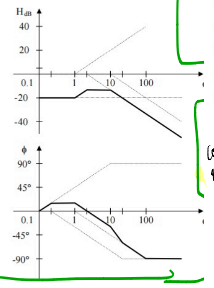
ter 14, Solution 15.

$$\mathbf{H}(\omega) = \frac{2(1+j\omega)}{(2+j\omega)(10+j\omega)} = \frac{0.1(1+j\omega)}{(1+j\omega/2)(1+j\omega/10)}$$

$$H_{dB} = 20\log_{10}0.1 + 20\log_{10}|1+j\omega| - 20\log_{10}|1+j\omega/2| - 20\log_{10}|1+j\omega/10|$$

$$\phi = \tan^{-1}\omega - \tan^{-1}\omega/2 - \tan^{-1}\omega/10$$

magnitude and phase plots are shown below.



2. Please analyze an **ideal transformer** circuit below. Assume all values in the question below are **RMS** values. [Total 16 points]

W = 100

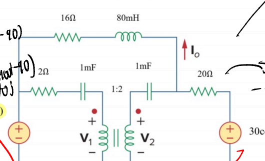
(1) Transform the circuit below to the phasor domain, e.g. L → jωL. [2 points]

(2) Please assign currents at the transformer terminals (any direction as you prefer). Determine polarities of turn ratio *n* of both voltage and current. [2 points]

(3) Please get values for V₁, V₂ and I₀ respectively in the circuit. [12 points]

$\cos(\theta) = \sin(\theta - 90)$

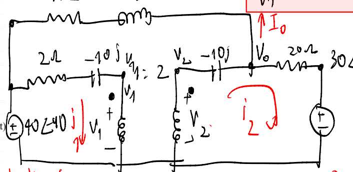
$40\sin(100t) \rightarrow 40\cos(100t - 90) = -40j$



→ −40j

80mH → 8j

1mF → −10j



look at the Cos and Sin and it need to be the Same.

for ideal transformer.

$$\boxed{\frac{V_2}{V_1} = n = 2 \;;\; \frac{I_1}{I_2} = 2}$$

$\uparrow I_0$

(3) KVL $I_1 \rightarrow (2-10j)I_1 + V_1 + 40j = 0$

$$I_1 = \frac{-V_1 - 40j}{2-10j} \rightarrow (1)$$

Eq for I_2

$$I_2 = \frac{V_2 - V_0}{-10j} \rightarrow (2)$$

Also

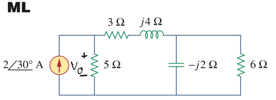
$$\frac{V_0 - V_2}{-10j} + \frac{V_0 - 30}{20} + \frac{V_0 + 40j}{16+8j} = 0 \rightarrow (3)$$

also We know → $I_0 = \frac{V_0 + 40j}{16+8j} \rightarrow (4)$ solve give

$V_1 = 7.56 - 15.80j \;;\; V_2 = 15.12 - 31.60j \;;\; I_0 \Rightarrow 1.54 + 0.44j$

1.56 Obtain the complex power delivered by the source in the circuit of Fig. 11.75.

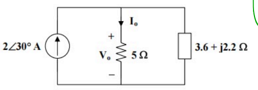
ML



$$-j2 \,\|\, 6 = \frac{(6)(-j2)}{6-j2} = \frac{12\angle -90°}{6.32456\angle -18.435°} = 1.897365\angle -71.565° = 0.6 - j1.8$$

$$3 + j4 + [(-j2)\,\|\,6] = 3.6 + j2.2$$

reduced to that shown below.



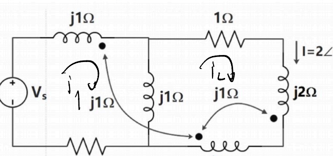
$$\mathbf{I}_o = \frac{3.6+j2.2}{8.6+j2.2}(2\angle 30°) = \frac{4.219\angle 31.4296°}{8.87694\angle 14.3493°}(2\angle 30°) = 0.95055\angle 47.08°$$

$$\mathbf{V}_o = 5\mathbf{I}_o = 4.75275\angle 47.08°$$

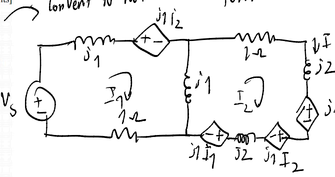
$$\mathbf{S} = \mathbf{V}_o\mathbf{I}_s^* = (4.75275\angle 47.08°)(2\angle -30°)$$

$$\mathbf{S} = 9.5055\angle 17.08° = \mathbf{(9.086{+}j2.792)\ VA}$$

Q3. For the magnetically coupled circuit below, please answer the following questions. [22 points]



Convert to Normal Phasor form.

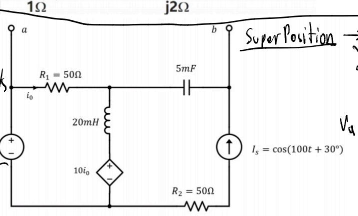


KVL eq Mesh $I_1 \rightarrow jI_1 + jI_2 + j1(I_1 - I_2) + 1I_1 - V_s = 0 \rightarrow (2j+1)I_1 = V_s \rightarrow (1)$

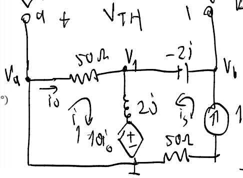
KVL eq Mesh I_2 $\;I_2 + 2jI_2 + jI_2 + jI_2 + 2jI_2 + jI_1 + j(I_2 - I_1) \rightarrow (7j+1)I_2 = 0 \rightarrow (2)$

but $I_2 = I_0 \rightarrow ?$

Find. V_{TH} and Z_{TH} for W of 100 rad/s and W = 200 rad/s



SuperPosition → for W = 100



$V_{TH} = V_a - V_b \rightarrow i_o = \frac{V_a - V_1}{50} \rightarrow (1)$

KCL at V_1 → $\frac{V_1 - V_a}{50} + \frac{V_1 - 10i_o}{2j} - 1\angle 30 = \frac{V_1 - V_a}{50} + \frac{V_1 - 10i_o}{2j} = \angle 30 \rightarrow (2)$

KCL at Vb → $\frac{V_b - V_1}{-2j} - 1\angle 30 = 0 \rightarrow V_b - V_1 = -2j\cdot\angle 30 \rightarrow (3)$

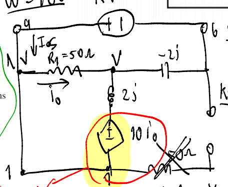
from $i_o = \frac{V_a - V_1}{50} \rightarrow V_a - V_1 = 50i_o$; also $V_1 - V_a - 25(V_1 j - 10i_o j) = 50\angle 30$; $V_b = V_1 + 2\angle -60$

solve give → $V_b = (2j+10)i_o$, $V_1 = (2j+10)i_o + 2jI_s$; $V_a \Rightarrow (2j+60)i_o + 2jI_s$

Now KVL at Mesh i_1 → $50i_1 + 2j(i_1 + 1\angle 30) + 10i_o = 0 \rightarrow (4) \rightarrow i_o = -\frac{j}{30+j}I_s \rightarrow (5)$

$V_{TH} = V_a - V_b \rightarrow 50i_o + 2jI_s$ (1∠30) $\rightarrow -0.216 + 0.263j = 0.340\angle 129.40$

Find Z_{TH} for W = 100



$= (10i_o + 1)$

$$Z_{TH} = \frac{V}{I_s} = \frac{1}{I_s}$$

KCL at 1 → $i_o = \frac{1-V}{50}$

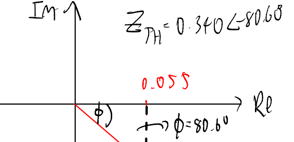
$50i_o = 1 - V \rightarrow (1)$

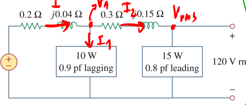
KCL at V →

$$\frac{V-1}{50} + \frac{V}{-2j} + \frac{V-(1+10i_o)}{2j} = 0 \rightarrow (2)$$

solve give $V = \frac{151-25j}{26}V_o \rightarrow I_o = \frac{V}{-2j} = \frac{151-25j}{-52j}V_o \rightarrow Z_{TH} = \frac{V_b}{I_o} \Rightarrow 0.055 - 0.335j \Rightarrow 0.340\angle -80.60$

Phasor for Z_{TH}





gure 11.81 Prob. 11.62.

$$\mathbf{S}_2 = 15 - j\frac{15}{0.8}\sin(\cos^{-1}(0.8)) = 15 - j11.25$$

$$\mathbf{S}_2 = \mathbf{V}_2\mathbf{I}_2^*$$

$$\mathbf{I}_2^* = \frac{\mathbf{S}_2}{\mathbf{V}_2} = \frac{15 - j11.25}{120}$$

$$\mathbf{I}_2 = 0.125 + j0.09375$$

$$\mathbf{V}_1 = \mathbf{V}_2 + \mathbf{I}_2(0.3 + j0.15)$$

$$\mathbf{V}_1 = 120 + (0.125 + j0.09375)(0.3 + j0.15)$$

$$\mathbf{V}_1 = 120.02 + j0.0469$$

$$\mathbf{S}_1 = 10 + j\frac{10}{0.9}\sin(\cos^{-1}(0.9)) = 10 + j4.843$$

$$\mathbf{S}_1 = \mathbf{V}_1\mathbf{I}_1^*$$

$$\mathbf{I}_1^* = \frac{\mathbf{S}_1}{\mathbf{V}_1} = \frac{11.111\angle 25.84°}{120.02\angle 0.02°}$$

$$\mathbf{I}_1 = 0.093\angle -25.82° = 0.0837 - j0.0405$$

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = 0.2087 + j0.053$$

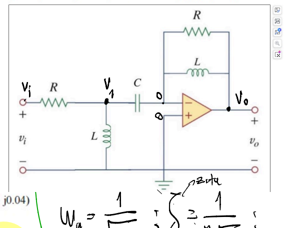
$$\mathbf{V}_s = \mathbf{V}_1 + \mathbf{I}(0.2 + j0.04)$$

$$\mathbf{V}_s = (120.02 + j0.0469) + (0.2087 + j0.053)(0.2 + j0.04)$$

$$\mathbf{V}_s = 120.06 + j0.0658$$

$$\mathbf{V}_s = \mathbf{120.06\angle 0.03°\ V}$$

5. Please draw a **Bode plot** of transfer function H(ω) = V₀/Vᵢ. RLC have the following values: R = 60 Ω, L = 12 H, and C = 3 F. Please use the **straight-line approximation**. Bode plot template and guide are provided in the next page. [18 points]



$$W_n = \frac{1}{\sqrt{LC}} \;;\; \zeta = \frac{1}{R\sqrt{LC}}$$

zeta

KCL at V_1 → $\frac{V_1 - V_i}{R} + \frac{V_1}{j\omega L} + \frac{V_1}{\frac{1}{j\omega C}} = 0 \rightarrow \frac{V_i - V_1}{R} = \frac{V_1}{j\omega L} + \frac{V_1}{\frac{1}{j\omega C}} \rightarrow V_i - V_1 = R\left(\frac{V_1}{j\omega L} + j\omega C V_1\right)$

$$V_i = \left[R\left(j\omega C + \frac{1}{j\omega L}\right) + 1\right]V_1$$

KCL at 0. → $\frac{0 - V_1}{\frac{1}{j\omega C}} + \frac{0 - V_o}{R//j\omega L} \Rightarrow \frac{V_1}{\frac{1}{j\omega C}} = \frac{-V_o}{R//j\omega L} \rightarrow V_o = -(R//j\omega L)\cdot j\omega C V_1$

$$H(\omega) = \frac{V_o}{V_i} \rightarrow \frac{-\left(\frac{Rj\omega L}{R+j\omega L}\right)\cdot j\omega C}{R\left(j\omega C + \frac{1}{j\omega L}\right)+1} \Rightarrow \frac{-RLC(j\omega)^2}{(R+j\omega L)\left(Rj\omega C + \frac{1}{j\omega L}+1\right)} \xrightarrow{\text{Multiply by}} \frac{-RLC(j\omega)^2}{\left(1+\frac{j\omega L}{R}\right)\left(Rj\omega C + \frac{1}{j\omega L}+1\right)}$$

$$\frac{-LC(j\omega)^3}{\left(1+\frac{j\omega}{\frac{R}{L}}\right)\left((j\omega)^2 RC + j\omega + \frac{1}{L}\right)} \rightarrow \frac{R}{L}\left(LC(j\omega)^2 + \frac{j\omega L}{R} + 1\right) \rightarrow \boxed{H(\omega) = \frac{-\frac{L^2C}{R}\cdot(j\omega)^3}{\left(1+\frac{j\omega}{\frac{R}{L}}\right)\left[1+2\zeta\left(\frac{j\omega}{\frac{1}{\sqrt{LC}}}\right)+\left(\frac{j\omega}{\frac{1}{\sqrt{LC}}}\right)^2\right]}}$$